DDS-100-S

DESIGN DATA SHEET
DEPARTMENT OF THE NAVY
NAVAL SHIP ENGINEERING CENTER

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DDS 9110-9

STRENGTH OF GLASS REINFORCED PLASTIC STRUCTURAL MEMBERS
PART I—SINGLE SKIN CONSTRUCTION

Contents

Part I

9110-9-a. References

(1) Mil. Spec. MIL-P-17549; Plastic Laminates, Fibrous Glass Reinforced Marine Structural

(2) Military Handbook MIL-HDBK 17; Plastics for Flight Vehicles, 1959 Edition

(3) Design Data Sheet DDS9110-4; Strength of Structural Members

(4) Buckling Strength of Metal Structures by F. Bleich, McGraw Hill Book Company

(5) Flat Plates of Plywood Under Uniform or Concentrated Loads—Forest Products Lab Reports 1300 and 1312

(6) Marine Design Manual—Gibbs and Cox Company, 1960, McGraw-Hill Book Company

9110-9-b. Purpose and scope

Part I of this Design Data Sheet is issued to provide guidance and uniform standards for design calculations of single skin Glass Reinforced Plastic (GRP) Ship Structures. Part II includes design data for sandwich panels (see definition of single skin in 9110-9-c). The formulas given are applicable to any grade of GRP laminate and in general are applicable to any orthotropic material. The formulas for flat panels assume that the natural axes of the laminate are parallel to the edges (see Figure 6). The references are given for additional information and provide the historical background for orthotropic plate theory and testing.

The formulas given in Appendix A are presented in the form of design curves for a commonly used grade of GRP laminate. In as much as GRP properties tend to vary with degrees of quality control and lay-up experience, these design curves must be checked against current and required properties in the Ship Specification for a specific design. Modification to these curves, or additional curves or both may be required.

In Appendix B some design examples are presented.

9110-9-c. Definitions

Glass Reinforced Plastic (GRP)—A structural material composed of woven glass fibers and a thermosetting resin binder. For ship application the glass is generally glass cloth, woven roving, and random mat combinations. The most commonly used resins are epoxy and polyester.

Glass cloth—A woven material made of glass yarns. Glass yarns are glass strands given a twist or as a combination of two or more of such twisted yarns plied together. Glass strands are a multiplicity of glass filaments drawn together and gathered into an approximately parallel arrangement without twist. A glass filament is a single thread of glass of very small diameter.

Woven roving—A woven material made of ropes of glass rovings. Rovings are groups of continuous strands wound into a cylindrical package. The number of strands can be varied.

Glass mat—Composed of numerous fibers, strands, or yarns of uniform length, unwoven but randomly distributed and resin bonded so as to form a uniformly thick and highly porous sheeting of glass fibers.

Laminate—A finished thickness of GRP material composed of a number of layers of woven glass and resin.

Ply—Each individual layer of glass in a laminate. Whenever thickness-per-ply is discussed, it is the thickness of the glass and resin. The glass thickness will remain constant or within a small tolerance for a given glass material. However, thickness associated with a given ply may vary since it is dependent on the fabrication. The strength of GRP varies as glass content varies in a given laminate.

Single skin construction—Term used to describe a relatively thin laminate composed entirely of GRP as distinguished from a sandwich panel which is composed of two skins or facings separated by a light weight core.

Orthotropic material—Material with different machanical properties in direction of three mutually perpendicular axes. These three axes are used for stress analysis and are called the natural axes. Two of the axes are in the laminate plane. The third axis is perpendicular to the laminate plane. Examples of orthotropic materials are GRP and plywood.

Isotropic material—Material with mechanical properties equal in all directions.

Bidirectional laminate—Orthotropic GRP material where the mechanical properties along the two natural axes in the laminate plane are about equal produced by placing equal amounts of glass fiber in both directions.

Unidirectional laminate—Orthotropic material with mechanical properties along one natural axis in the laminate plane much higher than the other axis.

9110-9-d. Symbols for formulas

SYMBOLS

A	cross sectional area of a laminate or structural member
a	length of one edge of a rectangular flat panel; subscript denoting direction parallel to the "a" edge of a rectangular flat panel.
a_e	effective width of a panel acting with a stiffener.
b	length of one edge of a rectangular flat panel.
B_1, B_2, B_3, etc.	coefficients for panel formulas.

SYMBOLS

C_c, C_f, C_1, C_2, etc.	coefficients for panel formulas.
cr	subscript denoting a critical condition of elastic buckling.
c	subscript denoting compression.
D	bending stiffness factor for flat panels.
E	Young's modulus of elasticity; with subscripts denotes modulus in tension, compression, etc.
F	ultimate strength of a GRP laminate or member; with the subscript cr, critical compressive buckling stress; with the subscript scr, the shear buckling stress.
F.S.	factor of safety on the ultimate strength.
f	induced stress; subscript denoting bending or flexural strength.
G	Modulus of rigidity (shear modulus)
H_c, H_s, h_c	coefficients for panel formulas.
I	moment of inertia of the cross section of a laminate or member.
K_f, K_m, K_1, K_2, etc.	coefficients for formulas.
L	unsupported length of a column or beam.
M	bending moment.
m	coefficient for panel formulas.
n	number of half-waves of a buckled column or rectangular flat panel; number of component laminates in a composite cross section.
N	shear stiffness factor.
p	unit load.
r	radius of gyration of a column section; stiffness factor for rectangular flat panels.
s	subscript denoting shear.
t	subscript denoting tension; thickness of a laminate.
U	shear stiffness factor

SYMBOLS

V	shearing force.
Z	section modulus of the cross section of a laminate or member in bending.
z	direction normal to plane of a flat panel.
λ_{fba}	$1-\mu_{fba}\mu_{fab}$
μ	Poisson's ratio; with two subscripts denoting direction, Poisson's ratio for strain when stress is in the direction of the first subscript.
δ, Δ	deflection of laminate or panel.

9110-9-e. Factors of safety

The formulas herein and design curves of Appendix A relate to the ultimate strength of the GRP materials. GRP has no discernible yield strength.

Factors of safety on the ultimate strength must be included in accordance with applicable Ship Specifications. Where not so specified, the following factors of safety are to be used.

Conditions	Factor of safety on ultimate strength
Stiffeners and stanchions	4.0
Flat panels	
(a) Panels loaded on the long edge from tension, compression, and shear, or combinations thereof, where failure would be catastropic.	4.0
(b) Compressive and/or shear stresses in panels loaded on short edges where buckling would not lead to catastrophic failure by transmittal of load to boundary stiffeners.	2.0

The large factor of safety (4.0) is primarily to account for fatigue, creep, and aging of GRP materials. In addition to stress levels, structures should be limited to a maximum deflection equal to L/200 (check applicable ship specifications). This deflection requirement is extremely important for GRP members because of the low modulus of elasticity.

9110-9-f. Columns

(1) Columns of uniform cross section

The critical buckling strength of a GRP column is given by the following formula. The coefficient, K_4, is given for varied end conditions. For pinned columns K_4 is unity and this value may be assumed for usual end conditions. If ends are held very rigidly, K_4 may be reduced, provided that allowance is made for all bending stresses, including any secondary bending. K_4 equal to unity should be used for stanchions. A plot of this formula for a specific grade of GRP laminate is given in Appendix A and a design example in Appendix B.

Critical compressive buckling stress,

$$F_{ccr} = \frac{\pi^2 E_f}{\left(K_4 \frac{L}{r}\right)^2}$$

where

r = radius of gyration

E_f is the flexural modulus of elasticity parallel to the column direction.

For pinned ends, $K_4 = 1.0$

For fixed ends, $K_4 = 0.5$

For fixed-pinned condition, $K_4 = 0.75$

For fixed-free condition, $K_4 = 2.0$

The limiting value of F_{ccr} is the compressive stress F_c (short column).

(2) Columns of non-uniform cross section

The formulas for uniform cross section as modified by the methods of reference (3) apply.

9110-9-g. Buckling strength of flat panels

The critical buckling strengths and finite panel coefficients for flat panels under uniform edge loadings are given below. For steel structures it has been normal practice to use simply supported edges since a standard steel Tee stiffener offers little clamping resistance to the plating. When buckling occurs, adjacent panels can buckle in opposite directions, rotating at the stiffener connection. Although many GRP structures are stiffened with a wide or "hi-hat" type stiffener (see Figure 6) and the edges are more "clamped" than a steel structure, a conservative approach is recommended by using simply supported edges. To accurately assess the edge restraint and modify the formulas accordingly for most stiffeners is unnecessarily tedious and time consuming. In the event, however, that other edge conditions are definitely known, the formulas for other edge conditions apply. These formulas are developed from reference (2).

(1) Critical Compressive Buckling Stress,

$$F_{ccr} = H_c \frac{\sqrt{E_{fa} E_{fb}}}{\lambda_{fba}} \left(\frac{t}{b}\right)^2$$

where

b = loaded edge of panel.
$H_c = h_c + C_c K_f$

The coefficient h_c is given in Figure 1. C_c is calculated as follows:

For edges simply supported or loaded edges clamped.

$$C_c = \frac{\pi^2}{6}$$

For loaded edges simply supported, other edges clamped or all edges clamped.

$$C_c = \frac{2\pi^2}{9}$$

Use of Figure 1 requires computation of edge stiffeners factor, r, as follows:

$$r = \frac{a}{b} \left(\frac{E_{fb}}{E_{fa}}\right)^{\frac{1}{4}}$$

Combined Flexural and torsional property:

$$K_f = \frac{E_{fb}\mu_{fab} + 2\lambda_{fba} G_{ba}}{\sqrt{E_{fa} E_{fb}}}$$

The column formula in 9110-9-f, applies for all panels having r less than 0.5. A plot of the formulas above for a specific grade of GRP is given in Appendix A and a design example in Appendix B.

(2) Critical shear buckling stress,

$$F_{scr} = \frac{H_s \left(E_f^3 E_{fa}\right)^{\frac{1}{4}}}{3\lambda_{fba}} \left(\frac{t}{b}\right)^2$$

The coefficient H_s is given by Figure 2. The stiffness r required for the curve is computed as for edge compression given above.

A plot of the formula for a specific grade of GRP is given in Appendix A and a design example in Appendix B.

9110-9-h. Rectangular flat panels under uniform loading

Formulas for rectangular flat panels subject to uniform loads, such as wind, sea slap, and hydrostatic pressures are given below. These panels develop combined bending and tensile stresses (large deflection theory) which act in the plane of the panel. Where deflections are small, the tensile stress will be negligible (small deflection theory). For panels simply supported on all edges, the maximum stresses occur at the center of the panel. For panels clamped on all edges, the maximum stresses occur at the edge. The maximum deflection occurs at the center of the panel for both clamped and simply supported edges. These formulas are developed from reference (5) and (6). Panels under uniform loads must satisfy the following formula:

$$\frac{f_{fb}}{F_{fb}} + \frac{f_{tb}}{F_{tb}} \le \frac{1}{F.S.}$$

where:

Edges simply supported:

$$f_{fa} = K_8 \left[C_f \frac{E_{fba}}{\lambda_{fba}} \left(\frac{t}{b}\right)^2 \left(\frac{\delta}{t}\right) \right]$$

$$f_{ta} = K_8^2 \, 2.572 \left[\frac{E_{tb}}{\lambda_{fba}} \left(\frac{t}{b}\right)^2 \left(\frac{\delta}{t}\right)^2 \right]$$

Edges clamped:

$$f_{fa} = K_8 \left[C_f \frac{E_{fb}}{\lambda_{fba}} \left(\frac{t}{b}\right)^2 \left(\frac{\delta}{t}\right) \right]$$

$$f_{ta} = K_8^2 \left[2.488 \frac{E_{tb}}{\lambda_{fba}} \left(\frac{t}{b}\right)^2 \left(\frac{\delta}{t}\right)^2 \right]$$

K_8 is a coefficient for finite panels where a corresponding δ for infinite panels does not exceed 0.5t. For deflections greater than 0.5t a value for K_8 does not exist. Therefore finite panels should be designed as infinite panels (K_8 = 1.0). To find K_8 use Figure 3.

The factor r required for the curve is:

$$r = \frac{a}{b} \left(\frac{E_{fb}}{E_{fa}}\right)^{\frac{1}{4}}$$

b = short edge of the panel

As can be seen by Figure 3 the panel is an infinite panel (K_8 = 1.0) when r is about 2.0.

C_f in the formulas above is given by Figure 4. The value m required for the figure is found by:

Edges simply supported:

$$m = 2.778 \left(\frac{E_{tb}}{E_{fb}}\right)^{\frac{1}{2}} \left(\frac{\delta}{t}\right)$$

Edges clamped:

$$m = 2.732 \left(\frac{E_{tb}}{E_{fb}}\right)^{\frac{1}{2}} \left(\frac{\delta}{t}\right)$$

$\frac{\delta}{t}$ in the above formulas is the ratio of the maximum deflection to the thickness of the panel, and is found with the aid of Figure 5. For the figure, $\frac{\Delta}{t}$ (the ratio of maximum deflection to the thickness of the panel assuming loads resisted by bending only) is calculated as follows:

Edges simply supported:

$$\frac{\Delta}{t} = \frac{5\lambda_{fba} p b^4}{32 E_{fb} t^4}$$

Edges clamped:

$$\frac{\Delta}{t} = \frac{\lambda_{fba} p b^4}{32 E_{fb} t^4}$$

Figure 5 also requires a factor C which is:

$$C = \frac{E_{tb}}{E_{fb}}$$

The deflection of the panel, δ, as computed for use with stress formulas, is the maximum deflection of an infinite panel and is used when deflection limit is a specification requirement. For finite panels multiply the infinite panel δ by the factor K_8.

Because of the membrane stress action, very low stresses may result for panels indicating a need for only small panel thicknesses. The designer is cautioned not to overlook the requirements for limiting deflection which may govern the design of many panels.

A plot of the above formulas for a specific GRP grade is given in Appendix A and a design example in Appendix B.

9110-9-i. Members in bending

The bending theory, as developed in mechanics of materials, is applicable to bending members made up of GRP laminates. It is emphasized, however, that the computation for section modulus must utilize theory developed for composite beams since the tensile and compressive strength of GRP laminates are usually different.

The modulus of rupture for members in bending will be either edgewise tension, edgewise compression, or the flatwise flexural strength of the laminate depending upon the nature of the cross section geometry (see Figure 6). When the bending member is the laminate itself (a panel of GRP), the modulus of rupture may be taken as the flatwise flexural strength of the laminate, F_f. When the bending member is a panel-stiffener combination or a deep member such as a plate girder, the modulus of rupture may be taken as the edgewise tensile strength, F_t, of the laminate on the tensile side and the edgewise compressive strength, F_c, of the laminate on the compressive side. Usually the compressive strength is low and will govern the design. When calculating section modulus of a panel-stiffener combination the following formula for effective width of panel is recommended:

$$a_e = 2a' + a''$$

where:

a_e = effective width

a' - 15t

a'' = width of stiffener

See Figure 6.

Stiffener spacing, if less than the above calculated effective width, should be used when adjacent stiffeners are similarly loaded.

Research to determine effective widths is incomplete and may be modified as future tests are conducted. Consult applicable Ship Specifications for current requirements.

9110-9-j. Combined stresses

The following formulas are recommended and may be used unless otherwise altered by Ship Specifications.

(1) Panels loaded on the short side such as a hull panel in a longitudinally framed ship.

(a) edgewise compression

$$f_c + f_{fc} \leq \frac{F_{ccr}}{F.S.}$$

where:

f_c is the applied edge compressive stress from primary loads such as longitudinal hull bending.

f_{fc} is the compressive stress in the panel caused by bending of a panel-stiffener combination under local loads.

The Factor of Safety is 2.0 (see 9110-9-e).

(b) edgewise tension

$$f_t + f_{ft} \leq \frac{F_t}{F.S.}$$

where:

f_t and f_{ft} are as defined above except loads are in tension.

The F.S. for this formula = 4.0. (See 9110-9-e.)

(c) combined edgewise shear and compression

$$\left(\frac{f_s}{F_{scr}}\right)^2 \frac{f_c}{F_{ccr}} \leq \frac{1}{F.S.}$$

where:

f_s are applied shear loads in panel.

f_c are applied compressive loads.

The F.S. is 2.0. (See 9110-9-e.)

An example for use of this formula is given in Appendix B.

(2) Panels loaded on the long side such as a hull panel of a transversely framed ship.

(a) edgewise tension

$$\frac{f_t}{F_t} + \frac{f_f}{F_f} \leq \frac{1}{F.S.}$$

where:

f_t is the applied edge tensile stress from primary loads such as longitudinal hull bending or other axial loads.

f_f is a tertiary flexural stress, i.e. flatwise bending of the panel from local loads. Note that since this is a relatively thin panel bending, the flexural strength, F_f is used which is unlike the tensile or compressive stress used where bending is applied to a deep stiffener. The factor of safety, F.S. is 4.0 for this formula. (See 9110-9-e.)

(b) edgewise compression

$$\frac{f_c}{F_{ccr}} \leq \frac{1}{F.S.}$$

The F.S. is 4.0. This high factor of safety is desirable since for plates loaded on their long edge, especially for transversely framed hulls, buckling results in catastrophic failure. For loading on short edges such as the longitudinally framed ship, the stiffeners will still carry load following buckling of the panel.

Unlike panels loaded on the short edge, the formula above does not combine compression caused by local bending since normal local loads tend to increase the buckling strength. See Reference (4). Without consideration of buckling strength, however, the combined stresses should not exceed the ultimate strength. For this condition the following formula may be used:

$$\frac{f_c}{F_c} + \frac{f_t}{F_f} \leq 1$$

(3) Combined bending and column stresses

(a) Stiffeners in tension

$$f_{ft} + f_t \leq \frac{F_t}{F.S.}$$

where:

f_t and f_{ft} are direct tension and tension from bending stresses applied directly to the stiffener or column. The F.S. is 4.0 (see 9110-9-e).

(b) Stiffeners in compression

$$\frac{f_{fc}}{F_c} + \frac{f_c}{F_{ccr}} \leq \frac{1}{F.S.}$$

where:

f_{fc} is the applied bending stress in the stiffener or column.

f_c is the applied axial load.

F_{ccr} is the critical column stress.

The factor of safety is 4.0. (See 9110-9-e.)

(c) compression of stanchions

Since bending loads are not usually applied to stanchions the following formula applies:

$$\frac{f_c}{F_{ccr}} \leq \frac{1}{F.S.} \text{ or } \frac{f_c}{F_c} \leq \frac{1}{F.S.}$$

where:

f_c and F_{ccr} are as defined for compression of stiffeners. The F.S. is 4.0. (See 9110-9-e.)

9110-9-k. Composite laminates

For this discussion, a composite laminate is defined as a laminate composed of several different types of reinforcement which have variable mechanical properties, such as a laminate composed of woven roving, glass cloth, and random mat. Although not often used for major structural elements, these composites have been used for small boats and ventilation ducts. A single layer of mat is often used with a GRP laminate to provide increased bond strength. This ply of mat is generally ignored, however, in determining properties of the laminate. If the mechanical properties of each type of reinforcement plus resin are known, the properties of the composite can be calculated as follows:

(a) For axial loading

Modulus of Elasticity:

$$E = \frac{1}{A} \sum_{i=1}^{i=n} E_i A_i$$

Modulus of Rigidity:

$$G = \frac{1}{A} \sum_{i=1}^{i=n} G_i A_i$$

Stress:

$$F = \frac{1}{A} \sum_{i=1}^{i=n} F_i A_i$$

(b) For bending

In the above formulas, substitute I for A.

where:

I, E, G, and F are properties of the total composite

A = total cross sectional area of the laminate

E_i, G_i, and F_i are properties of each type of reinforcement

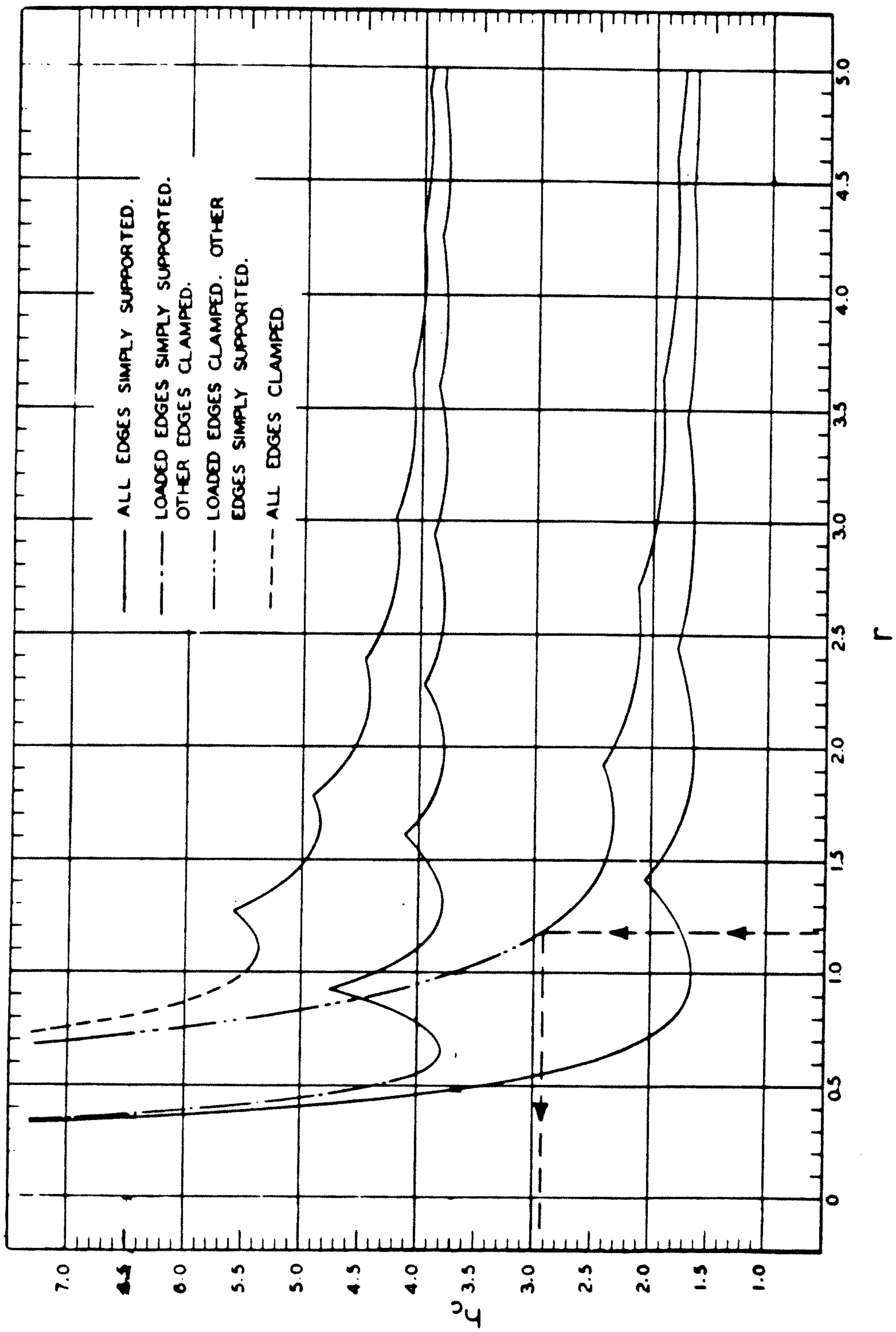
A_i is the cross sectional area of each type of reinforcement

Σ is the summation of the reinforcements

I_i is the moment of inertia of each type of reinforcement about the neutral axis of the entire laminate.

An example for one of these formulas is given in Appendix B.

9110-9-1. Figures for use with design formulas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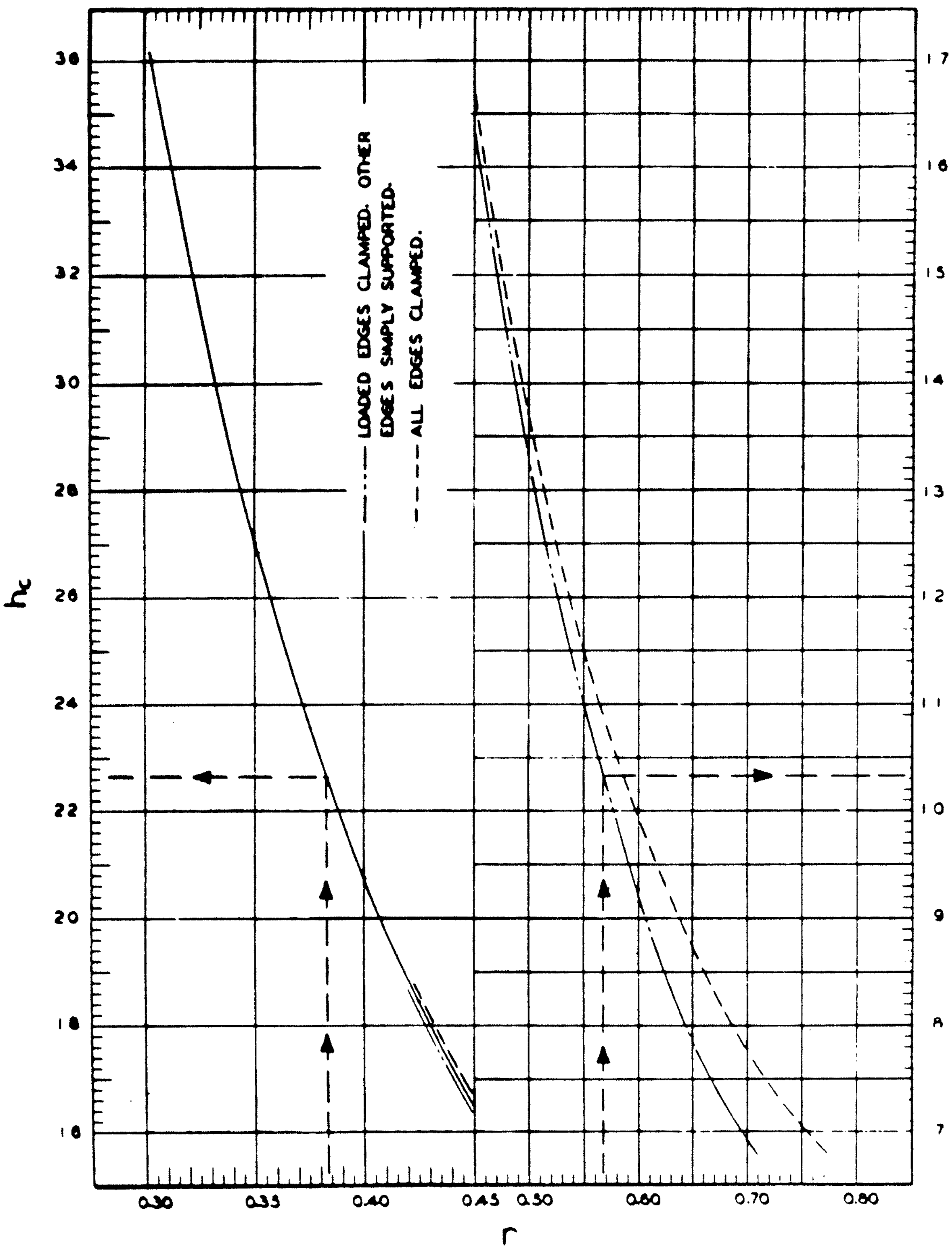
LOADED EDGES CLAMPED. OTHER EDGES SIMPLY SUPPORTED.
ALL EDGES CLAMPED.
r
k_c

FIGURE 1a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ALL EDGES CLAMPED.
h_c
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0.40
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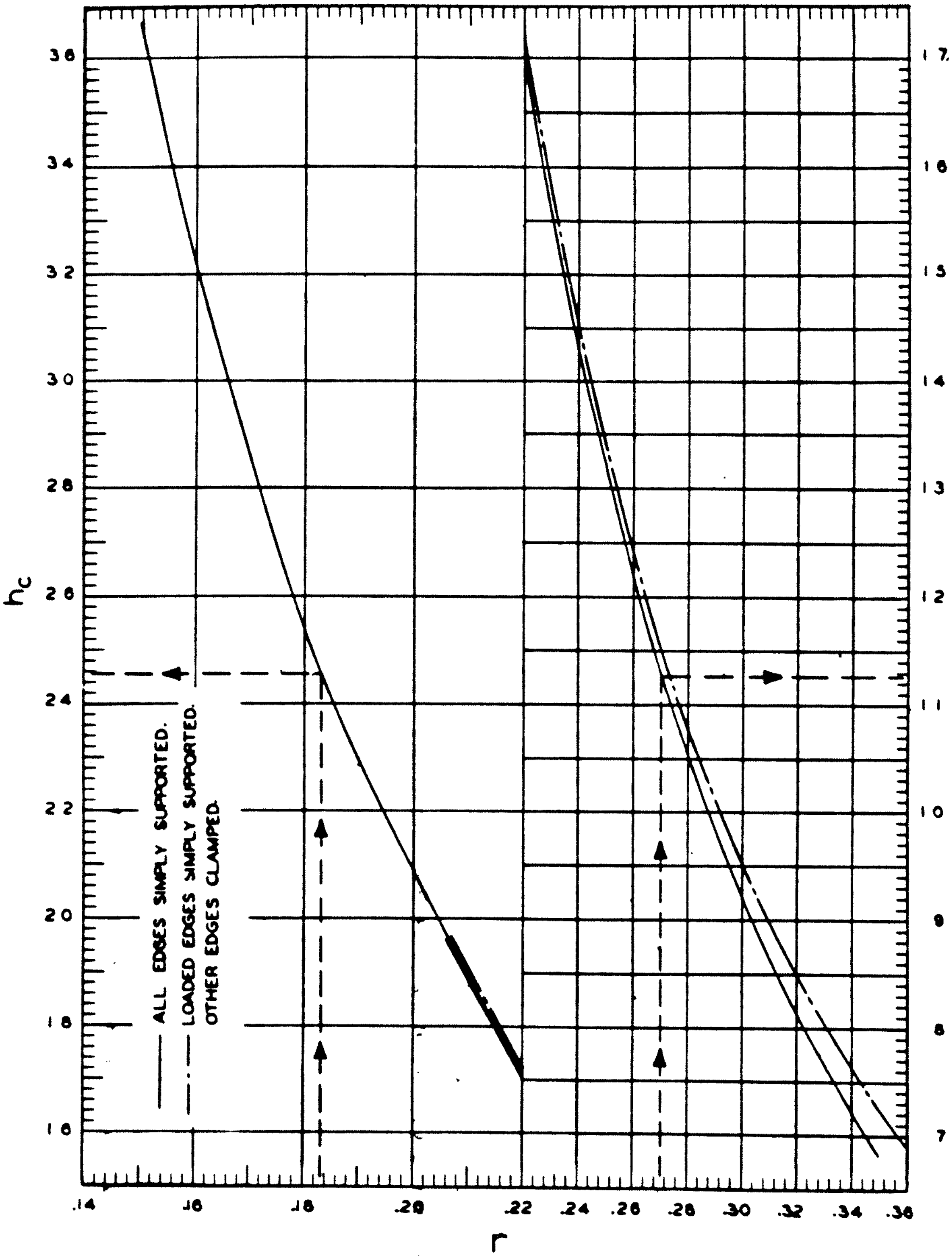
0.70
0.80

FIGURE 1b



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h_c
ALL EDGES SIMPLY SUPPORTED.
LOADED EDGES SIMPLY SUPPORTED.
OTHER EDGES CLAMPED.
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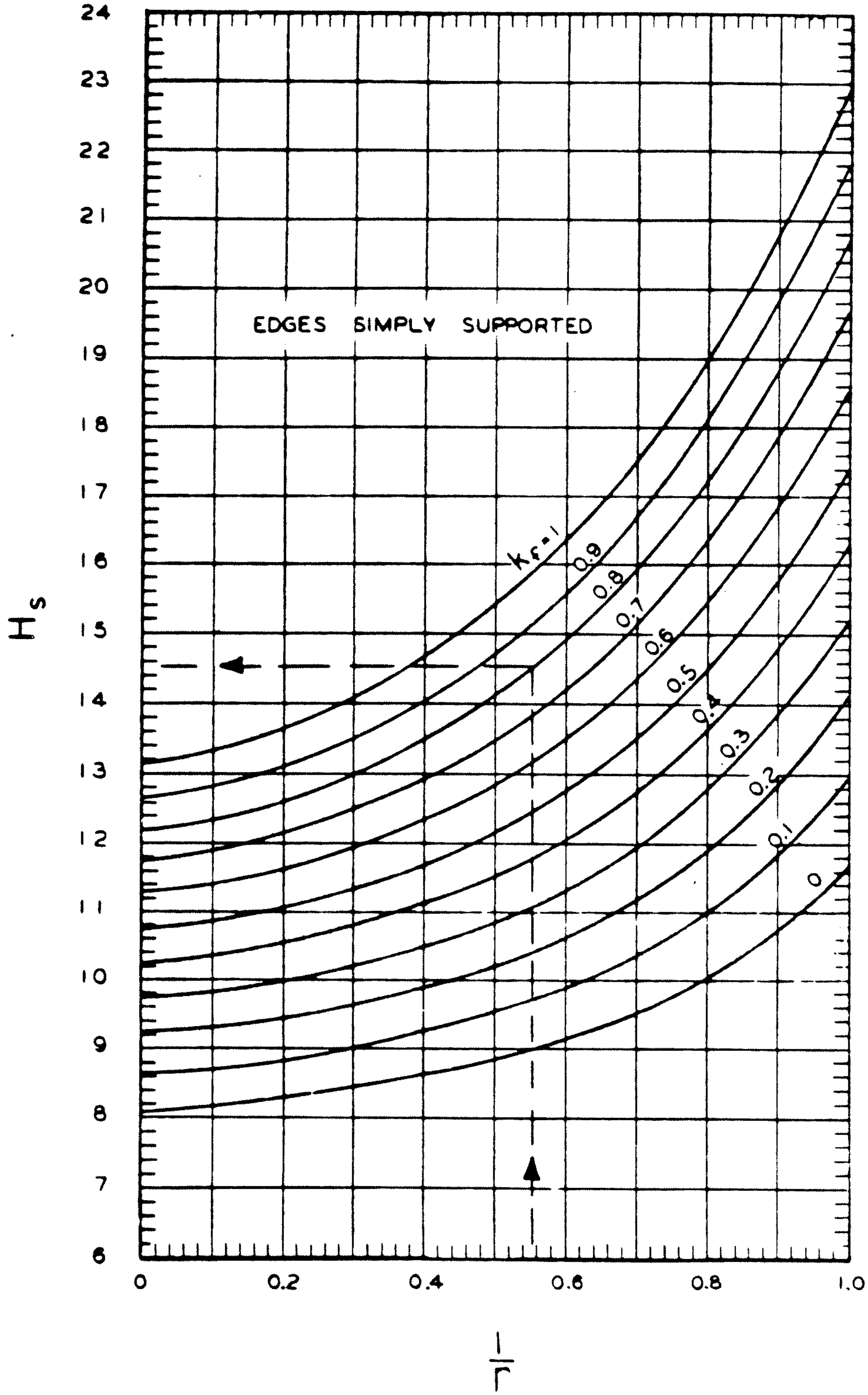
r

FIGURE 1c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0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
H_s
24
23
22
21
20
19
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17
16
15
14
13
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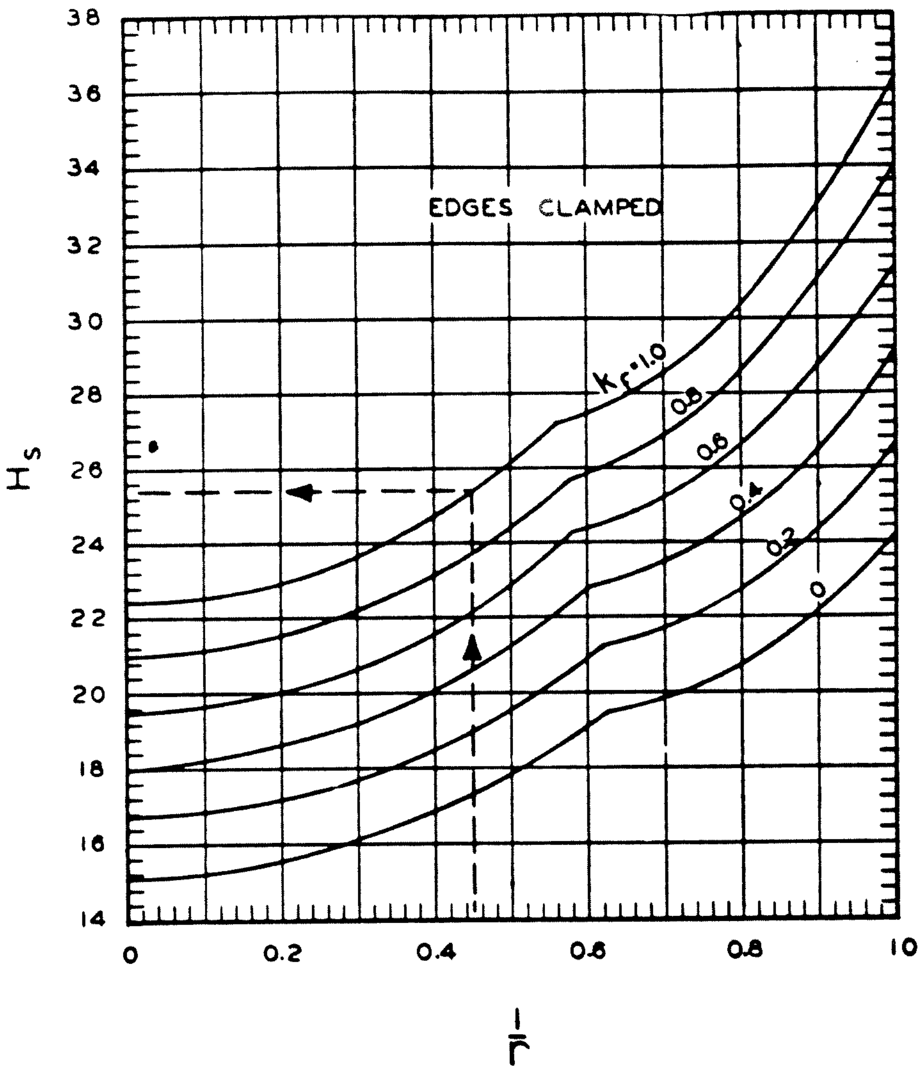
0.2
0.4
0.6
0.8
1.0
$\frac{l}{r}$

FIGURE 2a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K_f=1.0
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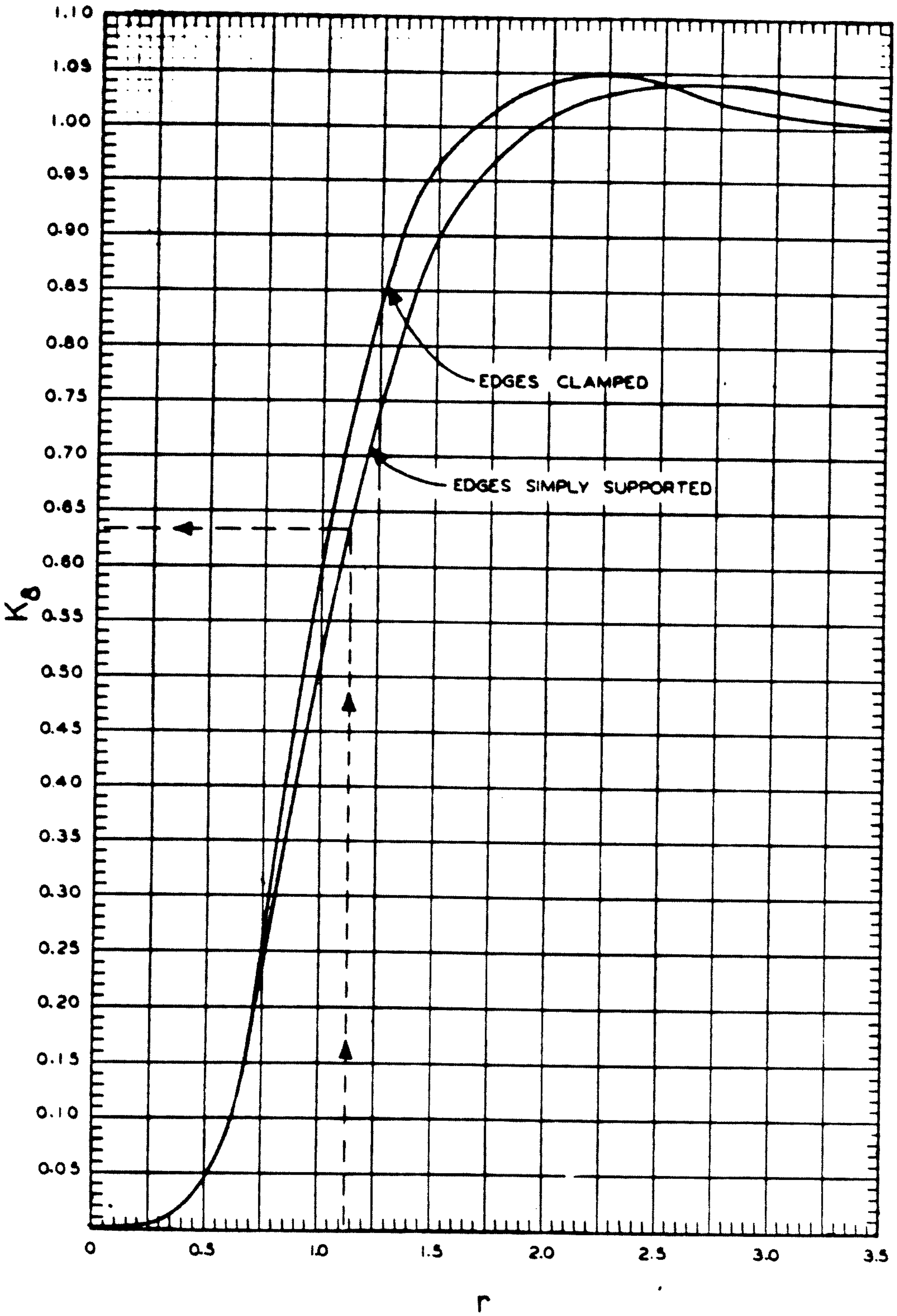
$\frac{l}{r}$

FIGURE 2b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K_δ
EDGES CLAMPED
EDGES SIMPLY SUPPORTED
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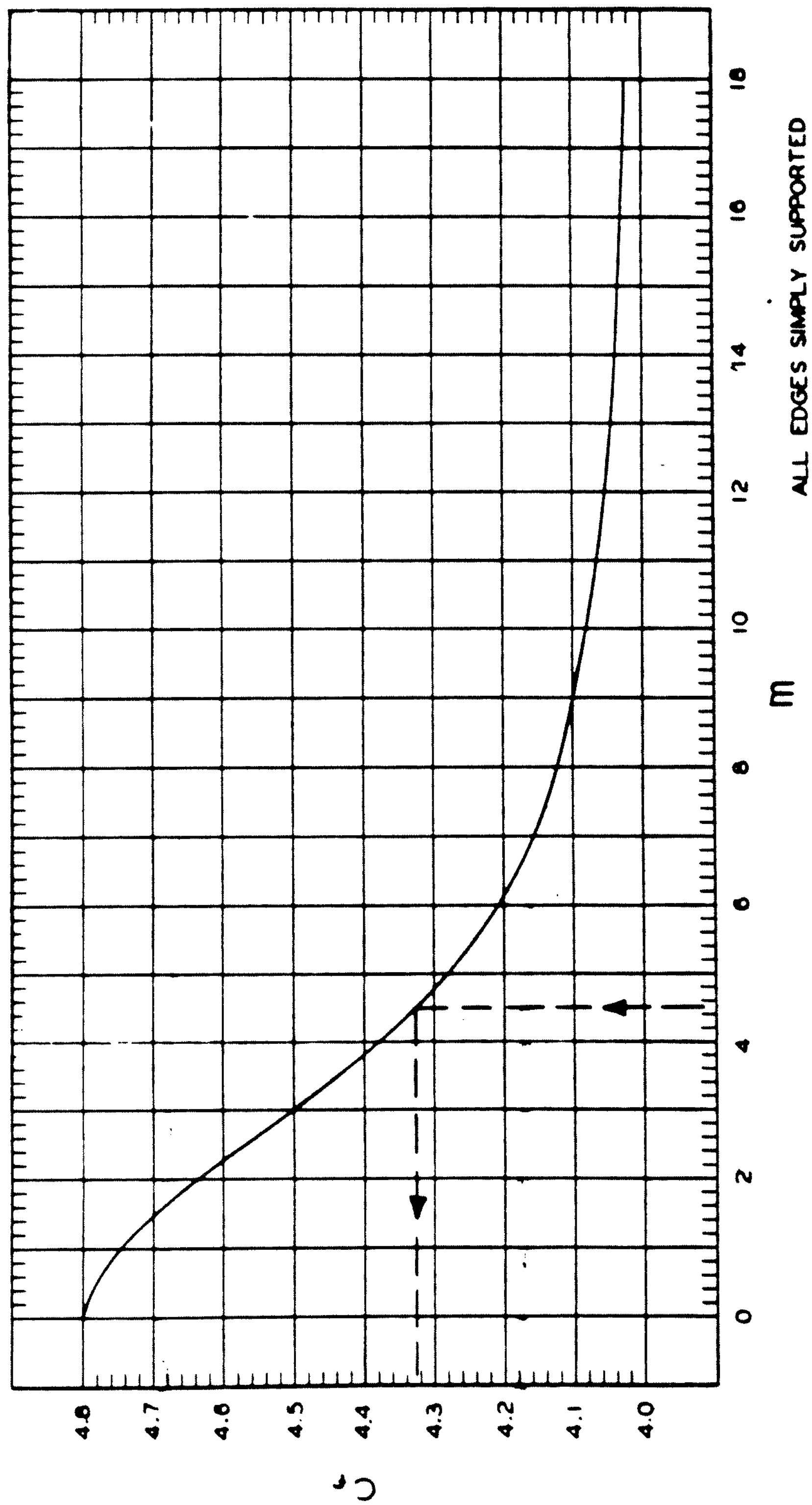
r

FIGURE 3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C_f
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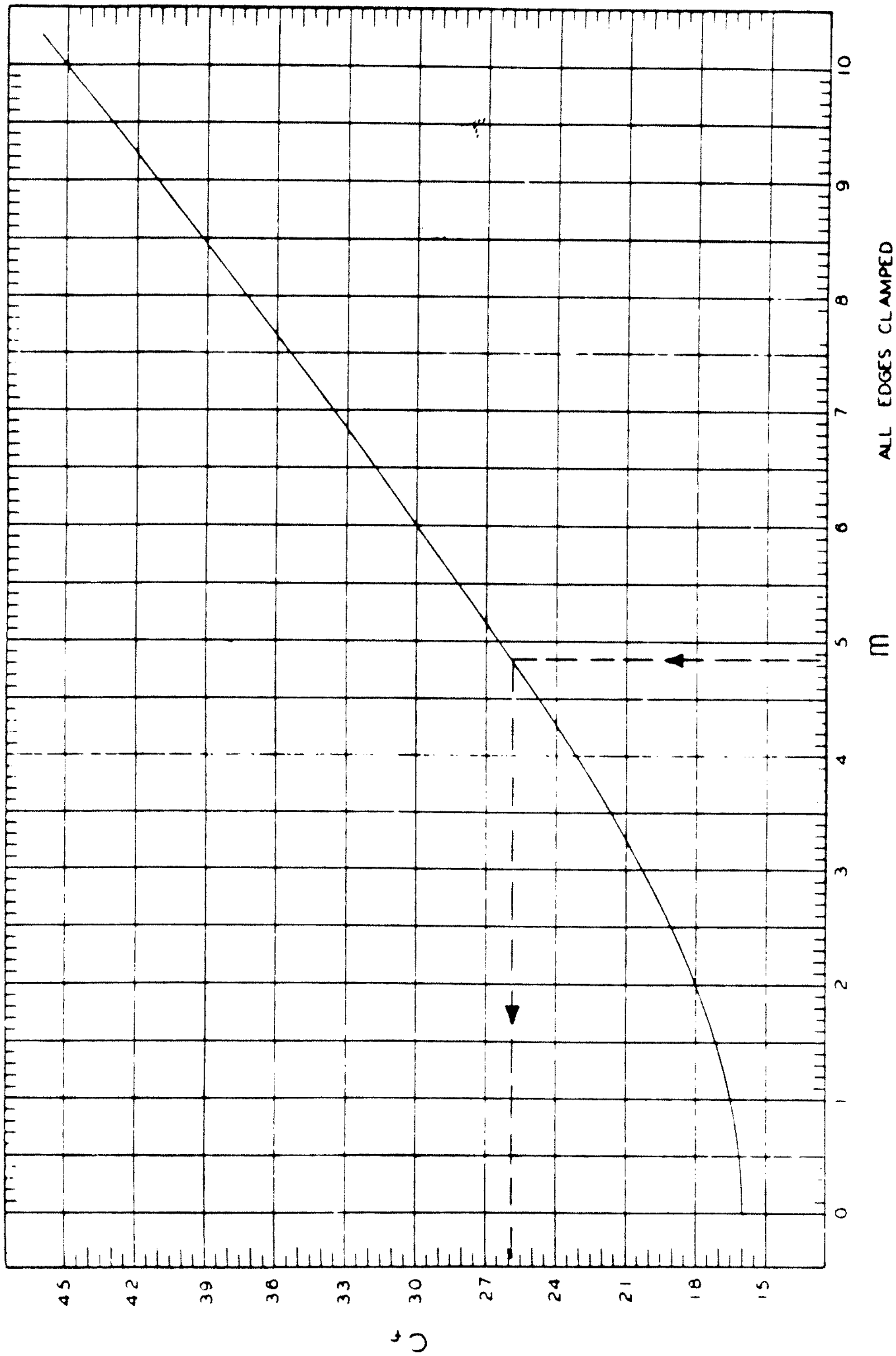
m
ALL EDGES SIMPLY SUPPORTED

FIGURE 4a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C_f
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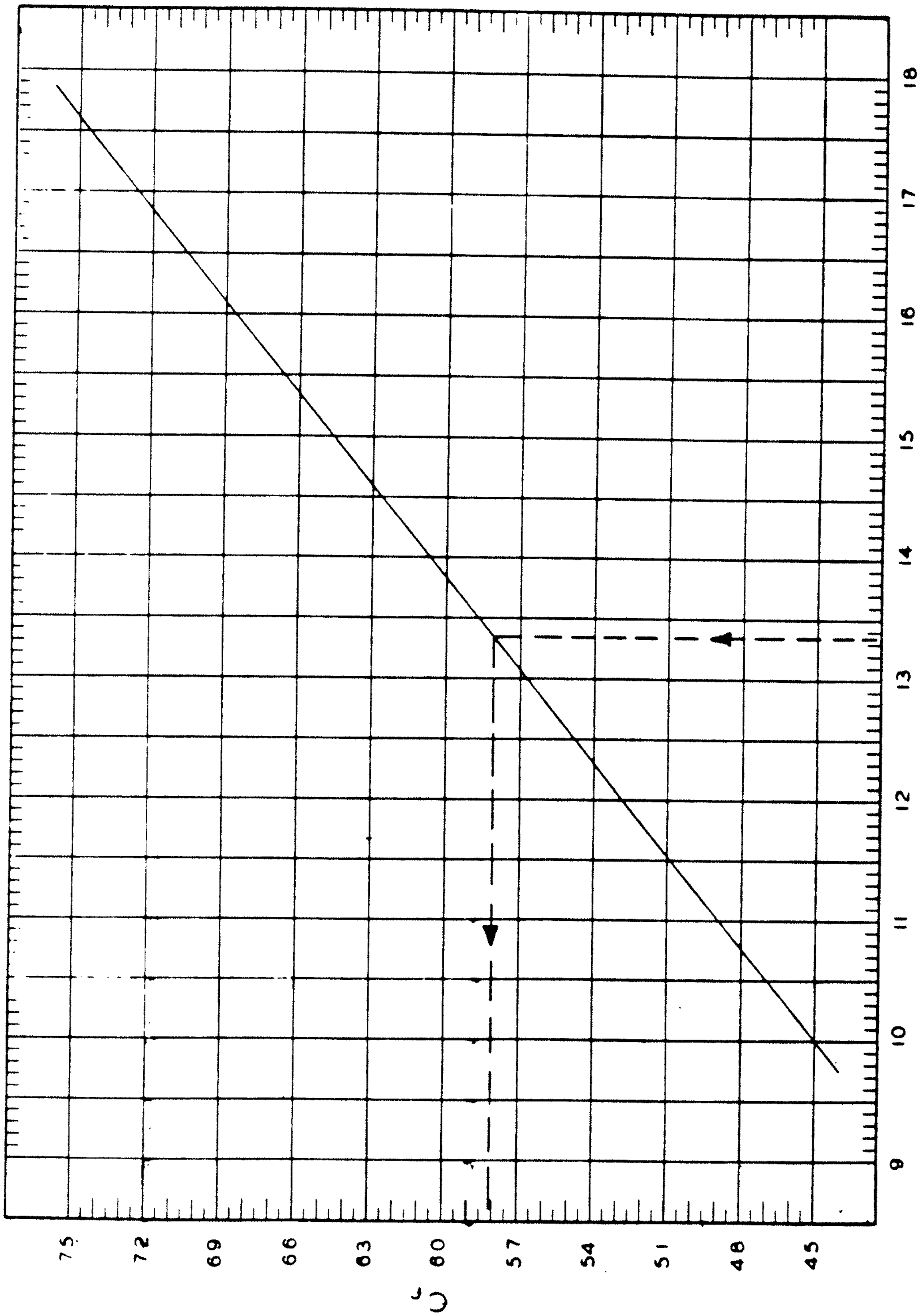
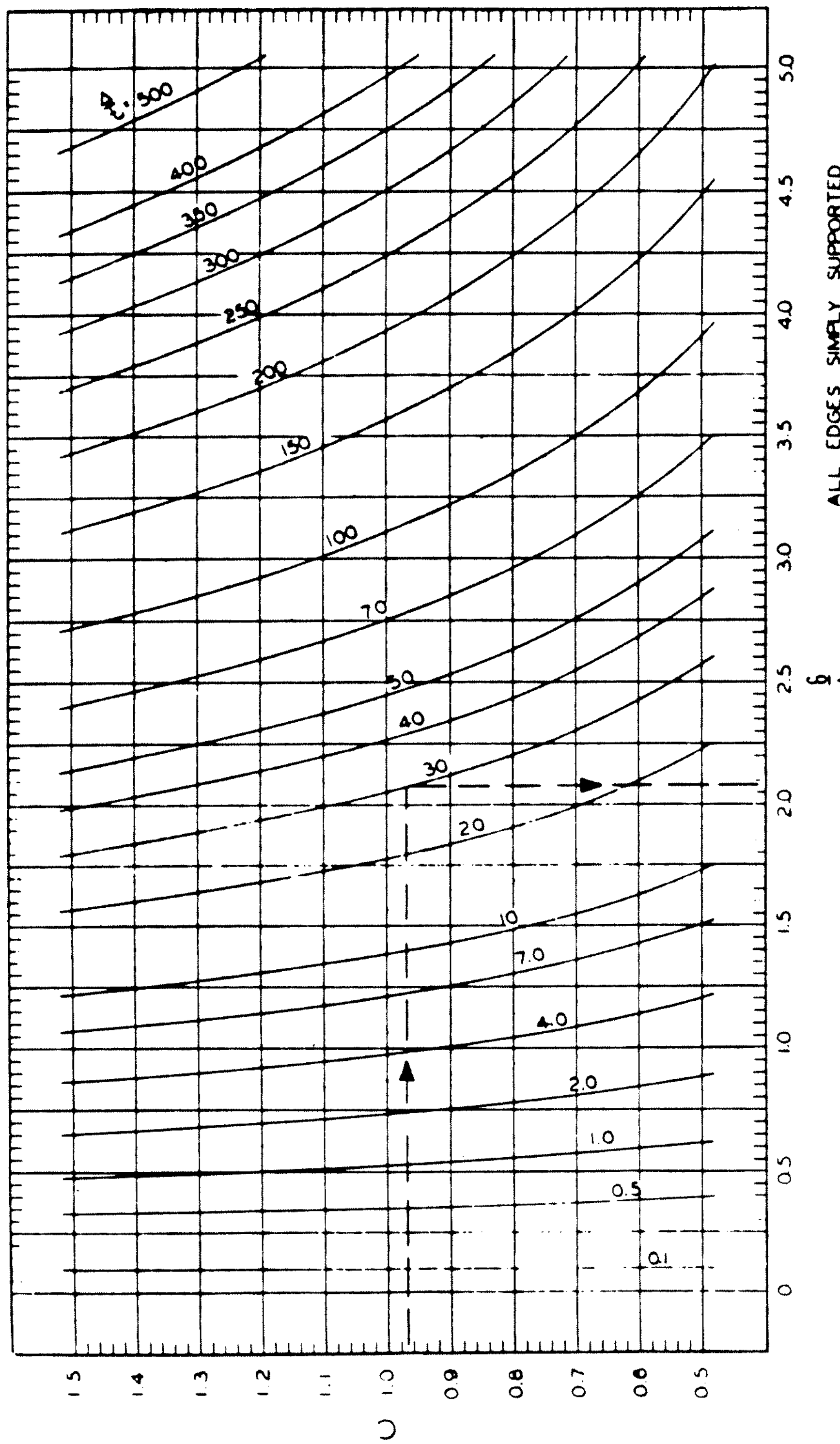
FIGURE 4b



ALL EDGES CLAMPED

FIGURE 4c


ALL EDGES SIMPLY SUPPORTED
δ/t
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a/t = 500
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0.8
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0.6
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FIGURE 5a

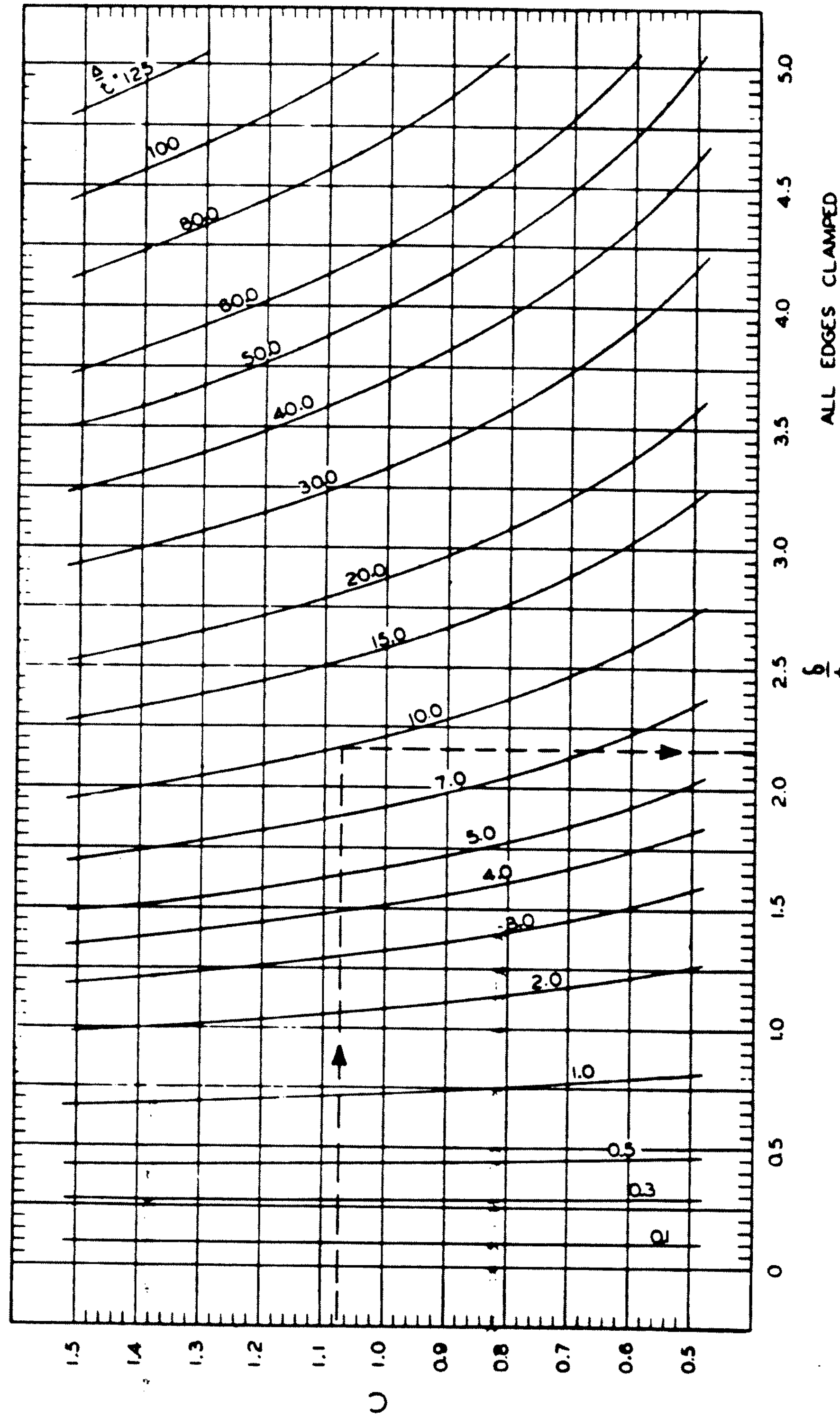
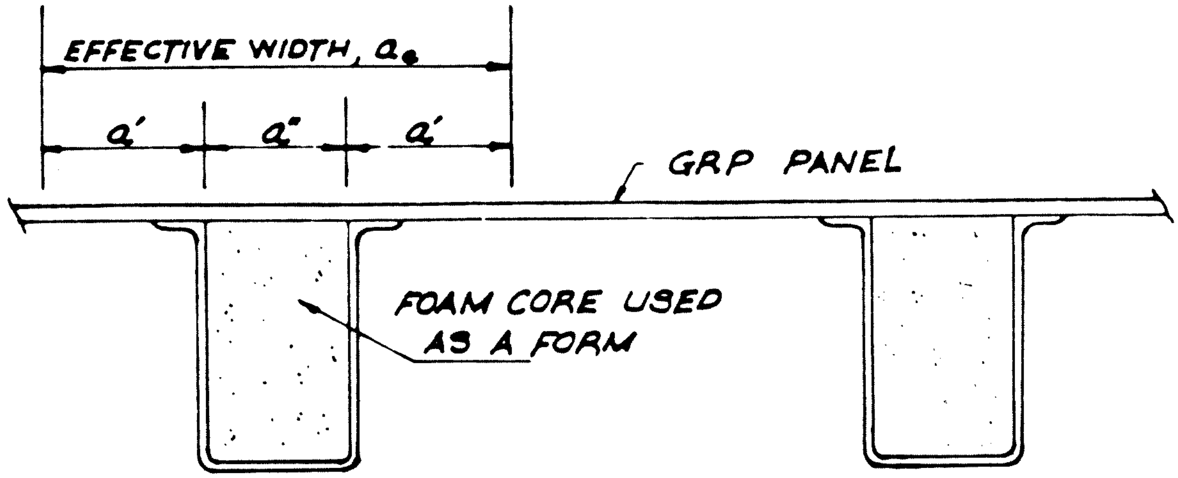
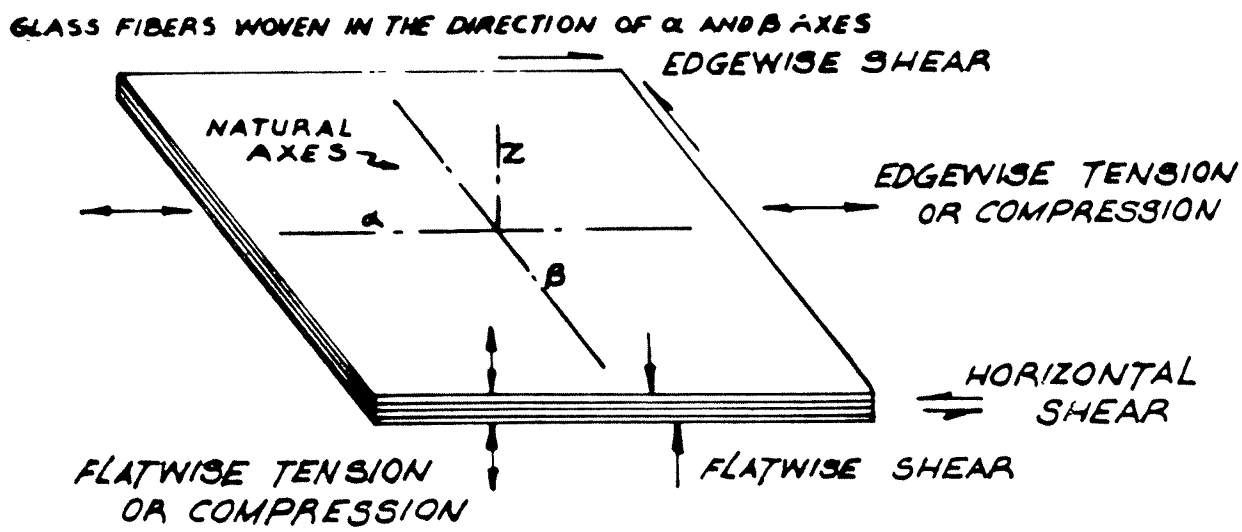

ALL EDGES CLAMPED
δ/t
C
Δ/t = 125
100
80.0
60.0
50.0
40.0
30.0
20.0
15.0
10.0
7.0
5.0
4.0
3.0
2.0
1.0
0.5
0.3
0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
1.5 1.4 1.3 1.2 1.1 1.0 0.9 0.8 0.7 0.6 0.5

FIGURE 5b



TYPICAL HI-HAT STIFFENER-PANEL COMBINATION



NATURAL AXES AND
STRESS DIRECTIONS OF A GRP PANEL

FIGURE 6

Appendix A

DDS 9110-9

Design curves for woven roving laminates

Appendix A to DDS 9110-9

Design curves for woven roving laminates

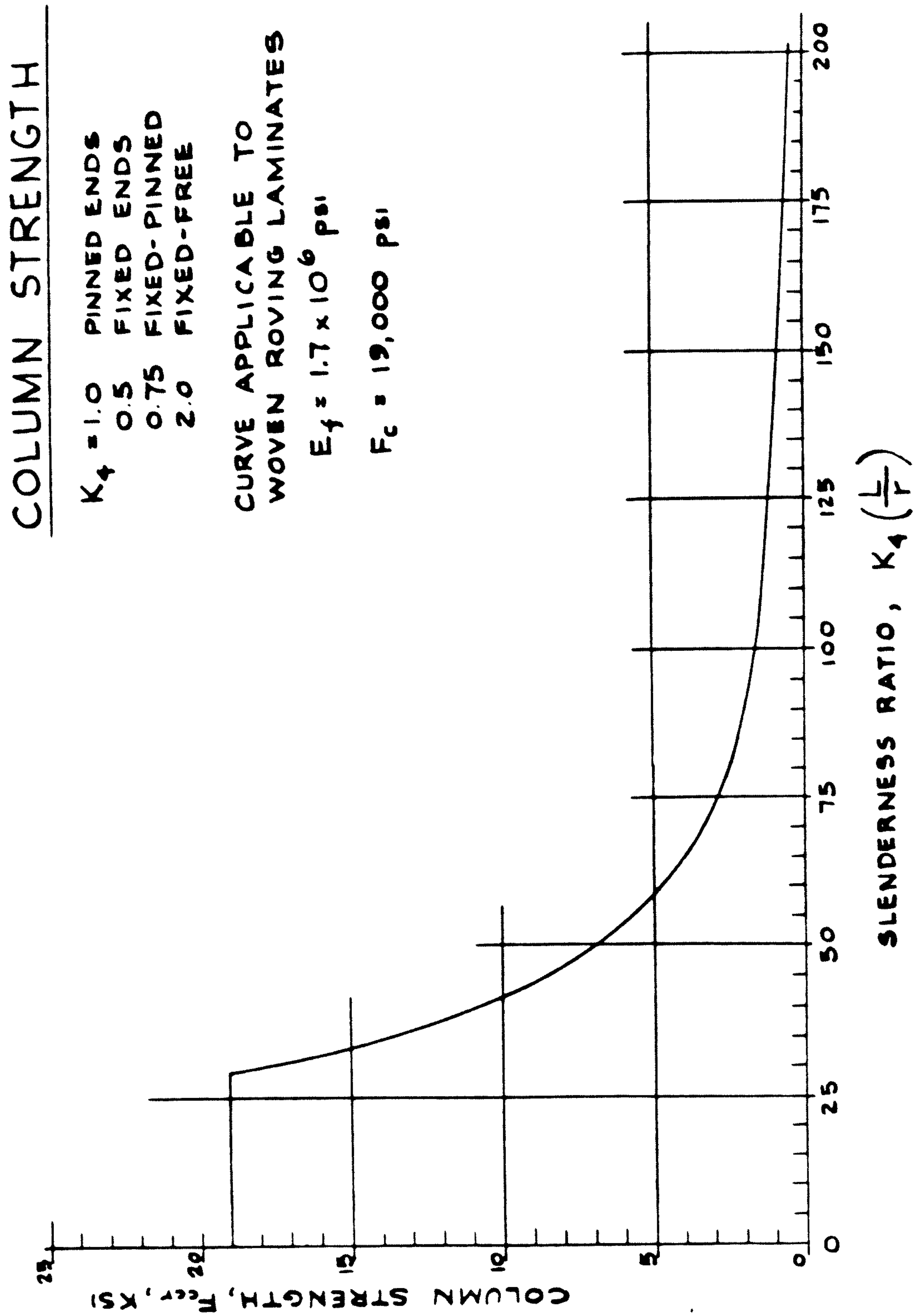
The following design curves have been developed from the formulas given in the basic text for woven roving-polyester laminates.

Woven roving is a medium strength, bi-directional reinforcement. These particular curves are presented primarily because woven roving has become a standard material for boat construction and will probably be used extensively for application to larger hulls. Woven roving-polyester laminates are economical laminates which provide sufficient strength for hulls and other structural elements under normal room temperature cure and by hand lay-up applications. Furthermore, considerable data is available on the mechanical properties.

The following properties were used in preparing the curves. These properties could vary and the Ship Specification for the design governs. Where the Ship Specifications do not specify values, these values may be used. The properties are average wet strength values based on laboratory tests and test specimens from actual fabricated boats. Note that because woven roving is bi-directional the properties in both flatwise axes are equal; $F_a = F_b$, $E_a = E_b$.

	ULTIMATE STRENGTH, PSI	MODULUS OF ELASTICITY, PSI
Tension	F_t 33,000	E_t 2.0×10^6
Compression	F_c 19,000	E_c 1.5×10^6
Flexure	F_f 40,000	E_f 1.7×10^6
Shear (Perpendicular to laminate)	F_s 12,000	
Shear (Horizontal and Secondary bonds)	F_s 700	
Shear modulus of Rigidity		G 0.45×10^6
Poissons Ratios, μ		

Tension 0.14
Compression 0.25
Flexure 0.19



COLUMN STRENGTH
K4 = 1.0 PINNED ENDS
0.5 FIXED ENDS
0.75 FIXED-PINNED
2.0 FIXED-FREE
CURVE APPLICABLE TO
WOVEN ROVING LAMINATES
Ef = 1.7 x 10^6 psi
Fc = 19,000 psi
COLUMN STRENGTH, Fccr, KSI
0
5
10
15
20
25
0
25
50
75
100
125
150
175
200
SLENDERNESS RATIO, K4 (L/r)

FIGURE 1

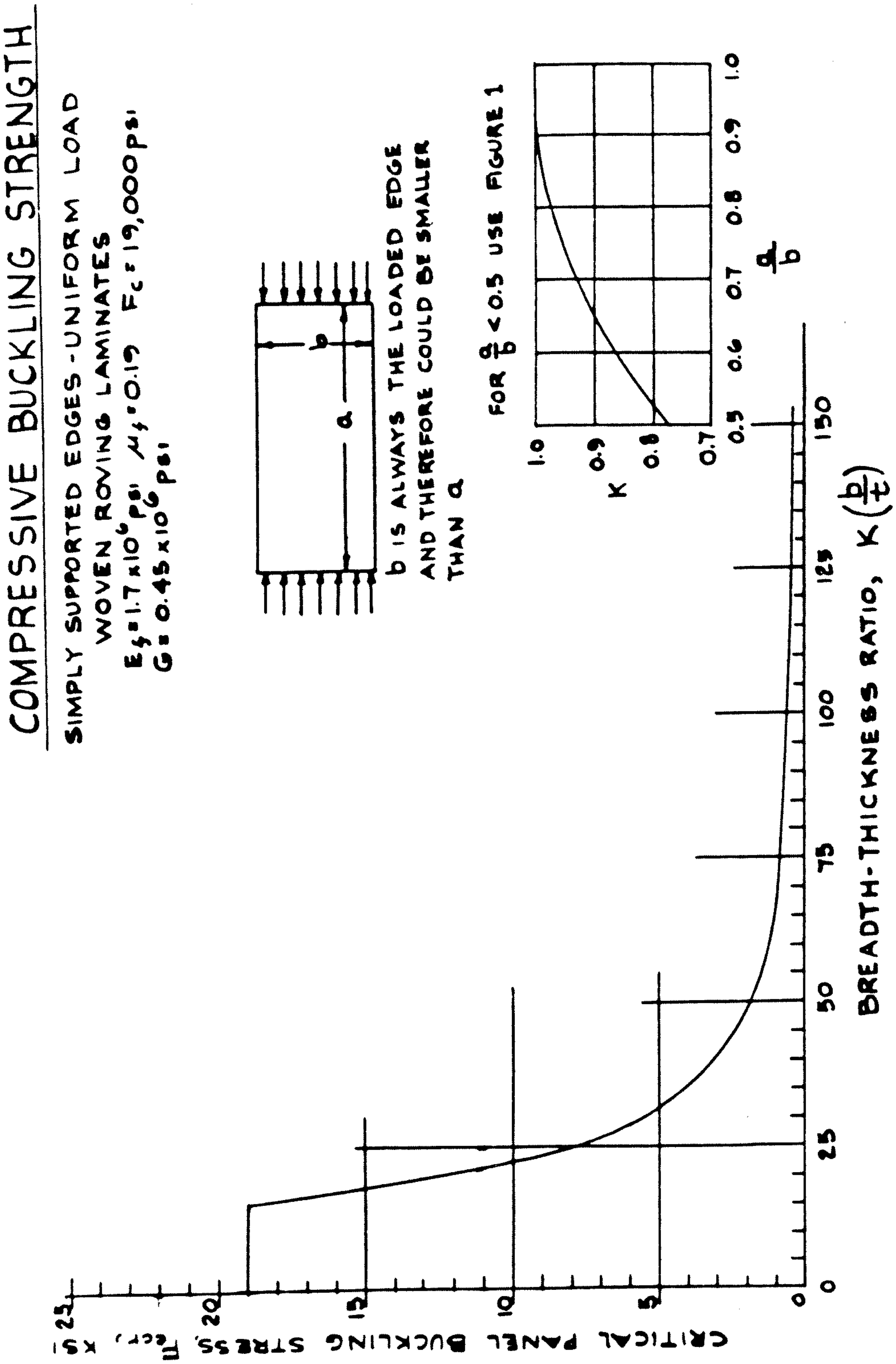

COMPRESSIVE BUCKLING STRENGTH
SIMPLY SUPPORTED EDGES-UNIFORM LOAD
WOVEN ROVING LAMINATES
E_f = 1.7x10^6 psi μ_f = 0.19 F_c = 19,000 psi
G = 0.45x10^6 psi
a
b
b IS ALWAYS THE LOADED EDGE
AND THEREFORE COULD BE SMALLER
THAN a
FOR a/b < 0.5 USE FIGURE 1
K
1.0
0.9
0.8
0.7
0.5 0.6 0.7 0.8 0.9 1.0
a/b
CRITICAL PANEL BUCKLING STRESS, F_ccr, KSI
25
20
15
10
5
0
0 25 50 75 100 125 150
BREADTH-THICKNESS RATIO, K(b/t)

FIGURE 2

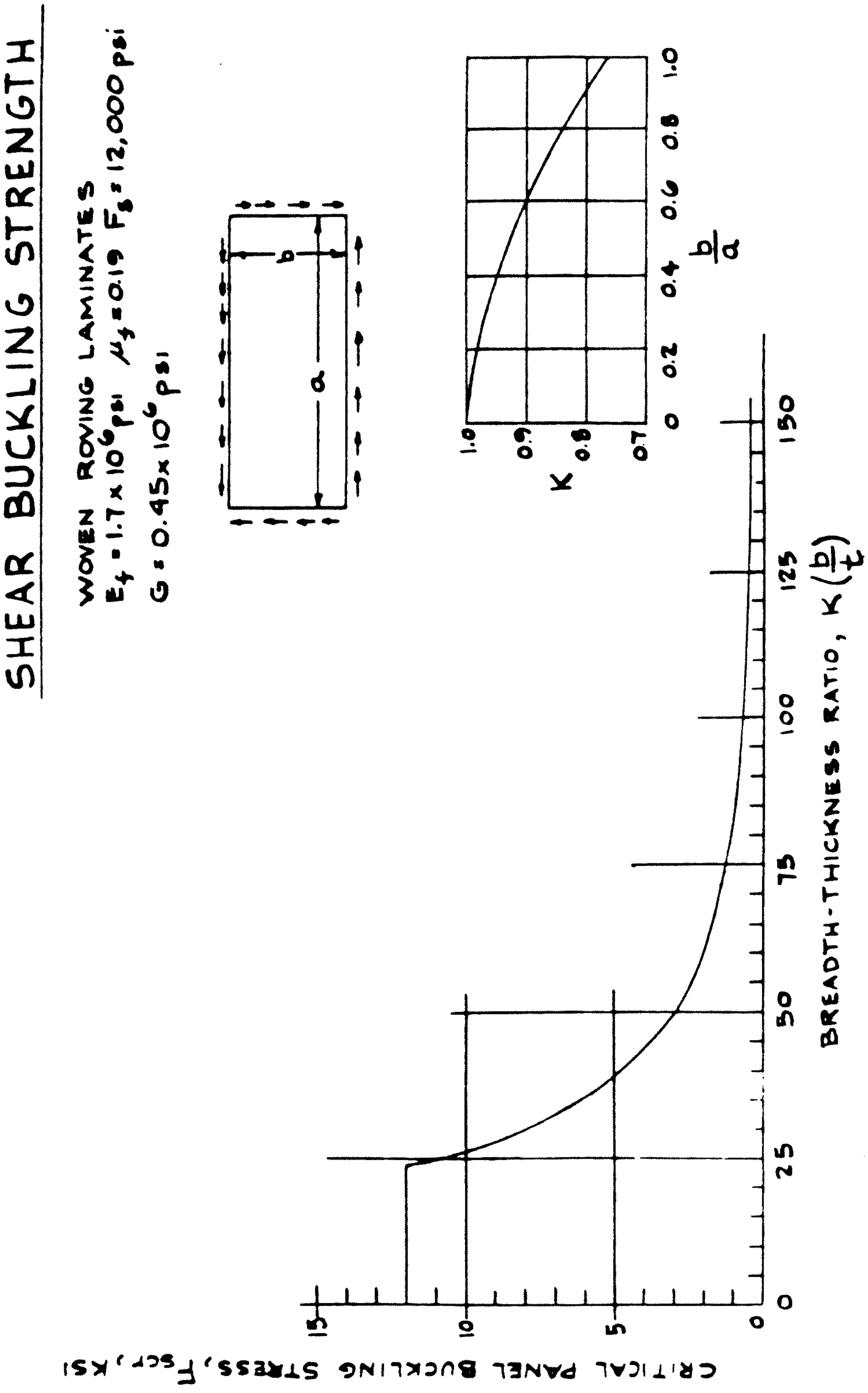

SHEAR BUCKLING STRENGTH
WOVEN ROVING LAMINATES
E_f = 1.7 x 10^6 psi μ_f = 0.19 F_s = 12,000 psi
G = 0.45 x 10^6 psi
a
b
1.0
0.9
0.8
0.7
K
0
0.2
0.4
0.6
0.8
1.0
b/a
0
25
50
75
100
125
150
BREADTH-THICKNESS RATIO, K(b/t)
0
5
10
15
CRITICAL PANEL BUCKLING STRESS, F_scr, KSI

FIGURE 3

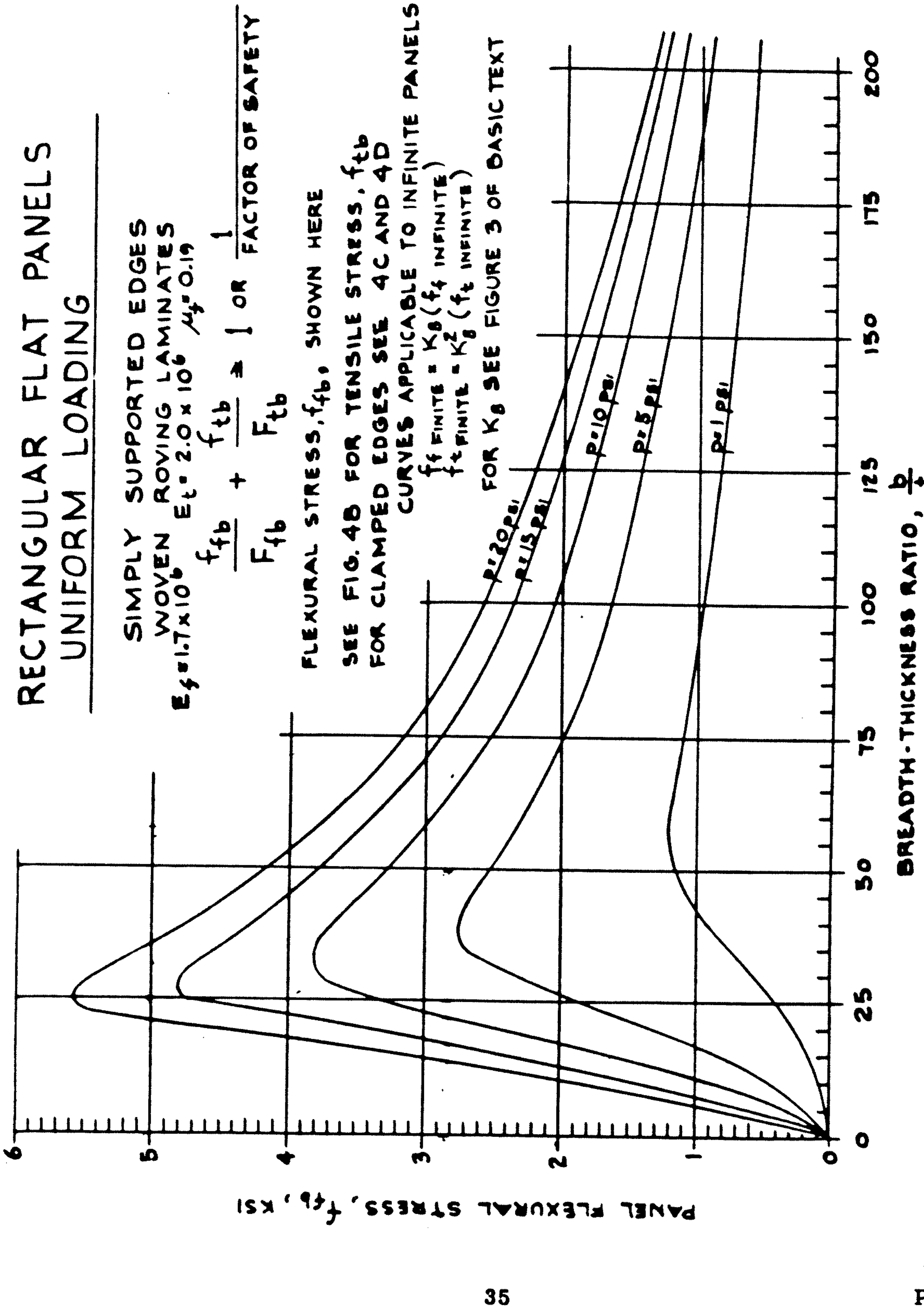

RECTANGULAR FLAT PANELS
UNIFORM LOADING
SIMPLY SUPPORTED EDGES
WOVEN ROVING LAMINATES
Ef = 1.7 x 10^6 Et = 2.0 x 10^6 μf = 0.19
ffb/Ffb + ftb/Ftb ≥ 1 OR 1/FACTOR OF SAFETY
FLEXURAL STRESS, ffb, SHOWN HERE
SEE FIG. 4B FOR TENSILE STRESS, ftb
FOR CLAMPED EDGES SEE 4C AND 4D
CURVES APPLICABLE TO INFINITE PANELS
ff FINITE = KB(ff INFINITE)
ft FINITE = KB^2(ft INFINITE)
FOR KB SEE FIGURE 3 OF BASIC TEXT
p=20 psi
p=15 psi
p=10 psi
p=5 psi
p=1 psi
0
25
50
75
100
125
150
175
200
BREADTH-THICKNESS RATIO, b/t
0
1
2
3
4
5
6
PANEL FLEXURAL STRESS, ffb, KSI

FIGURE 4A

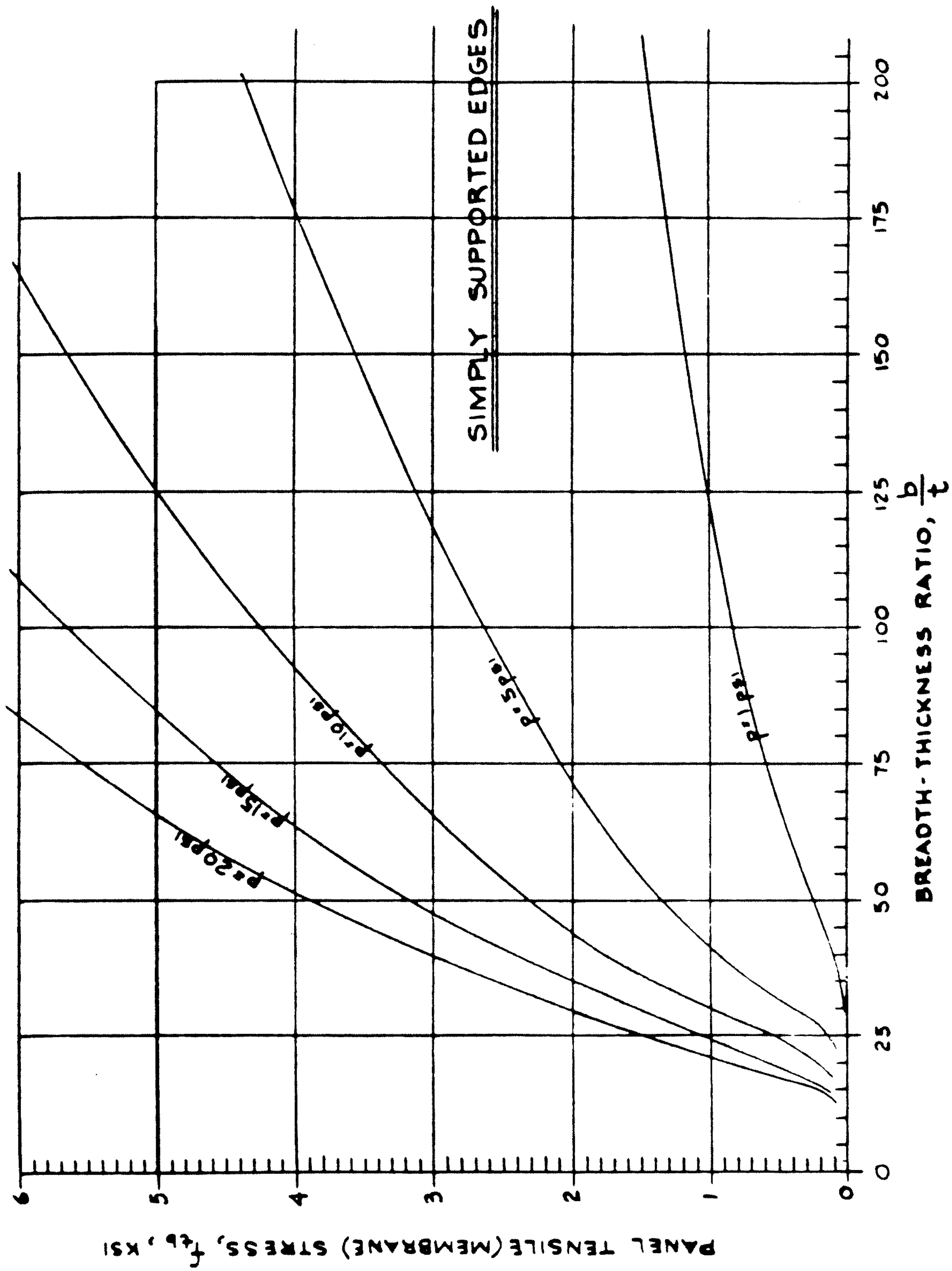

SIMPLY SUPPORTED EDGES
p=20psi
p=15psi
p=10psi
p=5psi
p=1psi
0
25
50
75
100
125
150
175
200
BREADTH-THICKNESS RATIO, b/t
0
1
2
3
4
5
6
PANEL TENSILE (MEMBRANE) STRESS, f_{tb}, KSI

FIGURE 4B

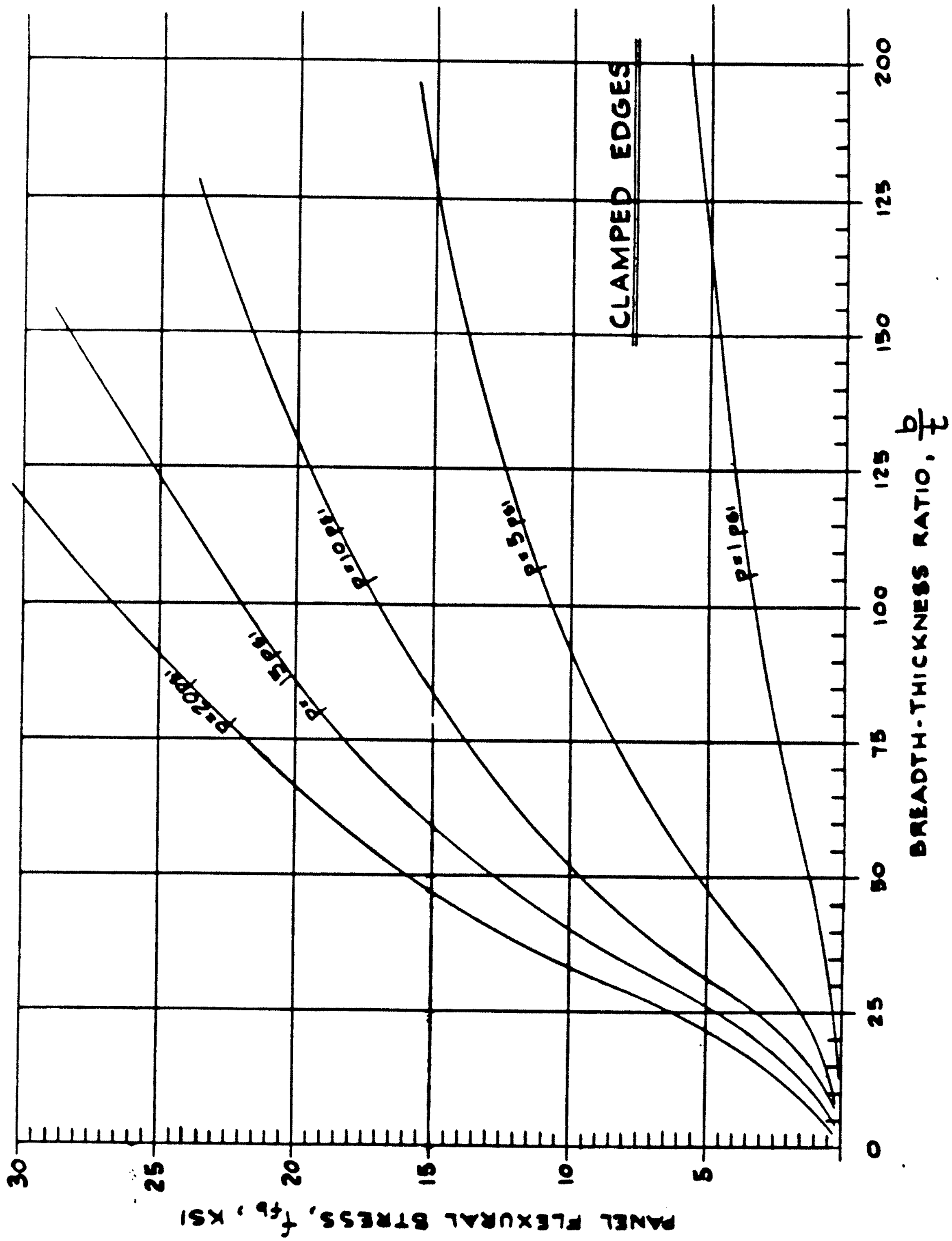

CLAMPED EDGES
p=20psi
p=15psi
p=10psi
p=5psi
p=1psi
BREADTH-THICKNESS RATIO, b/t
PANEL FLEXURAL STRESS, f_fb, KSI
0
25
50
75
100
125
150
200
5
10
15
20
25
30

FIGURE 4C

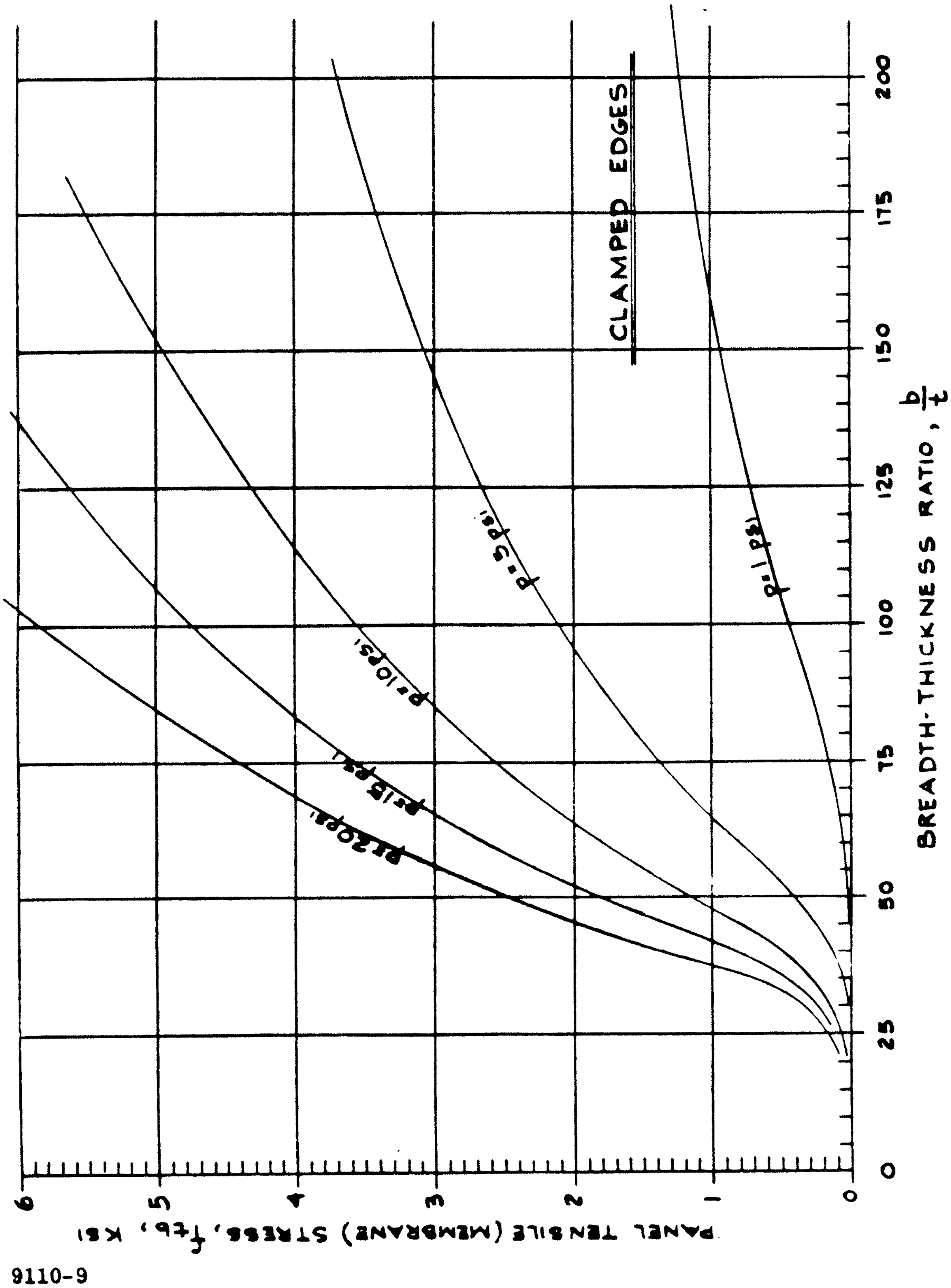

CLAMPED EDGES
p=1 psi
p=5 psi
p=10 psi
p=15 psi
p=20 psi
BREADTH-THICKNESS RATIO, b/t
PANEL TENSILE (MEMBRANE) STRESS, ftb, KSI
0
25
50
75
100
125
150
175
200
1
2
3
4
5
6

FIGURE 4D

Appendix B

DDS 9110-9

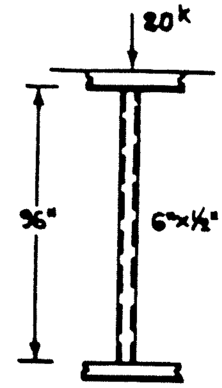
Design examples

Appendix B to DDS 9110-9

Design examples

1. Column formula

Given: A GRP stanchion where:



L = 96 inches

Diameter = 6 inch O.D.

Thickness = 1/2 inches

Material = woven roving, $E_f = 1.7 \times 10^6$ p.s.i.

Load = 20,000 pounds

Determine: Factor of safety

Computation:

radius of gyration, r = 1.953 (calculate or obtain from various standard tables)

A = 8.64 square inches

$\frac{L}{r} = \frac{96}{1.953} = 49.16$ $\left(\frac{L}{r}\right)^2 = 2417$ L = 1.0 (Pinned Ends)

$$F_{ccr} = \frac{\pi^2 E_f}{K\left(\frac{L}{r}\right)^2} = \frac{\pi^2 (1.7 \times 10^6)}{1\ (2417)} = 6942 \text{ p.s.i.}$$ = critical compressive buckling stress

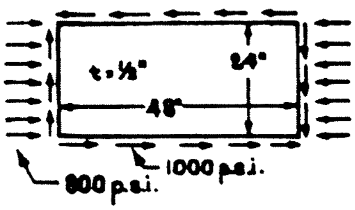
F_{ccr} can also be determined using Figure 1 of Appendix A for woven roving.

Axial Stress, $\frac{P}{A} = \frac{20,000}{8.64} = 2315$ p.s.i.

Factor of safety, F.S. $= \frac{6942}{2315} = \underline{\underline{3.00}}$ (answer)

2. Buckling formulas

Given: A GRP panel subjected to an edge compressive load of 800 p.s.i. and a shearing load of 1000 p.s.i.



b = Panel width = 24 inches (loaded edge)

a = Panel length = 48 inches

Material = woven roving; $G = 0.45 \times 10^6$, $E_f = 1.7 \times 10^6$

$\mu_f = 0.19$

Thickness = 1/2 inch

All edges simply supported

Determine: Factor of safety on ultimate strength

Computation:

Critical compressive buckling stress, $F_{ccr} = \dfrac{H_c\sqrt{E_{fa}E_{fb}}}{\lambda_{fba}}\left(\dfrac{t}{b}\right)^2$

$E_{fa} = E_{fb} = 1.7 \times 10^6$ for woven roving

$$\lambda_{fba} = 1 - \mu_{fab}\mu_{fba}$$
$$= 1 - (.19)^2$$
$$= 0.964$$

$$H_c = h_c + C_c K_f$$

$$C_c = \frac{\pi^2}{6} = 1.64 \text{ For simply supported edges}$$

$$K_f = \frac{E_{fa}\mu_{fba} + 2\lambda_{fab}G}{\sqrt{E_{fa}E_{fb}}}$$

$$Kf = \frac{1.7 \times 10^6 \ (.19) + 2(0.964)(0.45 \times 10)^6}{1.7 \times 10^6} = 0.700$$

$$r = \frac{a}{b}\left(\frac{E_{fb}}{E_{fa}}\right)^{\frac{1}{4}} = \frac{a}{b}(1) = \frac{48}{24} = 2$$

From Figure 1a of basic text, $h_c = 1.62$

$\therefore H_c = 1.62 + 1.64(0.695) = 2.76$

And $F_{ccr} = \dfrac{(1.7 \times 10^6)}{0.964}\left(\dfrac{0.5}{24}\right)^2 = 2094$ p.s.i.

F_{ccr} can also be obtained from Figure 2 of Appendix A for woven roving material.

Critical shearing stress, $F_{scr} = H_s \dfrac{\left(E_{fb}{}^3 E_{fa}\right)^{\frac{1}{4}}}{3\lambda_{fba}}\left(\dfrac{t}{b}\right)^2$

$\dfrac{1}{r} = \dfrac{1}{2} = 0.5$ $\qquad$ $K_f = 0.700$ (see previous calculation)

From Figure 2a of basic text, $H_s = 13.4$

$$F_{scr} = \frac{13.4\ (1.7 \times 10^6)}{3(0.964)}\left(\frac{0.5}{24}\right)^2 = 3388 \text{ psi.}$$

F_{scr} can also be determined from Figure 3 of Appendix A for woven roving.

Combined shear and compression:

$$\left(\frac{f_s}{F_{scr}}\right)^2 + \frac{f_c}{F_{ccr}} = \frac{1}{F.S.}$$

$$\left(\frac{1000}{3388}\right)^2 + \frac{800}{2094} = \frac{1}{F.S.}$$

$$0.087 + 0.382 = \frac{1}{F.S.}$$

$$F.S. = \frac{1}{0.469} = \underline{\underline{2.13}} \text{ (Answer)}$$

3. Panel bending formulas

Given: A GRP panel 24 inches wide, infinitely long, p = Load = 10 p.s.i. normal to surface

Thickness = 1/4 inch

Material: Woven roving

F_f = 40,000 p.s.i.

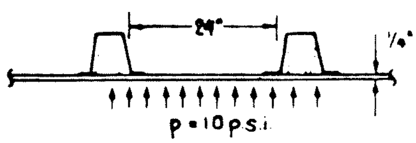
F_t = 33,000 p.s.i.

$E_f = 1.7 \times 10^6$ p.s.i.

$E_t = 2.0 \times 10^6$ p.s.i.

λ = 0.964 (See example 2)

All edges simply supported



Determine: Factor of safety on ultimate strength and the deflection of the panel.

Computation:

$$\frac{f_{fb}}{F_{fb}} + \frac{f_{tb}}{F_{tb}} = \frac{1}{F.S.}$$

$$f_{fb} = C_f \frac{E_{fba}}{\lambda_{fba}} \left(\frac{t}{b}\right)^2 \left(\frac{\delta}{t}\right)$$

$$f_{tb} = 2.572 \frac{E_{tb}}{\lambda_{fba}} \left(\frac{t}{b}\right)^2 \left(\frac{\delta}{t}\right)^2$$

C is found from Figure 4 of basic text and requires computation of m.

$$m = 2.778 \left(\frac{E_{tb}}{E_{fb}}\right)^{\frac{1}{2}} \frac{\delta}{t}$$

$\frac{\delta}{t}$ is found by Figure 5 of basic text and requires computation of

$\frac{\Delta}{t}$ and C.

$$\frac{\Delta}{t} = \frac{5}{32} \frac{\lambda_{fba}\, p b^4}{E_{fb} t^4}$$

$$\frac{\Delta}{t} = \frac{5\,(0.964)\,(10)\,(24)^4}{32\,(1.7 \times 10^6)\,(.25)^4} = 75.37$$

$$C = \frac{E_{tb}}{E_{fb}} = \frac{2.0 \times 10^6}{1.7 \times 10^6} = 1.176$$

From Figure 5, $\frac{\delta}{t} = 2.68$

Maximum deflection, $\delta = 2.68(.25) = \underline{\underline{0.67}}$ inches (Answer)

Continuing with computation:

$$m = 2.778(1.176)^{\frac{1}{2}}(2.68) = 8.07$$

From Figure 4, $C_f = 4.12$

$$f_{fb} = 4.12 \frac{(1.7 \times 10^6)}{0.964} \left(\frac{0.25}{24}\right)^2 (2.68) = 2099 \text{ p.s.i.}$$

$$f_{tb} = 2.572 \frac{(1.7 \times 10^6)}{0.964} \left(\frac{0.25}{24}\right)^2 (2.68)^2 = 3511 \text{ p.s.i.}$$

Factor of safety:

$$\frac{2099}{40{,}000} + \frac{3511}{33{,}000} = \frac{1}{F.S.}$$

$$0.052 + 0.106 = \frac{1}{F.S.}$$

$$F.S. = \frac{1}{0.158} = \underline{\underline{6.33}} \text{ (Answer)}$$

Note that for this example the factor of safety of 6.33 is more than the 4.0 usually required. However, if deflection limits of L/200 are imposed:

L/200 = 24/200 = 0.12 inches

$\delta = 0.67$ inches > 0.12 inches

Therefore panel would not be acceptable if this criteria was imposed.

4. Composite laminates

Given: A simple composite beam simply supported at each end, 20 inches long.

Load 100 pounds concentrated at center

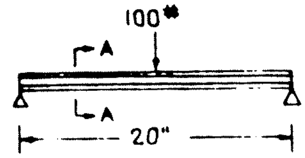
Overall thickness of beam 1 inch

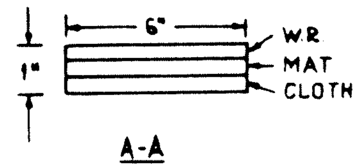
Width of beam = 6 inches

Composition: Top 1/3 of beam is woven roving; E_f = 1.7 x 10^6 p.s.i.

Middle 1/3 of beam is random mat; E_f = 0.86 x 10^6 p.s.i.

Bottom 1/3 of beam is glass cloth; E_f = 1.9 x 10^6 p.s.i.





A-A

Determine: Deflection of the beam

Computation:

$$\text{Composite } E_f = \frac{1}{I} \sum_{i=1}^{i=n} E_i I_i$$

Where: I_i = moment of inertia of each different material area about the neutral axis of the total composite.

I = the total composite moment of inertia

For this example assume that the neutral axis is at the center.

$$\text{Total } I = \frac{bd^3}{12} = \frac{6(1)^3}{12} = 0.5 \text{ inches}^4$$

$$I \text{ of mat} = \frac{bt^3}{12} = \frac{6(1/3)^3}{12} = 0.0185 \text{ inches}^4$$

I of woven roving and cloth

$$I = \frac{bt^3}{12} + \text{Area} \times y^2 \qquad y = 0.34 \text{ inches}$$

$$I = 0.0185 + 2(0.34)^2 = 0.2497 \text{ inches}^4$$

$\Sigma E_i I_i$	$1.7 \times 10^6 (0.2497)$ =	0.4245×10^6
	$1.9 \times 10^6 (0.2497)$	0.4744×10^6
	$0.86 \times 10^6 (0.0185)$ =	0.0159×10^6
	Total	0.9148×10^6 p.s.i.

$$\text{Composite } E_f = \frac{0.9148 \times 10^6}{0.5} = 1.83 \times 10^6 \text{ p.s.i.}$$

$$\text{Deflection of simple beam} = \frac{PL^3}{48\ EI}$$

$$= \frac{100(20)^3}{48\ (1.83 \times 10^6)(0.5)}$$

$$= \underline{\underline{0.0182}} \text{ inches (Answer)}$$

DESIGN DATA SHEET
DEPARTMENT OF THE NAVY
NAVAL SHIP ENGINEERING CENTER

1 August 1969
DDS 9110-9

STRENGTH OF GLASS REINFORCED PLASTIC STRUCTURAL MEMBERS
PART II - SANDWICH PANELS

Contents

Part II

9110-9-a. References

(a) Military Handbook MIL-HDBK 23 - Structural Sandwich Composites

(b) Mil. Spec. MIL-P-17549; Plastic Laminates, Fibrous Glass Reinforced Marine Structural

(c) Sandwich Construction for Aircraft, ANC-23 Bulletin - Second Edition 1955

(d) Shear Stability of Flat Panels of Sandwich Construction - U.S. Forest Products Lab Report 1560-1962.

(e) Deflection and Stresses in a Uniformly Loaded, Simply Supported, Rectangular Sandwich Plate - U.S. Forest Products Lab Report 1847-1962.

(f) Design Data Sheet DDS 9110-6 - Structural Design of Aircraft Handling Decks.

9110-9-b. Purpose and Scope

Part II of this Design Data Sheet is issued to provide guidance and uniform standards for design calculations of glass reinforced plastic (GRP) sandwich panels. Part I includes design data for single skin construction (see definitions in 9110-9-c).

The design data presented in the following paragraphs are taken almost entirely from reference (a). Reference (a) was developed primarily for flight vehicles and contains considerable data on core configuration and properties and is an excellent source from which to obtain data on other configurations not included herein such as circular panels and cylinders.

Because of the many possible variables in core configuration and mechanical properties, sandwich construction presents a complex design problem. Much of the data has been developed through actual tests. The design data presented herein is applicable to most core configurations presently used for ship and boat structures especially those of a "solid" nature such as balsa wood and foams and combinations of foam and GRP webbing (see Figure 27). Data on the foam-GRP webbing core is presently limited but this type of core can be designed as a simple beam structure or with data on the core shear modulus, designed as a solid core configuration. The advantage of this particular type of core is the high shear strength provided by the GRP web, which is required especially for panels subject to high concentrated type loads such as a helicopter landing platform. Cores such as honeycomb paper and aluminum, or corrugated and waffle configurations, not generally applicable to ship structures, are not covered. Reference (a) may be used, however, in the event data are required on these sandwich panels.

The data presented are applicable to sandwich panels with isotropic faces or orthotropic faces which are bi-directional. Since most ship type laminates are bi-directional or have properties nearly bidirectional, the formulas given should be adequate. In extreme cases, where a unidirectional face is used with extreme variance in properties in the a and b direction, data presented in reference (c) can be used.

Appendix A gives some design examples for sandwich panels.

Design graphs for woven roving laminates as given in Part I are not given herein for sandwich panels. Although woven roving reinforcement is still considered an ideal facing material, the presentation of simplified graphs is complicated because of the many possible combinations of core thickness and properties. It is therefore suggested that instead of graphs, the formulas be programmed for computer application.

Formulas for stiffeners supporting sandwich panels and combined stresses as given in Part I of this DDS are applicable to sandwich panels.

9110-9-c. Definitions

Glass Reinforced Plastic (GRP) - A structural material composed of woven glass fibers and a thermosetting resin binder. For ship application the glass is generally glass cloth, woven roving, and random mat combinations. The most commonly used resins are epoxy and polyester.

Glass cloth - A woven material made of glass yarns. Glass yarns are glass strands given a twist or as a combination of two or more of such twisted yarns plied together. Glass strands are a multiplicity of glass filaments drawn together and gathered into an approximately parallel arrangement without twist. A glass filament is a single thread of glass of very small diameter.

Woven roving - A woven material made of ropes of glass rovings. Rovings are groups of continuous strands wound into a cylindrical package. The number of strands can be varied.

Glass mat - Composed of numerous fibers, strands, or yarns of uniform length, unwoven but randomly distributed and resin bonded so as to form a uniformly thick and highly porous sheeting of glass fibers.

Laminate - A finished thickness of GRP material composed of a number of layers of woven glass and resin.

Ply - Each individual layer of glass in a laminate. Whenever thickness-per-ply is discussed, it is the thickness of the glass and resin. The glass thickness will remain constant or within a small tolerance for a given glass material. However, thickness associated with a given ply may vary since it is dependent on the fabrication. The strength of GRP varies as glass content varies in a given laminate.

Single skin construction - Term used to describe a relatively thin laminate composed entirely of GRP and distinguished from a sandwich panel which is composed of two skins of facings separated by a light weight core.

Orthotropic material - Material with different mechanical properties in direction of three mutually perpendicular axes. These three axes are used for stress analysis and are called the natural axes. Two of the axes are in the laminate plane. The third axis is perpendicular to the laminate plane. Examples of orthotropic materials are GRP and plywood.

Isotropic material - Material with mechanical properties equal in all directions.

Bidirectional laminate - Orthotropic GRP material where the mechanical properties along the two natural axes in the laminate plane are about equal, produced by placing equal amounts of glass fiber in both directions.

Unidirectional laminate - Orthotropic material with mechanical properties along one natural axis in the laminate plane much higher than the other axis.

Sandwich panel - A composite formed by bonding two strong thin facings to a low density thick core. The facings resist nearly all the edgewise loads and flatwise bending moments. The core provides the shear rigidity of the composite and spaces the facings and transmits shear between them so they are effective about a common neutral axis. The facings of sandwich panels should be spaced far enough apart to achieve a high ratio of stiffness to weight.

Core - The low density material separating facings in a sandwich panel. For ship applications cores are generally composed of balsa wood and various foam plastics, or a combination of light weight foam and glass webbing.

Facing - The outer surfaces of the sandwich composite. For this DDS the facings are composed of GRP laminates.

9110-9-d. Symbols for formulas

SYMBOLS

A	cross sectional area of a sandwich panel; coefficient for sandwich panel formulas.
a	length of one edge of a rectangular flat panel; subscript denoting direction parallel to the "a" edge of a rectangular flat panel.
b	length of one edge of a rectangular flat panel.
B	coefficient for sandwich panel formulas.
C	subscript denoting core of a sandwich panel.
cr	subscript denoting a critical condition of elastic buckling.
c	subscript denoting compression; coefficient for edge conditions of sandwich panels.
D	bending stiffness factor for flat panels.
d	total depth (thickness) of a sandwich panel.

E	Young's modulus of elasticity; with subscripts denotes modulus in tension, compression, etc.
F	ultimate strength of a GRP laminate or member; with the subscript cr, critical compressive buckling stress; with the subscript scr, the shear buckling stress; subscript denoting facing of a sandwich panel.
F.S.	factor of safety on the ultimate strength.
f	induced stress; subscript denoting bending of flexural strength.
G	Modulus of rigidity (shear modulus)
H	extensional stiffness
h	distance between facing centroids of a sandwich panel.
I	moment of inertia of the cross section of a laminate or member.
K, K_M, etc.	coefficients for formulas.
L	unsupported length of a column or beam, core axis for defining sandwich panel core properties.
M	bending moment
n	number of half-waves of a buckled column or rectangular flat panel.
N	shear stiffness factor; load per unit length of panel edge.
p	unit load.
Q	coefficient for sandwich panel formulas.
r	radius of gyration of a column section; stiffness factor for rectangular flat panels; subscript denoting reduced.
R	coefficient for sandwich panel formulas.
s	subscript denoting shear.
T	core axis for defining sandwich panel core properties.
t	subscript denoting tension; thickness of sandwich facings with subscripts 1 and 2 indicating each face.
U	shear stiffness factor.
V	shearing force.
W	weight; core axis for defining sandwich panel core properties.

Z	section modulus of the cross section of a panel in bending.
α, β, γ	coefficients for sandwich panel formulas, factor relating to properties of materials.
μ_{fba}	$1 - \mu_{fba}\mu_{fab}$
μ	Poisson's ratio; with two subscripts denoting direction, Poisson's ratio for strain when stress is in the direction of the first subscript.
δ, Δ	deflection of laminate or panel.

9110-9-e. Factors of safety

The formulas herein and design curves of Appendix A relate to the ultimate strength of the CRP materials. GRP has no discernible yield strength.

Factors of safety on the ultimate strength must be included in accordance with applicable Ship Specifications. Where not so specified, the following factors of safety are to be used.

Conditions	Factor of safety on ultimate strength
Stiffeners and stanchions	4.0
Flat panels	
(a) Panels loaded on the long edge from tension, compression, and shear, or combinations thereof, where failure would be catastrophic.	4.0
(b) Compressive and/or shear stresses in panels loaded on short edges where buckling would not lead to catastrophic failure by transmittal of load to boundary stiffeners.	2.0

The large factor of safety (4.0) is primarily to account for fatigue, creep, and aging of GRP materials. In addition to stress levels, structures should be limited to a deflection equal to L/200. This deflection requirement is extremely important for GRP members because of the low modulus of elasticity.

9110-9-f. Modes of failure of sandwich panels

The modes of failure that may occur in sandwich panels are shown in Figure 1. These failures are obviously different than simple laminate failures because of the presence of the core.

(1) General Buckling-Figure 1-A - The panel may fail by a local failure under edgewise direct or flatwise bending loads.

(2) Shear crimping-Figure 1-B - This failure appears to be a local mode of failure but is actually a form of general overall buckling in which the wave-length of the buckles is very small because of low core shear modulus. The crimping of the sandwich occurs suddenly and usually causes the core to fail in shear at the crimp; it may also cause shear failure in the bond between the facing and core. Crimping may also occur in cases where the overall buckle begins to appear and then the crimp occurs suddenly because of severe local shear stresses at the ends of the overall buckle. As soon as the crimp appears, the overall buckle may disappear. Therefore, although examination of the failed sandwich indicates crimping or shear instability, failure may have begun by overall buckling that finally caused crimping.

(3) Face buckling-Figure 1-C - If the core is of cellular (honeycomb) or corrugated material, it is possible for the facing to buckle or dimple into spaces between core walls or corrugation as shown in Figure 1-C. Dimpling may be severe enough so that permanent dimples remain after removal of load and the amplitude of dimples may be large enough to cause dimples to grow across the core cell walls and result in a wrinkling of the facings.

For corrugated cores, consideration must also be given to secondary bending of the core webs. This mode of failure is not considered for "solid" cores covered by this DDS.

(4) Wrinkling-Figure 1-D - Wrinkling may occur if a sandwich facing subjected to edgewise compression buckles as a plate on an elastic foundation. The facings may buckle inward or outward, depending on the flatwise compressive strength of the core relative to the flatwise tensile strength of the bond between the facing and core. If the bond between facing and core is strong, facings can wrinkle and cause tension failure in the core. Thus, the wrinkling load depends upon the elasticity and

strength of the core and the bond between the facing and core. Since the facing is never perfectly flat, the wrinkling load will also depend upon the eccentricity of the facing or original waviness.

The local modes of failure may occur in sandwich panels under edgewise loads or normal loads. In addition to overall buckling and local modes of failure, sandwich panels are designed so that facings do not fail in tension, compression, shear, or combined stresses due to edgewise loads or normal loads, and cores and bonds do not fail in shear, flatwise tension or flatwise compression due to normal loads.

9110-9-g. Fundamental formulas

In the development of formulas for deflection, stresses, and buckling of sandwich components, mathematical expressions for bending, extensional, and shear stiffness often appear as parameters involving these stiffnesses. It is convenient to present the fundamental stiffness formulas at the outset. Here also are discussed the effects of facing and core stiffness on sandwich bending stiffness so that the degree of approximation implied by simplified formulas neglecting facing and core stiffness is known.

Note that in this section subscripts are given for facing, F and core, C. F1 is one facing, F2 the other facing. No subscripts are given for tension, compression, flexure, etc. The text will define which condition is applicable. This is done for clarity only.

Sandwich bending stiffness

A structural sandwich under forces normal to its facings has a bending stiffness, per unit width, given by the formula:

$$D = \frac{1}{\frac{E_{F1}t_{F1}}{\lambda_{F1}} + \frac{E_C t_C}{\lambda_C} + \frac{E_{F2}t_{F2}}{\lambda_{F2}}}\left[\frac{E_{F1}t_{F1}}{\lambda_{F1}}\left(\frac{E_{F2}t_{F2}}{\lambda_{F2}}\right)h^2 + \frac{E_{F1}t_{F1}}{\lambda_{F1}}\left(\frac{E_C t_C}{\lambda_C}\right)\left(\frac{t_{F1}+t_C}{2}\right)^2 + \frac{E_{F2}t_{F2}}{\lambda_{F2}}\left(\frac{E_C t_C}{\lambda_C}\right)\left(\frac{t_{F2}+t_C}{2}\right)^2\right] + \frac{1}{12}\left[\frac{E_{F1}t_{F1}^3}{\lambda_{F1}} + \frac{E_C t_C^3}{\lambda_C} + \frac{E_{F2}t_{F2}^3}{\lambda_{F2}}\right]$$

Where faces are composed of bi-directional laminates, i.e., $E_a = E_b$, then D is the same in both the a and b direction. If a and b directions have different properties, substitute the appropriate E_F and E_C values for the a and b direction; for E use the lowest modulus. For GRP this is usually compression.

(See sketch of Figure 2 for notation.) For many combinations of facing materials it will be found advantageous to choose thicknesses such that $E_{F1}t_{F1} = E_{F2}t_{F2}$

For sandwich with facings of the same material and thickness, the formula reduces to:

$$D = \frac{E_F t_F h^2}{2\lambda_F} + \frac{1}{12}\left(\frac{2E_F t_F^3}{\lambda_F} + \frac{E_C t_C^3}{\lambda_C}\right)$$

The second term of the above formula incorporating facing stiffness and core stiffness is neglected for most sandwich. The effect of this second term in increasing basic sandwich stiffness is obtained from values of K shown graphically in Figure 3, with the above formula reduced to:

$$D = K\frac{E_F t_F h^2}{2\lambda_F}$$

If the sandwich has thin facings on a core of negligible bending stiffness, as is usually the case, and after assuming $\lambda_{F1} = \lambda_{F2} = \lambda$, the bending stiffness is given by the formula:

$$D = \frac{E_{F1}t_{F1}E_{F2}t_{F2}h^2}{(E_{F1}t_{F1} + E_{F2}t_{F2})\lambda_F} \quad \text{(for unequal facings)}$$

$$D = \frac{E_F t_F h^2}{2\lambda_F} \quad \text{(for equal facings)}$$

<u>Sandwich extensional stiffness</u>

The extensional stiffness of a sandwich, stretched or compressed by force in its plane, is given by the formula:

$$H = E_{F1}t_{F1} + E_{F2}t_{F2} + E_C t_C \quad \text{(for unequal facings)}$$

$$H = 2E_F t_F + E_C t_C \quad \text{(for equal facings)}$$

For E and E_C use compression or tension modulus.

Sandwich shear stiffness

A sandwich that has fairly thin facings on a thick core has a transverse shear stiffness per unit width given approximately by the formula:

$$U = \frac{h^2}{t_C} G_C \approx hG_C$$

G_C is the core shear modulus associated with the distortion of the $\gamma\alpha$ or $\gamma\beta$ plane.

9110-9-h. Rectangular flat sandwich panels under edgewise loads

Overall buckling of the sandwich or wrinkling of the facings cannot occur without possible total collapse of the panel. Detailed procedures giving theoretical formulas and graphs for determining dimensions of the facings and core for resisting compressive and shear edge loads are given in following paragraphs. Double formulas are given, one formula for sandwich with facings of different materials and thicknesses and another formula for sandwich with each facing of the same material and thickness.

(1) Panels loaded in edgewise compression

Formulas to determine minimum face thickness

Facing stresses are related to the edge load by the equations:

$$t_{F1}F_{F1} + t_{F2}F_{F2} = N \text{ (for unequal facings)}$$

$$t_F = \frac{N}{2F_F} \qquad \text{(for equal facings)}$$

F_F is the chosen design facing compressive stress; N is the design compression load per unit length of panel edge. These formulas assume both faces are of the same material. If not, see reference (a).

Critical compressive buckling stress

The following formulas are for determining core thickness and core shear modulus so that overall buckling of the sandwich panel will not occur.

The load per unit panel width at which buckling of a sandwich panel will occur is given by the theoretical formula:

$$N_{cr} = K \frac{\pi^2}{b^2} D$$

where D is sandwich bending stiffness. This formula, solved for the facing stress, becomes:

$$F_{Fcr_{1,2}} = \pi^2 K \frac{E_{F1} t_{F1} E_{F2} t_{F2}}{(E_{F1} t_{F1} + E_{F2} t_{F2})^2} \left(\frac{h}{b}\right)^2 \frac{E_{F1,2}}{\lambda_F} \quad \text{(unequal facings)}$$

E_F is the chosen compression modulus of elasticity.

$$F_{Fcr} = \frac{\pi^2 K}{4} \left(\frac{h}{b}\right)^2 \frac{E_F}{\lambda_F} \quad \text{(for equal facings)}$$

For orthotropic faces, $E_F = \sqrt{E_{Fa} E_{Fb}}$ in the above formulas. For bi-directional laminates the formulas may be used in the given form.

b is the length of loaded panel edge; $K = K_F + K_M$; K_F is a theoretical coefficient dependent on facing stiffness and panel aspect ratio; and K_M is a theoretical coefficient dependent on sandwich bending and shear rigidities and panel aspect ratio.

The above formulas assume core has negligible bending stiffness and therefore were developed by substituting the D value corresponding to negligible bending stiffness. If this is not the case modify by substituting the appropriate D value given in 9110-9-g.

Determination of coefficient K

$$K = K_F + K_M$$

Values of K_F shall be determined by the formula:

$$K_F = \frac{\left(E_{F1} t_{F1}^3 + E_{F2} t_{F2}^3\right)\left(E_{F1} t_{F1} + E_{F2} t_{F2}\right)}{12 E_{F1} t_{F1} E_{F2} t_{F2} h^2} K_{MO} \quad \text{(unequal facings)}$$

$$K_F = \frac{t_F^2}{3h^2} K_{MO} \quad \text{(for equal facings)}$$

Where K_{MO} is determined from Figure 4. ($K_{MO} = K_M$ when V = 0.) For a/b ratios greater than shown on Figure 4, assume $K_F = 0$.

Values of K_M may be determined for most sandwich panels by using Figures 5 to 16. These figures are applicable to isotropic facings $\alpha = 1.0$; $\beta = 1.0$; $\gamma = 0.375$ and orthotropic facings where $\alpha = 1.0$; $\beta = 0.6$ and $\gamma = 0.2$ where α, β, γ are as follows:

$$\alpha = \sqrt{\frac{E_b}{E_a}} \; ; \; \beta = \alpha\mu_{ab} + 2\gamma \; ; \; \gamma = \frac{G_{ba}}{\sqrt{E_a E_b}}$$

G_{ba} is the face shear modulus.

Use of the figure requires a parameter V which incorporates bending and shear rigidity:

$$V = \frac{\pi^2 D}{b^2 U}$$

which can be written as:

$$V = \frac{\pi^2 t_C E_{F1} t_{F1} E_{F2} t_{F2}}{\lambda_F b^2 G_C \left(E_{F1} t_{F1} + E_{F2} t_{F2}\right)} \quad \text{(unequal facings)}$$

$$V = \frac{\pi^2 t_C E_F t_F}{2\lambda_F b^2 G_C} \quad \text{(equal facings)}$$

For each value of V, there is a cusped curve giving values of K_M for various a/b, b/a ratios. These cusps are indicated by dotted lines for the top curve in each figure. The cusps show the sandwich panel buckling coefficients calculated for different values of n; the number of half waves into which the panel buckles. Only the portions of each cusped curve for which K_M is a minimum are shown. Envelope curves indicate values of K_M for use in design.

If the Figures do not apply because ratios of core shear moduli or α, β and γ values are far different from what is given on the Figures, or it is desired to check by a more accurate analysis, the formulas given in the following shall be used:

$$K_M = \frac{\psi_1 K_2 + \left(1 + \frac{R}{c_4}\right) B_2 V}{\psi_2 + \psi_3 Q_2 V + \frac{R}{c_4} B_2 V^2}$$

$$K_i = \alpha_i c_1 + 2\beta_i c_2 + \frac{c_3}{\alpha_i}$$

where

$$\psi_1 = T + (1 - T)\frac{K_1}{K_2}\left(\frac{B_2}{B_1}\right)$$

$$\psi_2 = T^2 + 2T(1 - T)\frac{B_{12}}{B_1} + (1 - T)^2\frac{B_2}{B_1}$$

$$\psi_3 = T + (1 - T)\frac{Q_1}{Q_2}\left(\frac{B_2}{B_1}\right)$$

$$B_i = c_1 c_3 - \beta_i^2 c_2^2 + \alpha_i c_2 K_i$$

$$B_{12} = \left(\frac{\alpha_1^2 + \alpha_2^2}{2\alpha_1\alpha_2}\right) c_1 c_3 - \beta_1 \beta_2 c_2^2 + \frac{c_2}{2}(\gamma_1 K_2 + \gamma_2 K_1)$$

$$Q_i = \alpha_i c_1 \frac{R}{c_4} + \left(1 + \frac{R}{c_4}\right)\gamma_i c_2 + \frac{c_3}{\alpha_i}$$

The parameters of these formulas are given by the following expressions:

$$T = \frac{A_1}{A_1 + A_2}$$

$$V = \frac{A_1 A_2}{A_1 + A_2}\left(\frac{\pi^2 t_C}{b^2 G_{Ca}}\right)$$

$$R = \frac{G_{Ca}}{G_{Cb}}$$

$$A_i = \frac{t_{Fi}}{\lambda_{Fi}}\sqrt{E_{Fai} E_{Fbi}}$$

where G_{Cb} and G_{Ca} are the moduli of transverse rigidity of the core associated with the directions of the loaded and unloaded edges of the panel.

The values of c_1, c_2, c_3, and c_4 depend upon the panel aspect ratio, b/a, the integral number of longitudinal half waves, n, into which the panel buckles, and the panel edge conditions. Values of n are chosed to produce minimum values of N.

For a panel with all edges simply supported:

$$c_1 = c_4 = \frac{a^2}{n^2 b^2}, \; c_2 = 1, \text{ and } c_3 = \frac{n^2 b^2}{a^2}$$

For a panel with loaded edges simply supported and other edges clamped:

$$c_1 = \frac{16a^2}{3n^2 b^2}, \; c_2 = \frac{4}{3}, \; c_3 = \frac{n^2 b^2}{a^2}, \text{ and } c_4 = \frac{4a^2}{3n^2 b^2}$$

For a panel with loaded edges clamped and other edges simply supported:

$$\text{For } n = 1 \quad c_1 = c_4 \frac{3a^2}{4b^2}, \; c_2 = 1, \; c_3 = 4\frac{b^2}{a^2}$$

$$\text{For } n \geq 2 \quad c_1 = c_4 = \frac{a^2}{(n^2 + 1)b^2}, \; c_2 = 1, \; c_3 = \frac{n^4 + 6n^2 + 1}{n^2 + 1}\left(\frac{b^2}{a^2}\right)$$

For a panel with all edges clamped:

$$\text{For } n = 1 \quad c_1 = 4c_4 = 4\frac{a^2}{b^2}, \; c_2 = \frac{4}{3}, \; c_3 = 4\frac{b^2}{a^2}$$

For $n \geq 2$

$$c_1 = 4c_4 = \frac{16a^2}{3(n^2 + 1)b^2}, \; c_2 = \frac{4}{3}, \; c_3 \; \frac{n^4 + 6n^2 + 1}{n^2 + 1}\left(\frac{b^2}{a^2}\right)$$

Wrinkling of Sandwich facings under edgewise compressive load

Wrinkling of sandwich facings, as shown in Figure 1-D, may occur if a sandwich facing buckles as a plate on an elastic foundation. Analysis of this localized buckling behavior is complicated by unknown waviness of sandwich facings. Thus, the designer must, in effect, consider the buckling of a column (facing) that is supported on an elastic foundation (core) and that is not initially straight. The initial curvature or deflection (waviness) is not easily defined or easily measured, and attempts to correlate wrinkling data, including measured facing waviness, with theory have not been very successful. Growth of initial waves causes stresses in the core and in the bond between facings and core. Final failure may occur suddenly and the facing may buckle inward or outward, depending on the flatwise compressive strength of the core relative to the flatwise tensile strength of the bond between the facing and core. Information given here should not be used as a primary means of sandwich design, but should be used in conjunction with information on general buckling, deflection, etc. The final design should be checked to ascertain whether wrinkling of the sandwich facings might occur at design load. Because of uncertainties in analysis and values of material properties, it is recommended that the final design be checked by tests of a few small specimens.

The stress at which wrinkling of sandwich facings on a continuous core such as foams and balsa will occur is given approximately by the formula:

$$\text{Facing wrinkling stress, } F_W = Q\left(\frac{E_F E_C G_C}{\lambda_F}\right)^{\frac{1}{3}}$$

E_F is compressive facing elastic modulus in the direction of the applied load.

E_C is core elastic compression modulus in a direction normal to the sandwich facings; G_C is core shear modulus associated with shear distortion in the plane perpendicular to the facings and parallel to the direction of applied load; and Q is the relative minimum with respect to ζ of the expression:

$$Q = \frac{\dfrac{\zeta^2}{30q^2} + \dfrac{16q}{\zeta}\left(\dfrac{\cosh\zeta - 1}{11\sinh\zeta + 5}\right)}{1 + 6.4K\zeta\left(\dfrac{\cosh\zeta - 1}{11\sinh\zeta + 5}\right)}$$

where

$$q = \frac{t_C}{t_F} G_C \left(\frac{\lambda_F}{E_F E_C G_C}\right)^{\frac{1}{3}}$$

$$K = \frac{\delta E_F}{t_F F_C}$$

δ is initial deflection of facing waviness; and F_C is flatwise sandwich strength (the lesser of flatwise core compression or sandwich flatwise tension). The parameter ζ is proportional to the fourth root of the ratio of the core elastic moduli and to the ratio of the core thickness to the ideal buckle wavelength.

A graphical presentation of Q is given in Figure 17. The graph can be entered at known values of the abscissa, q, and the ordinate, Q, determined after choosing an estimated K curve. Present state of the art does not permit a suitable choice for values of δ. If test values of wrinkling stresses are known, the graph of Figure 17 can be used to determine which K curve fits the data and then compute values of δ from the above formula. Changes in design for similar sandwich can then be made by assuming δ to be a constant for that particular type of sandwich and then using the graph of Figure 17 to redesign.

In the event data is not available it is recommended that the value q be calculated. Then if q is greater than 0.5 use the value of Q at q = 0.5 which is 0.26 as a maximum value. If q is less than 0.5, Q is not dependent on the K factor.

Wrinkling will generally not occur on most sandwich panels composed of relatively thick facings such as those used for most ship structures unless a very soft core is used. The above formulas are more applicable to very thin faces such as isotropic aluminum panels or GRP with only one or two plies of reinforcement.

(2) Flat rectangular sandwich panels under edgewise shear loads

Formulas to determine minimum face thickness

$$t_{F1} F_{F1} + t_{F2} F_{F2} = N_s \quad \text{(unequal facings)}$$

$$t_F = \frac{N_s}{2 F_F} \quad \text{(equal facings)}$$

F_F is the chosen design facing shear stress; N_s is the design shear load per unit length of panel edge.

These formulas assume both faces are of the same material. If not, see reference (2).

Critical shear buckling

Formulas given for compression buckling apply also to shear buckling except for coefficients K_M and K_{MO} use Figures 18 to 23.

On the figures note that $K_{MO} = K_M$ for $V = 0$.

Curves for clamped sandwich panels with orthotropic cores are approximate because they were obtained by multiplying buckling coefficients for simply supported orthotropic sandwich by the ratio of clamped to simply supported buckling coefficients for isotropic sandwich.

The curves given are applicable to the same α, β, and γ values noted for compression buckling. For isotropic facings it was assumed that $\mu = 0.25$. For orthotropic facings it was assumed that $\mu_{ab} = \mu_{ba} = 0.2$, $E_{Fa} = E_{Fb}$, and $G_{Fba} = 0.21 E_{Fa}$.

If the figures do not apply because of large variances in the properties assumed, the formulas given in reference (d) can be used.

9110-9-i Rectangular flat sandwich panels under uniform normal loading

Detailed procedures giving theoretical formulas and graphs for determining dimensions of the facings and core for resisting bending from normal loads for simply supported panels are given in the following paragraphs. Double formulas are given, one formula for sandwich with isotropic facings of different materials and thicknesses and another formula for sandwich with each isotropic facing of the same material and thickness. Facing moduli of elasticity, $E_{F1,2}$, and stress values, $F_{F1,2}$, shall be compression or tension. Data for clamped edge panels are not presently available.

The following formulas are for determining sandwich facing and core thicknesses and core shear modulus so that chosen design facing stresses and allowable panel deflections will not be exceeded. The facing stresses, produced by bending moment, are maximum at the center of a simply supported panel under uniformly distributed normal load. If restraint exists at panel edges, a redistribution of stresses may cause higher stresses near panel edges. The procedures given apply only to panels with simply supported edges. Because facing stresses are caused by bending moment, they depend not only upon facing thickness but also upon the distance the facings are spaced, hence core thickness. Panel stiffness, hence deflection, is also dependent upon facing and core thickness.

If the panel is designed so that facing stresses are at chosen design levels, the panel deflection may be larger than allowable, in which case the core or facings must be thickened and the design facing stress lowered in order to meet deflection requirements. A solution is presented in the form of charts with which, by iterative process, the facing and core thicknesses and core shear modulus can be determined.

The average facing stress, F (stress at facing centroid), is given by the theoretical formula:

$$F_{F1,2} = K_2 \frac{pb^2}{ht_{F1,2}} \quad \text{(unequal facings)}$$

$$F_F = K_2 \frac{pb^2}{ht_F} \qquad \text{(equal facings)}$$

where p is the intensity of the distributed load; b is the panel width; h is the distance between facing centroids; t is facing thickness; 1 and 2 are subscripts denoting facings 1 and 2; and K_2 is a theoretical coefficient dependent on panel aspect ratio, and sandwich bending and shear rigidities. If the core is isotropic (shear moduli alike in the two principal directions), K_2 values depend only upon panel aspect ratio. The values of K_2 for sandwich with orthotropic core and dependent not only on panel aspect ratio but also upon sandwich bending and shear rigidities as incorporated in the parameter V. Formulas for V are as given previously herein.

The deflection, δ, of the panel center is given by the theoretical formula:

$$\delta = \frac{K_1}{K_2}\left(\frac{F_{F1,2}}{E_{F1,2}}\right)\left(1 + \frac{E_{F1,2}t_{F1,2}}{E_{F2,1}t_{F2,1}}\right)\frac{b^2}{h} \quad \text{(unequal facings)}$$

$$\delta = 2\frac{K_1}{K_2}\left(\frac{\lambda F_F}{E_F}\right)\left(\frac{b^2}{h}\right) \qquad \text{(equal facings)}$$

where K_1 is a coefficient dependent upon panel aspect ratio and the value of V.

Core shear stress, F_{Cs} is given by:

$$F_{Cs} = K_3 p \frac{b}{h}$$

Where K_3 is a theoretical coefficient dependent on panel aspect ratio and the parameter V. If the core is isotropic, values of V do not affect the core shear stress.

Computation of coefficients K_1, K_2, and K_3

Coefficients necessary to compute facing stress, core shear stress and panel deflection are given in Figures (24), (25) and (26).

If the charts do not apply because ratios of core shear moduli are far different from those given on the charts, or it is desired to check by a more accurate analysis, the formulas given if reference (e) shall be used.

Special core configurations

If the core is composed of a foam-GRP web configuration as shown in Figure 27 the above formulas apply if combined foam-web properties are known. If not, the formulas can be modified to include the section modulus of the actual panel face and GRP webs and ignoring the light weight core.

9110-9-j Rectangular flat sandwich panels under concentrated loads

Data have not been developed for concentrated type loads on sandwich panels but these members can be designed by assuming the panel as a simple plate. Bending moments are determined using any standard strength of material text. For concentrated loads such as aircraft landing loads the methods given in reference (f) may be used to determine the moments.

Stresses are determined by computing the section modulus of a unit strip of the panel ignoring the core. For computing deflection, however, the shear deflection of the core must be included. If the core is a foam-GRP webbing type as shown on Figure 27, the web is also included in the section modulus computation.

In addition to the overall bending moment and panel shear consideration the panel core must be sufficient to resist direct compression under the concentrated load. Generally, data is available on the compressive strength of solid cores. Once the load is known the core density can be determined to provide the required compressive strength. For the foam-webbing type core the webs can be designed as a short column, ignoring the core. The cores for these panels are usually a low density core, used more as a form for the webs and to prevent web buckling.

9110-9-k Figures for use with design formulas

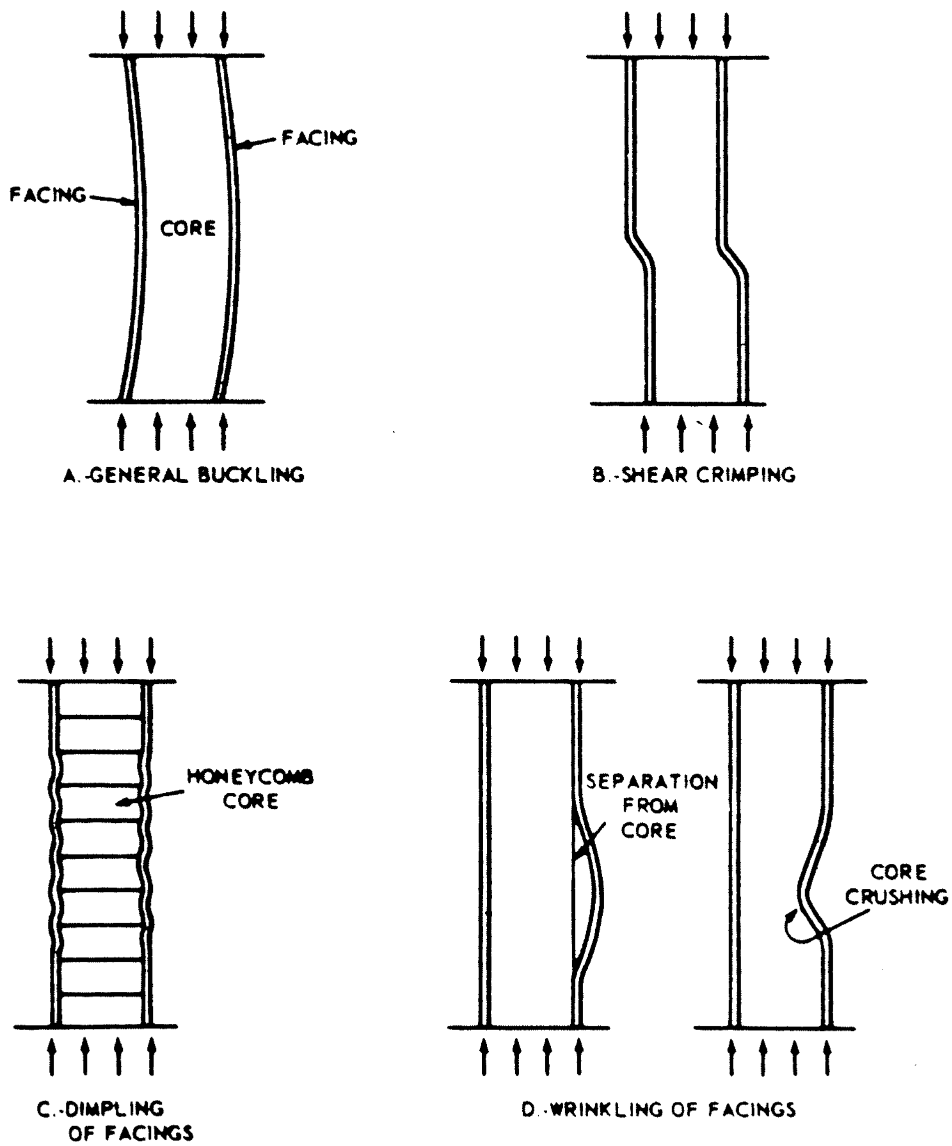


FIGURE 1

Modes of failure of sandwich panels

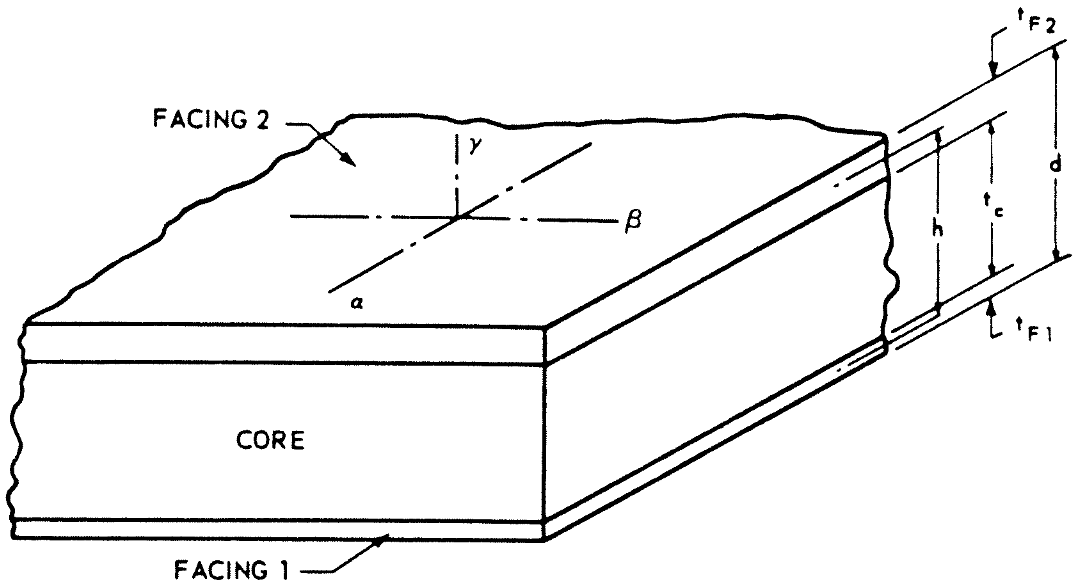


FIGURE 2

Notation for sandwich panels

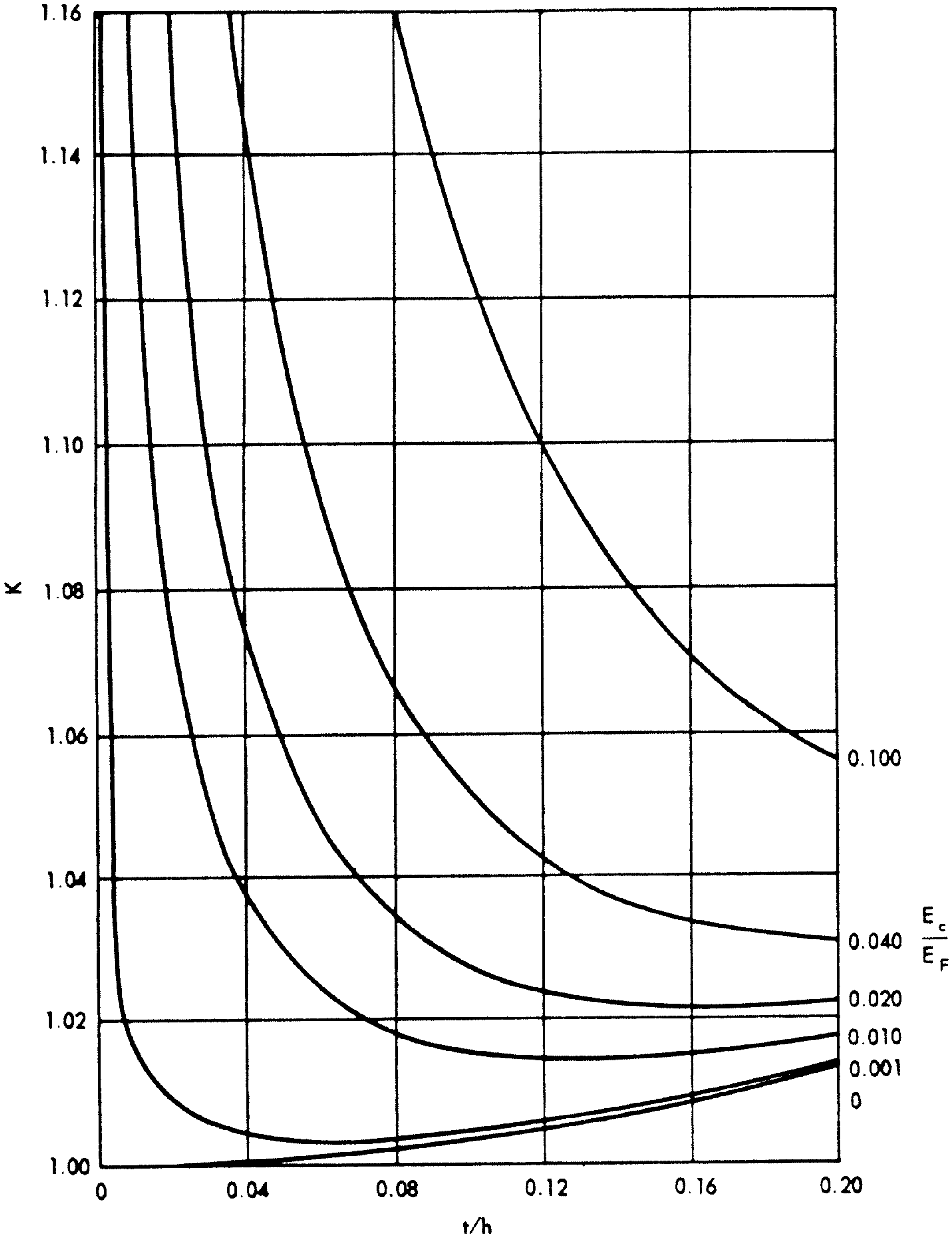


FIGURE 3

Coefficient for bending stiffness factor

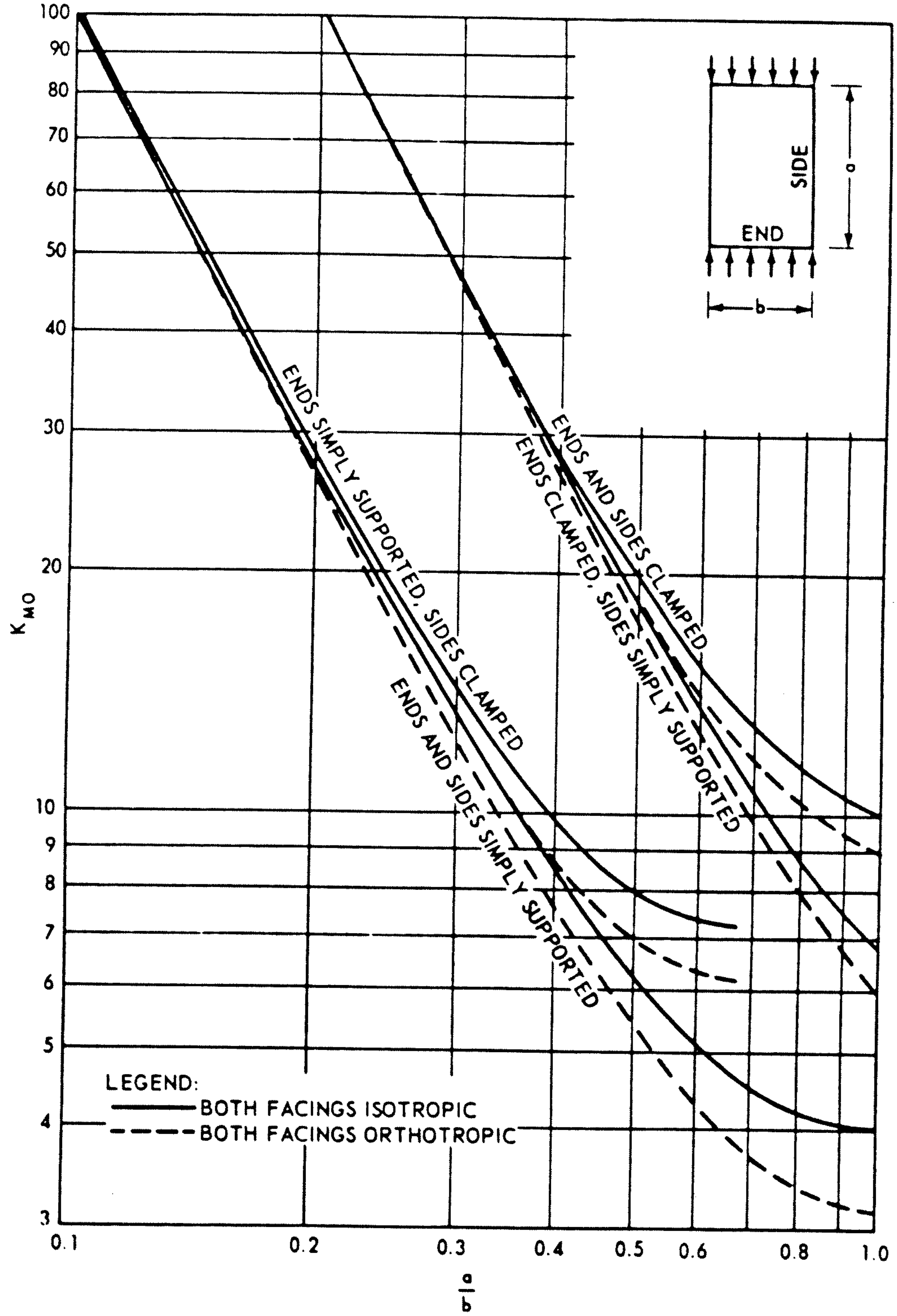


FIGURE 4

Values of K_{MO} for sandwich panels in edgewise compression

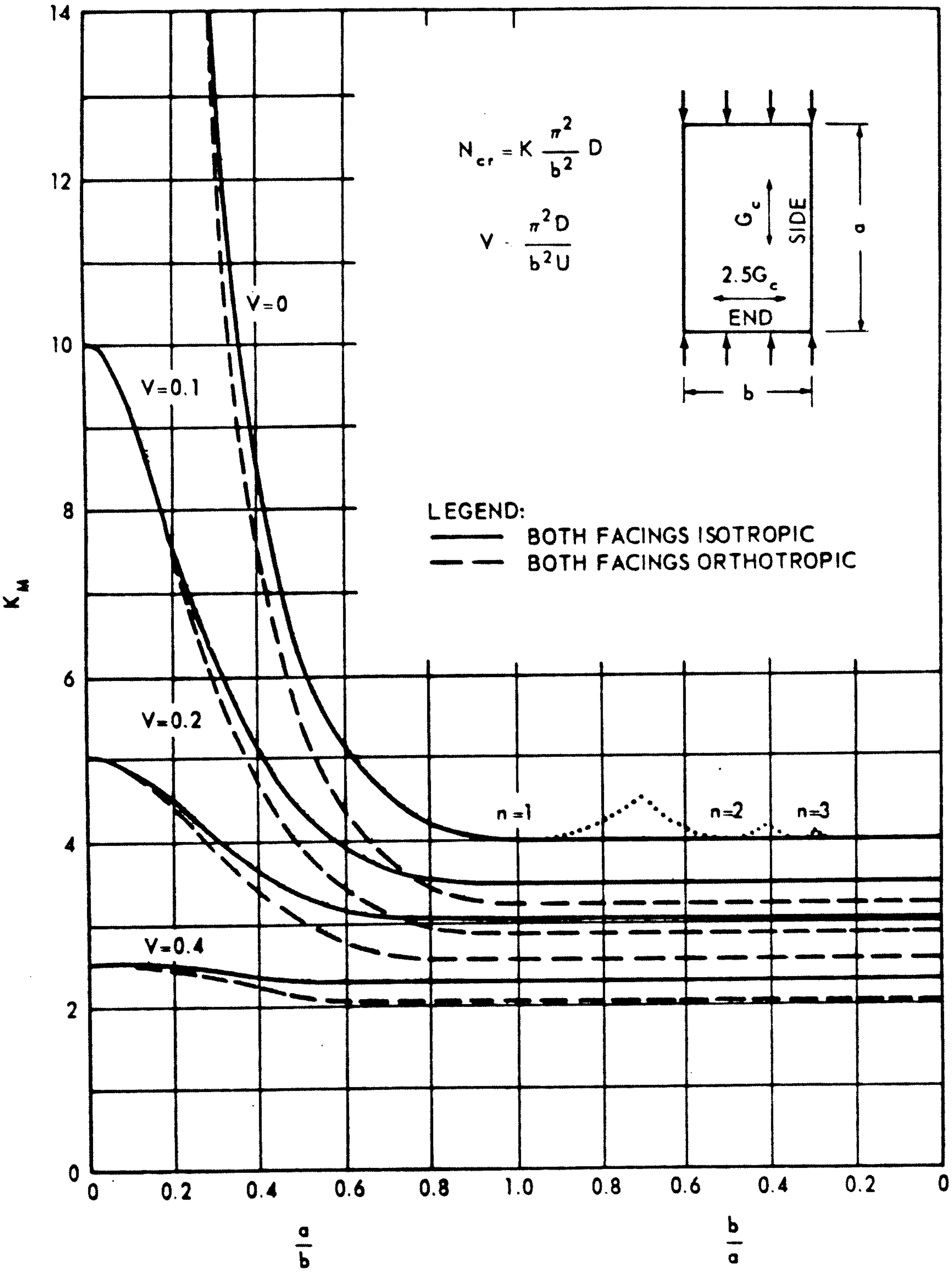


FIGURE 5

K_M for sandwich panel with ends and sides simply supported and orthotropic core. ($G_{Cb} = 2.5\ G_{Ca}$).

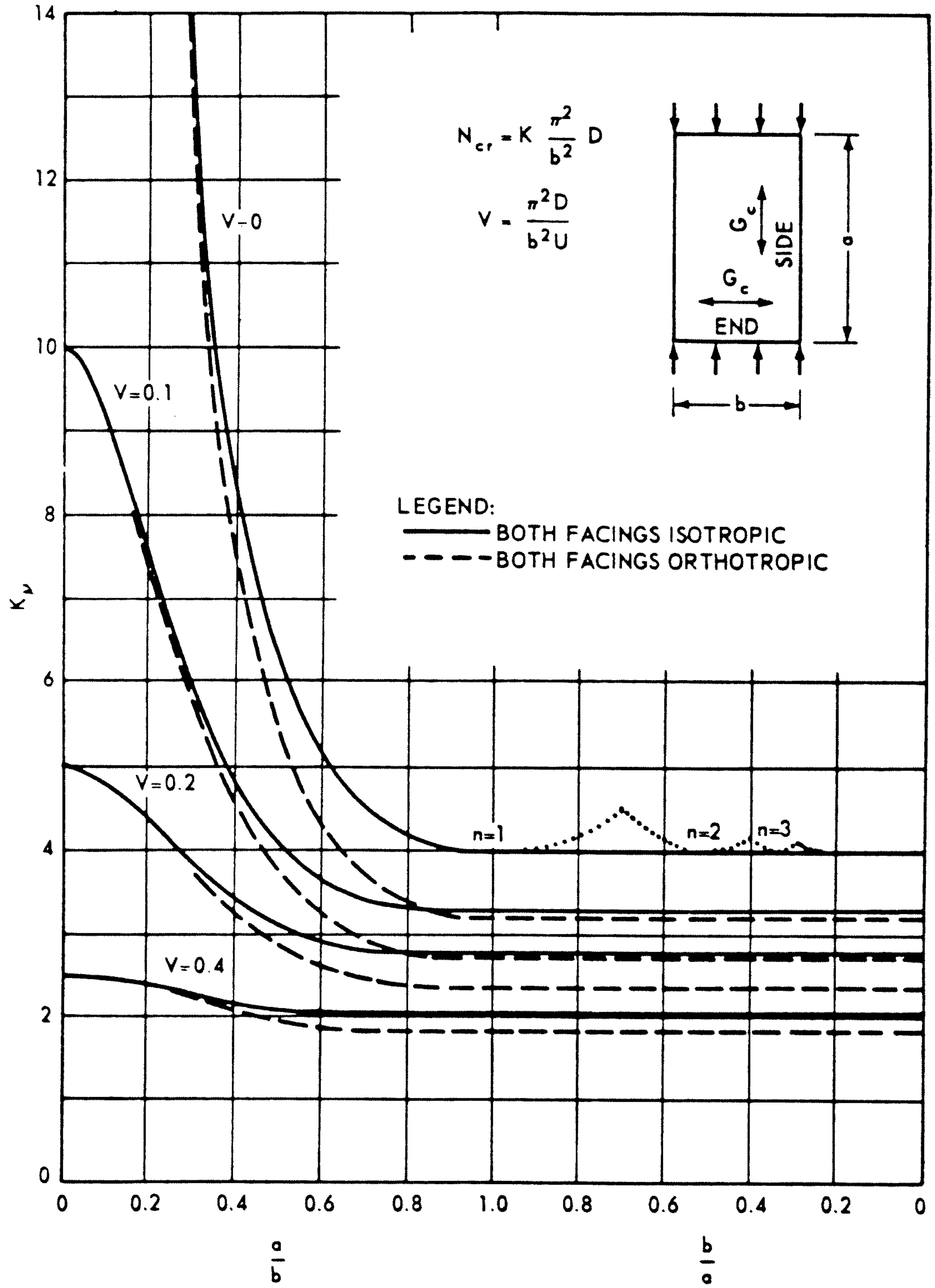


FIGURE 6

K_M for sandwich panel with ends and sides simply supported and isotropic core. $(G_{Cb} = G_{Ca})$.

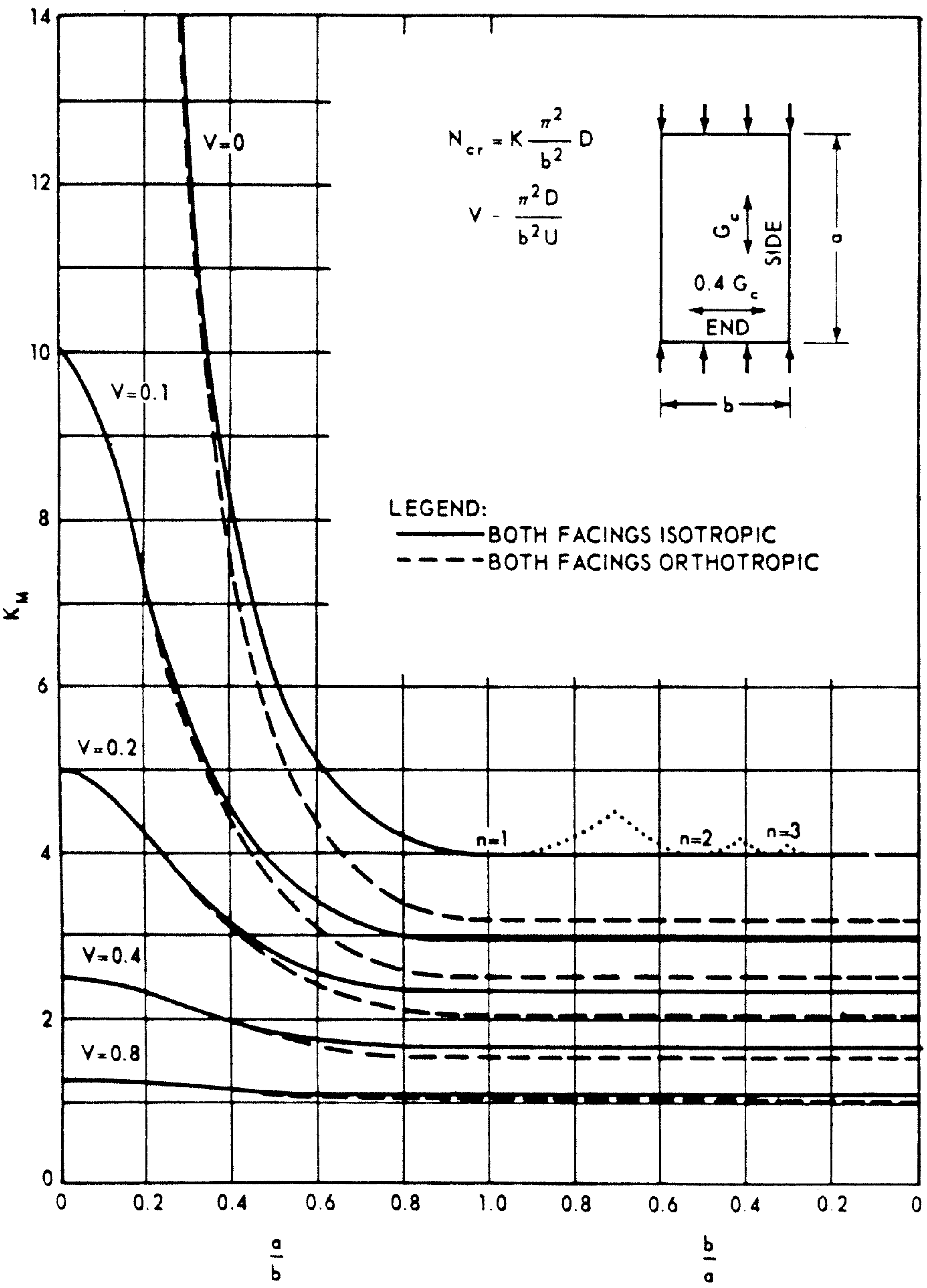


FIGURE 7

K_M for sandwich panel with ends and sides simply supported and orthotropic core. ($G_{Cb} = 0.4\ G_{Ca}$).

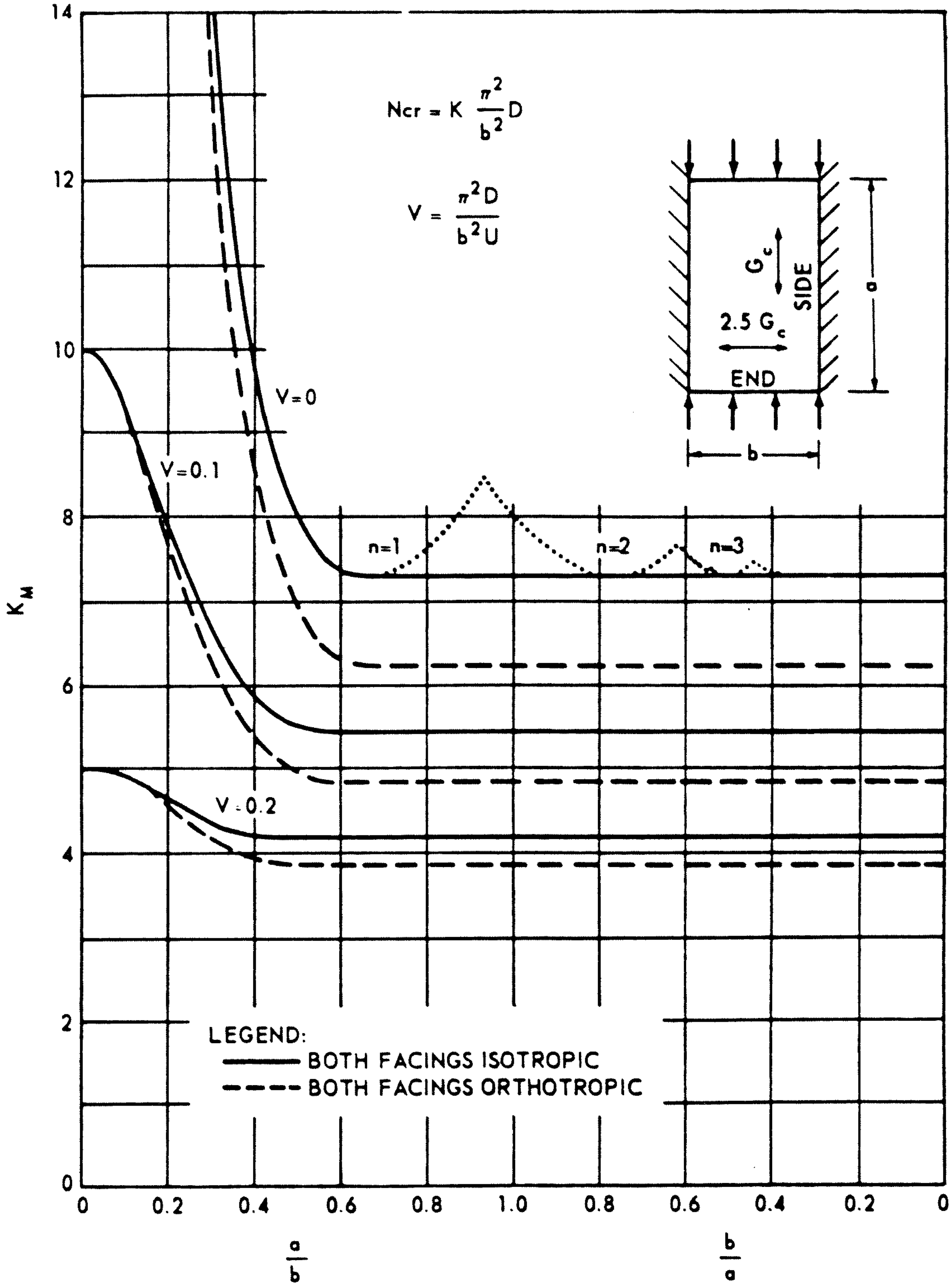


FIGURE 8

K_M for sandwich panel with ends simply supported and sides clamped, and orthotropic core. (G_{Cb} = 2.5 G_{Ca}).

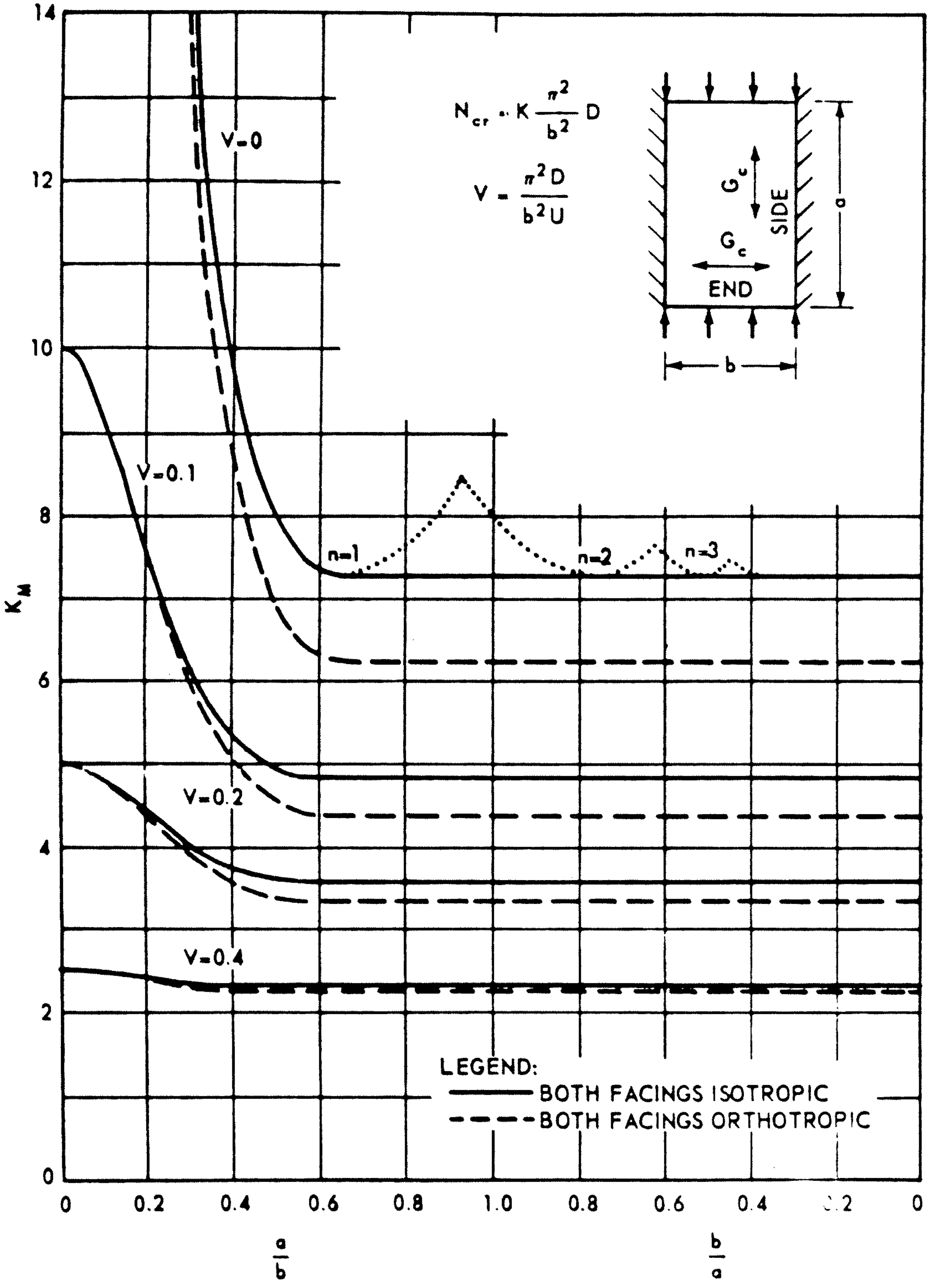


FIGURE 9

K_M for sandwich panel with ends simply supported and sides clamped, and isotropic core. ($G_{Cb} = G_{Ca}$).

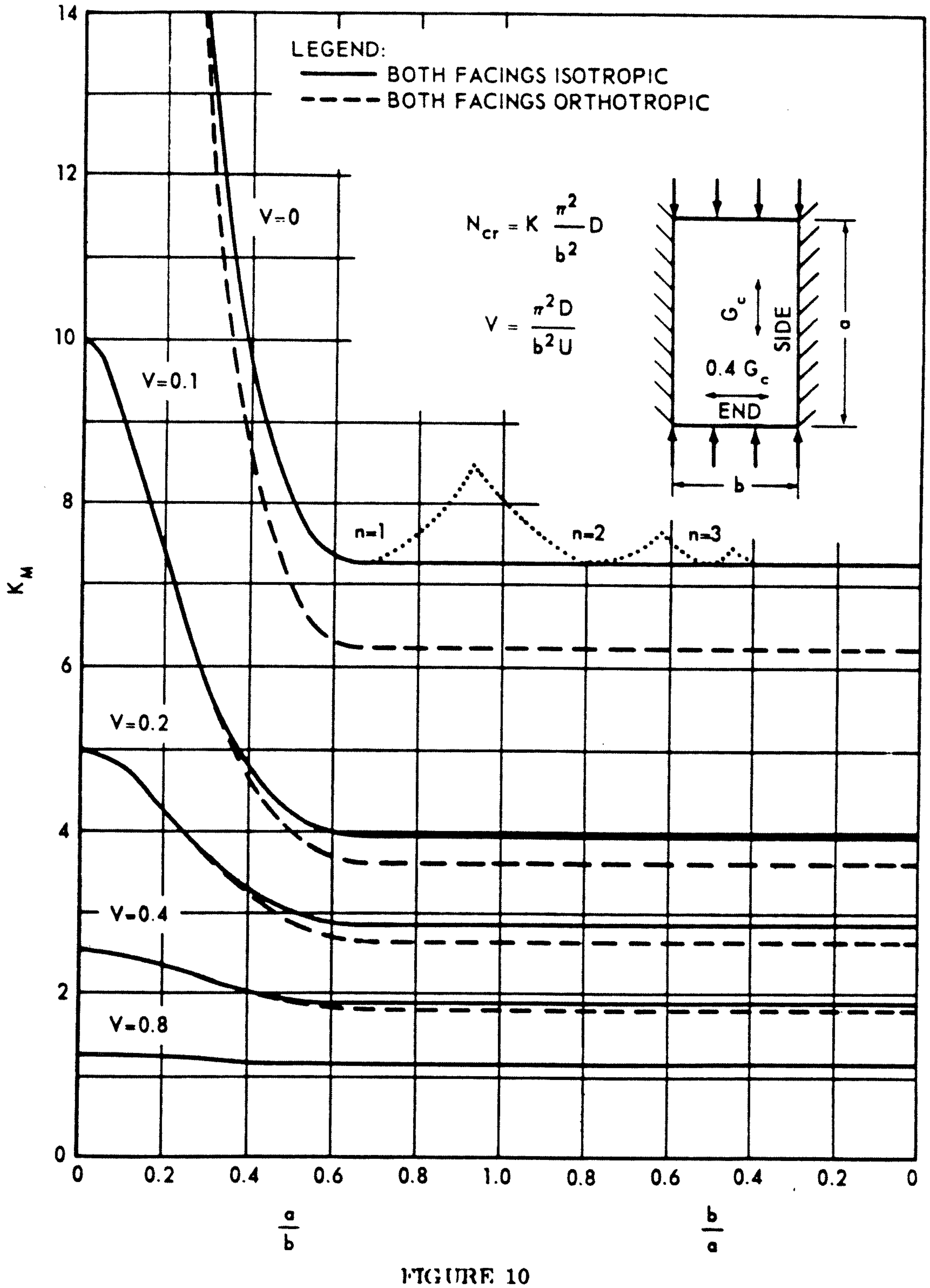


FIGURE 10

K_M for sandwich panel with ends simply supported and sides clamped, and orthotropic core. (G_{Cb} = 0.4 G_{Ca}).

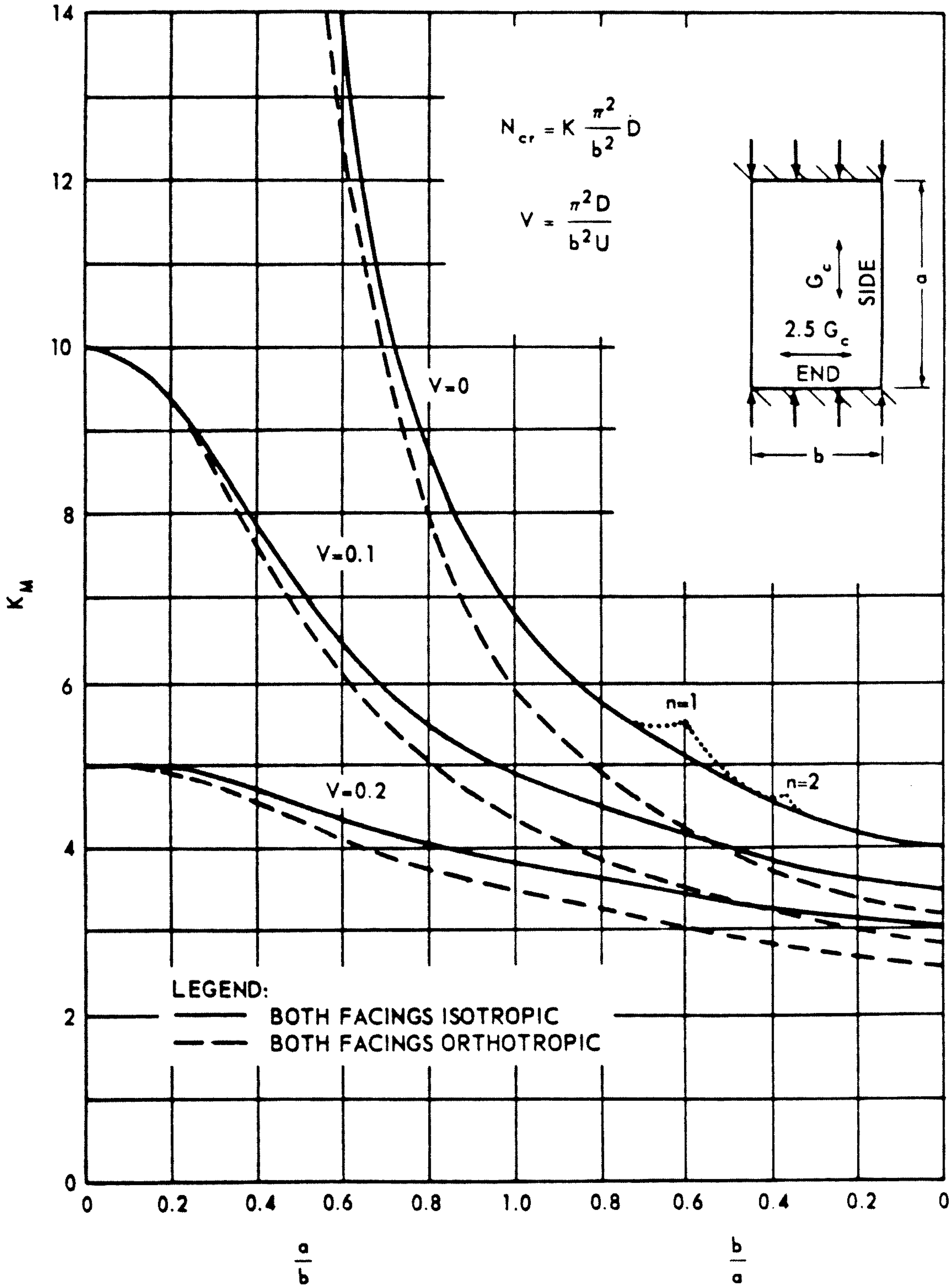


FIGURE 11

K_M for sandwich panel with ends clamped and sides simply supported, and orthotropic core. (G_{Cb} = 2.5 G_{Ca}).

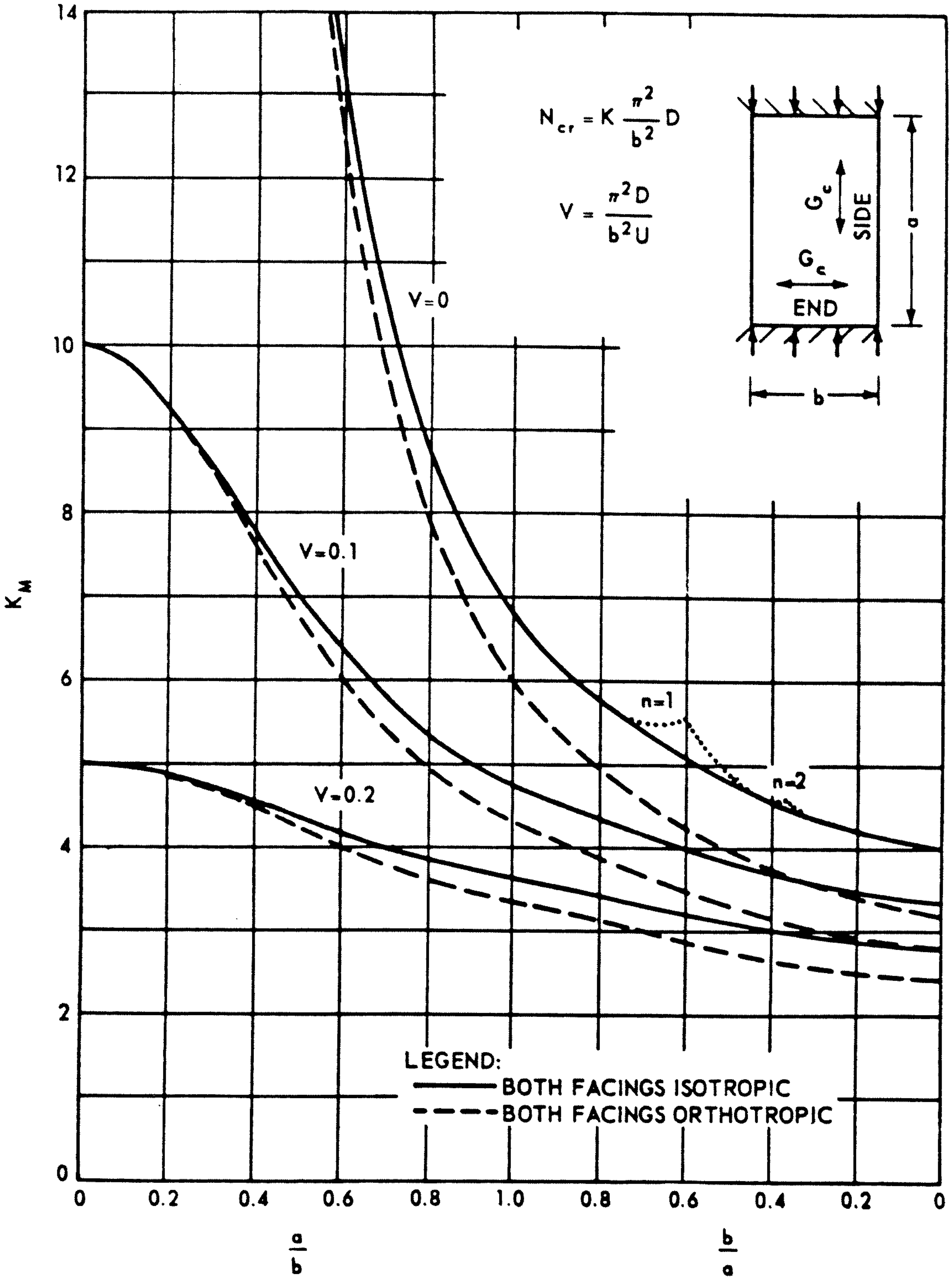


FIGURE 12

K_M for sandwich panel with ends clamped and sides simply supported, and isotropic core. ($G_{Cb} = G_{Ca}$).

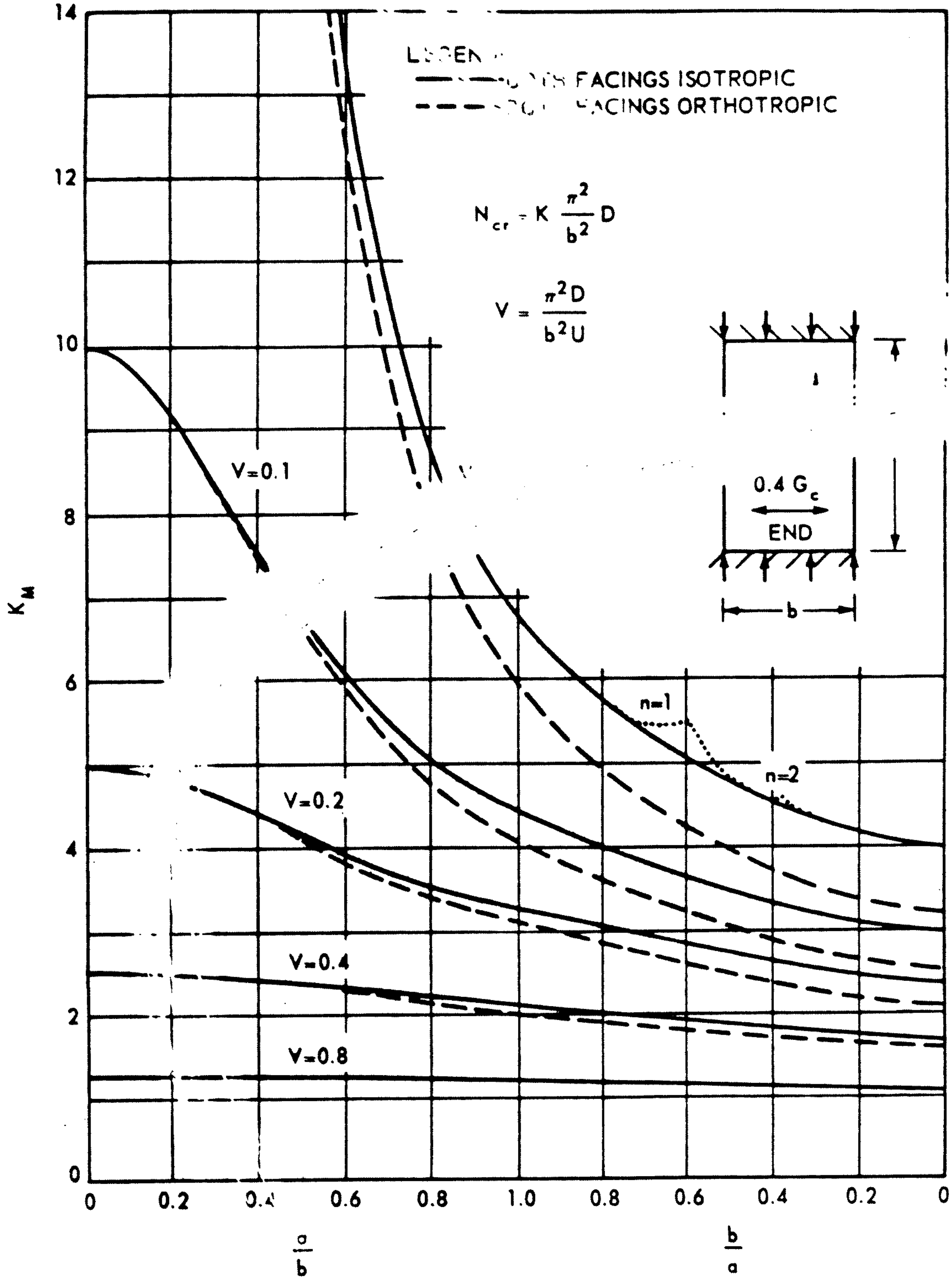


FIGURE 13

K_M for sandwich panel with ends clamped and sides simply supported, and orthotropic core. ($G_{Cb} = 0.4\ G_{Ca}$).

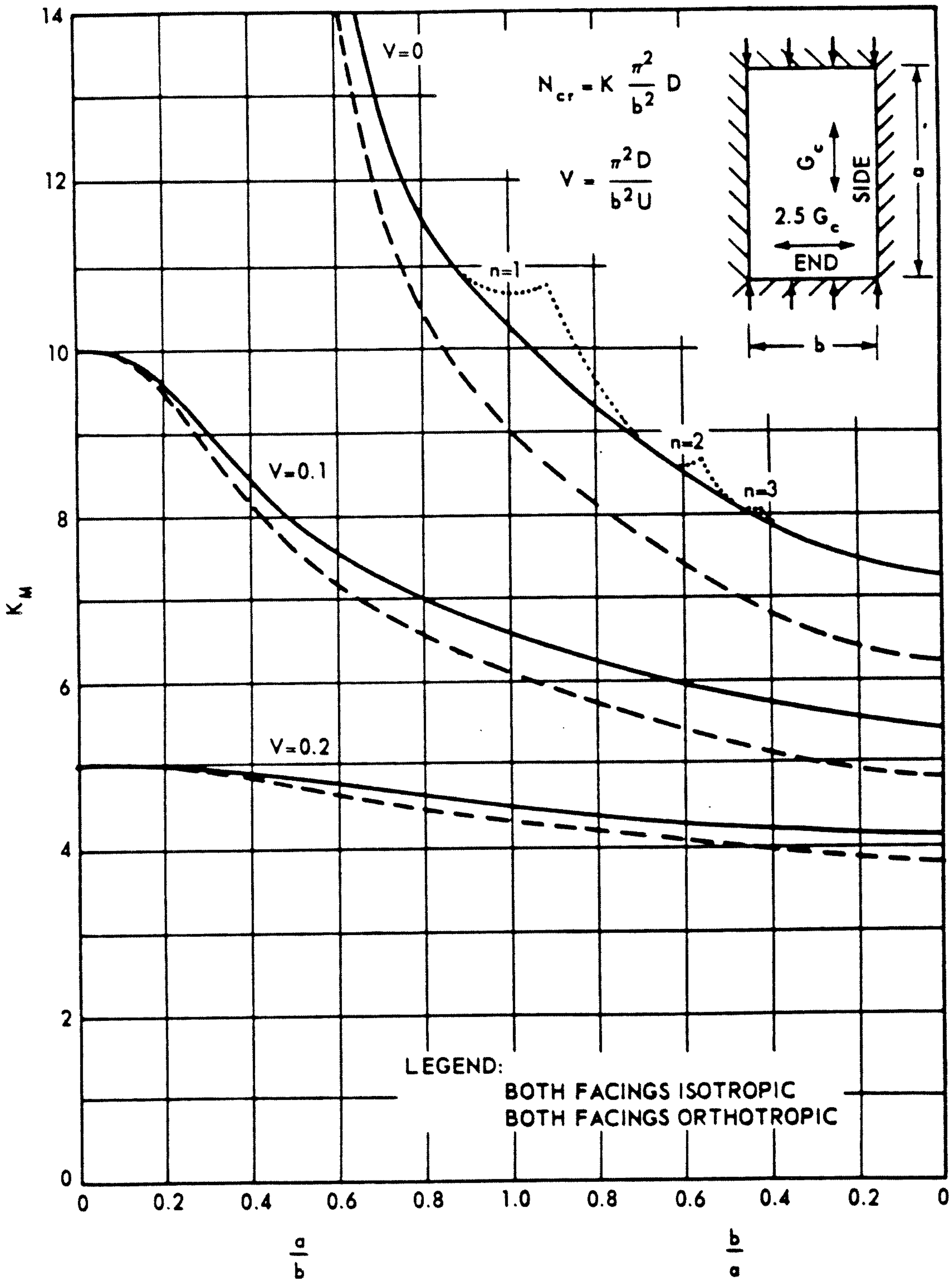


FIGURE 14

K_M for sandwich panel with ends and sides clamped, and orthotropic core. ($G_{Cb} = 2.5\ G_{Ca}$).

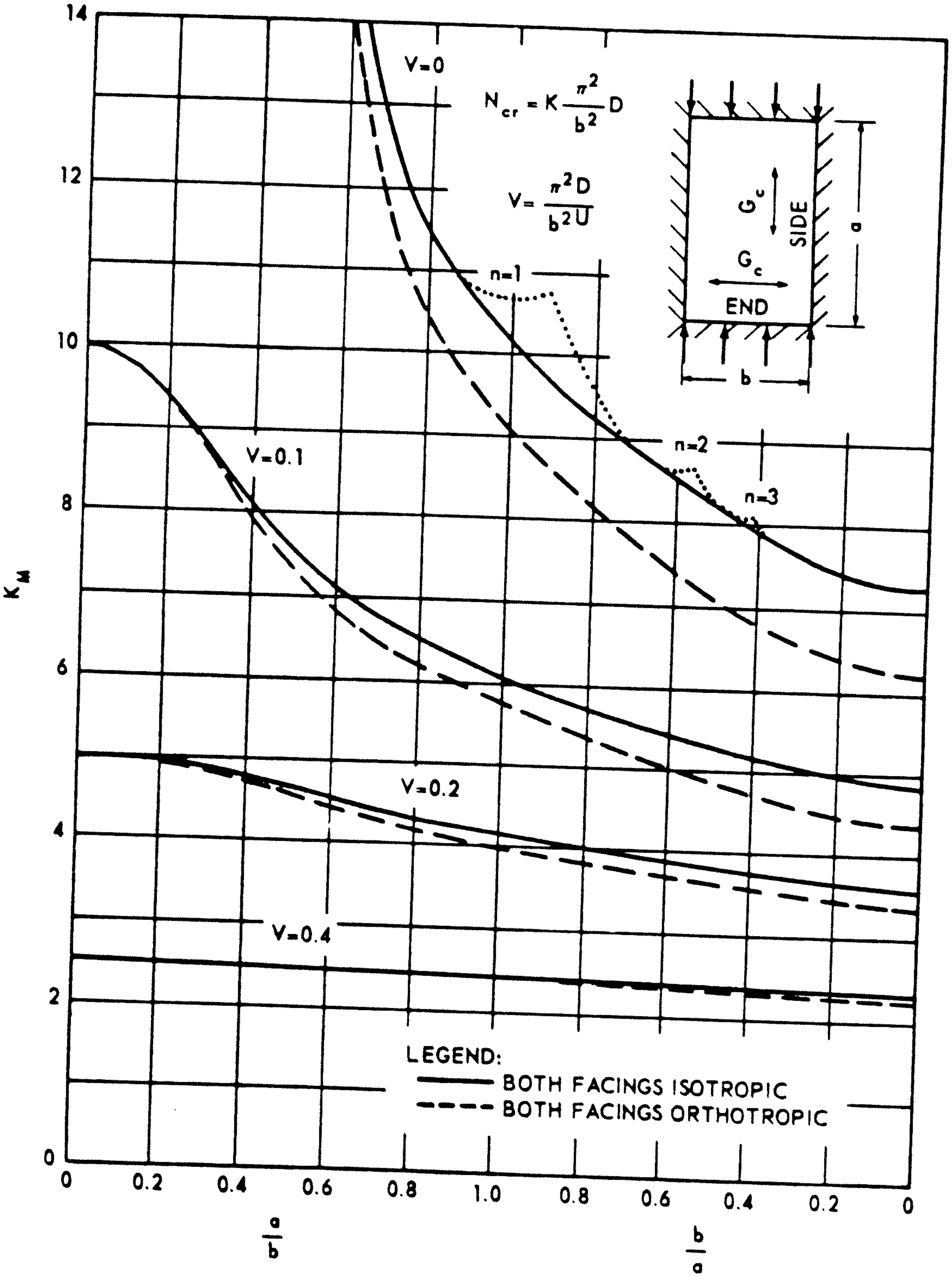


FIGURE 15

K_M for sandwich panel with ends and sides clamped, and isotropic core. ($G_{Cb} = C_{Ca}$).

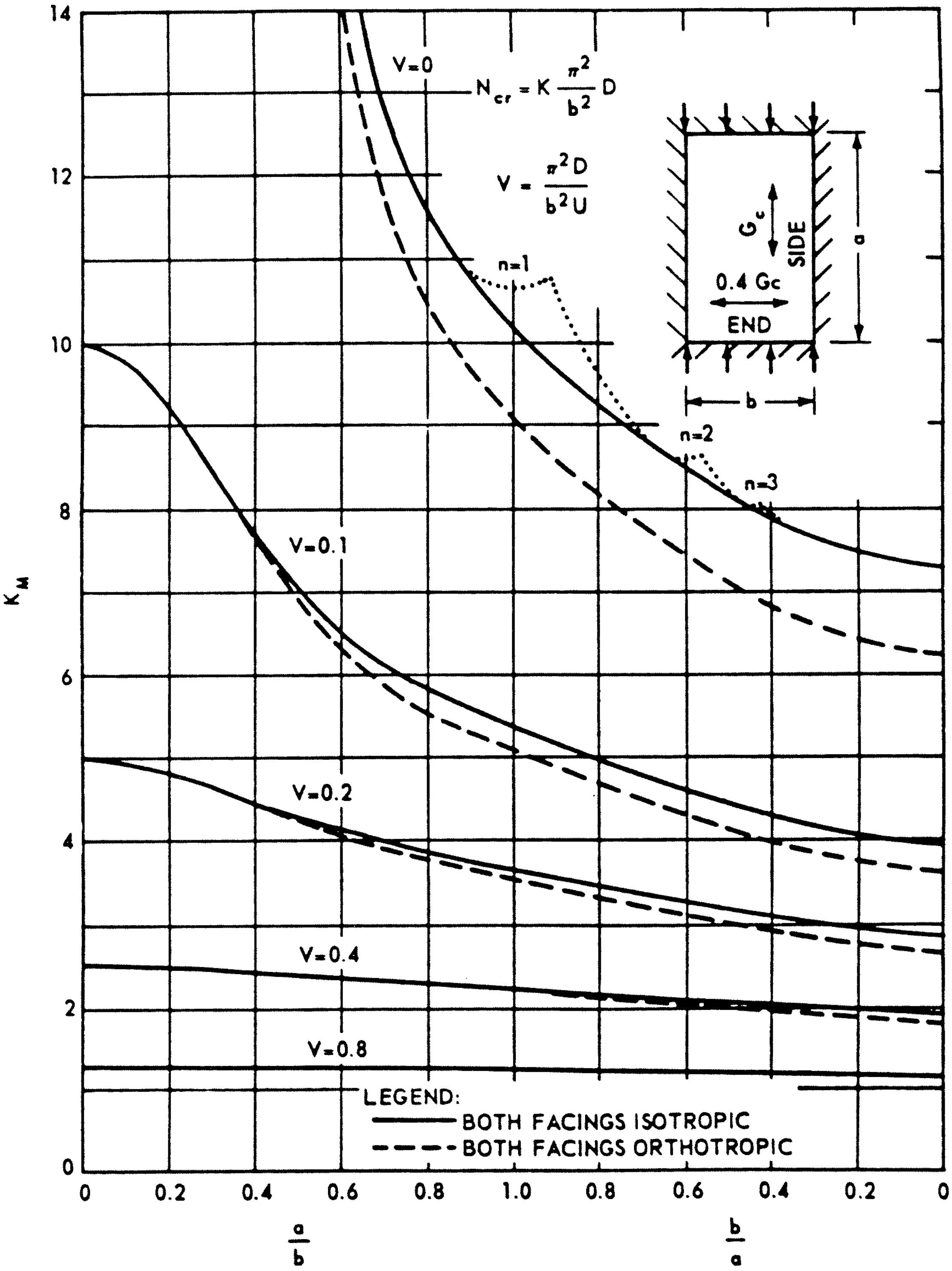


FIGURE 16

K_M for sandwich panel with ends and sides clamped, and orthotropic core. ($G_{Cb} = 0.4\ G_{Ca}$).

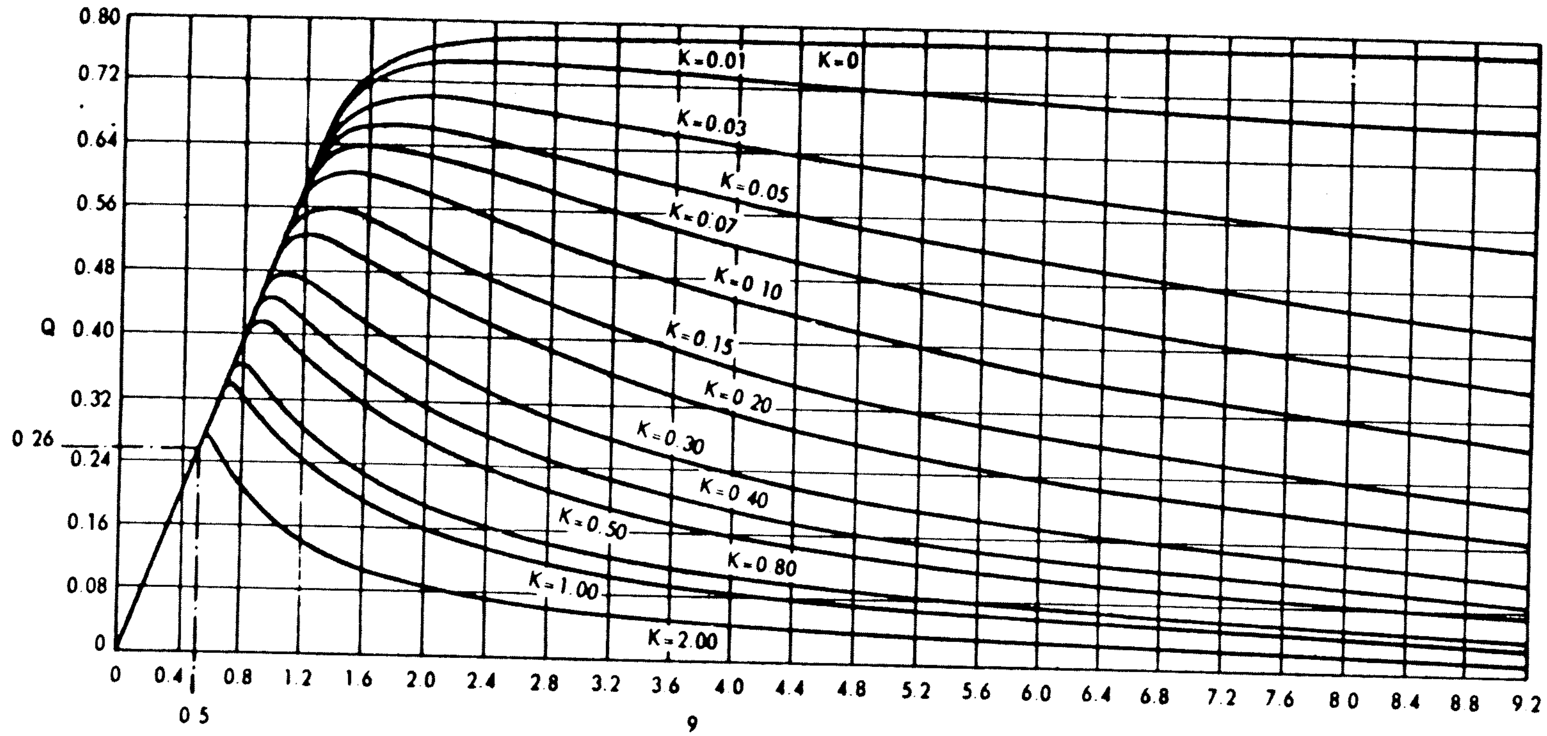


FIGURE 17

Parameters for face wrinkling formulas

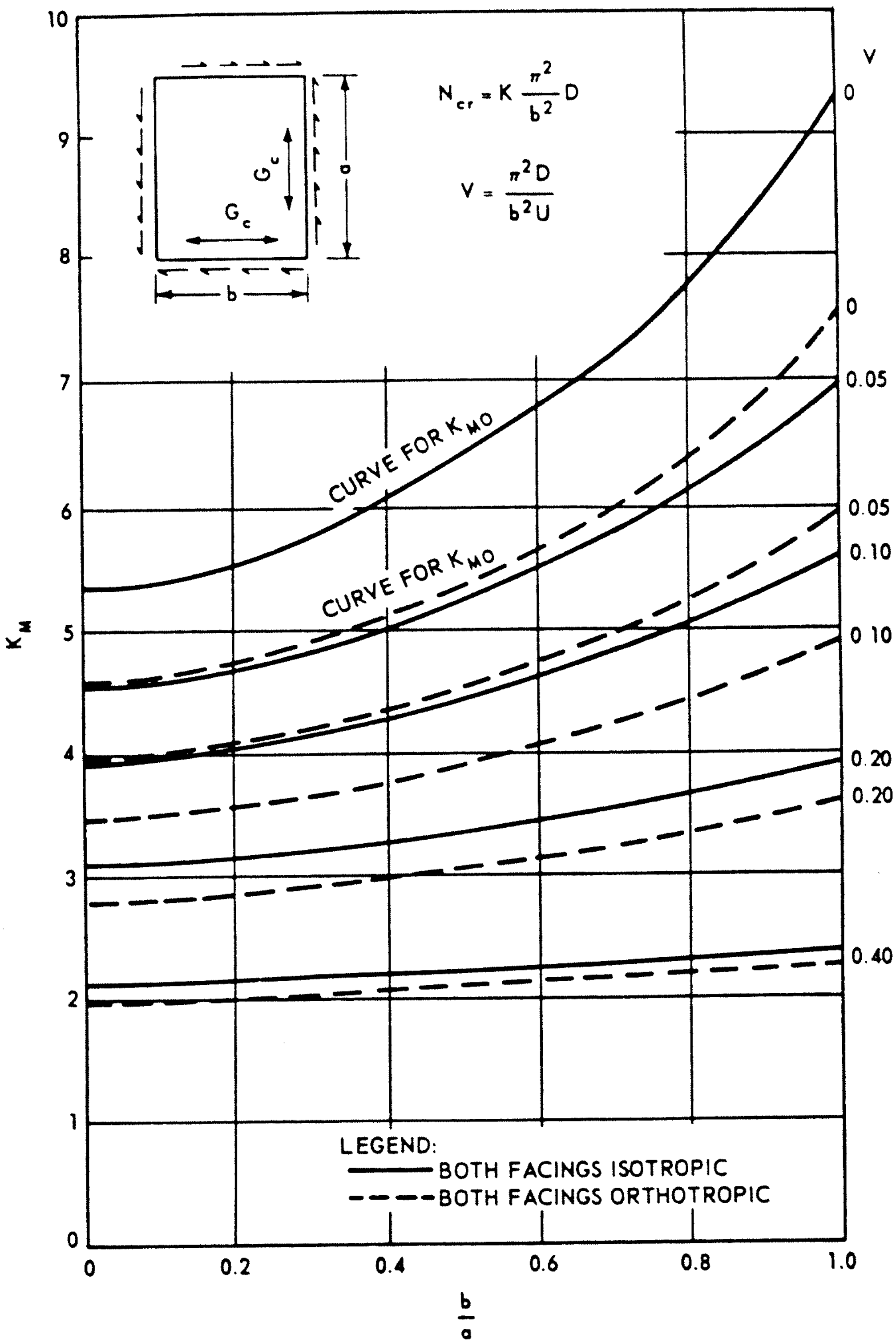


FIGURE 18

K_M for sandwich panel with all edges simply supported, and isotropic core.

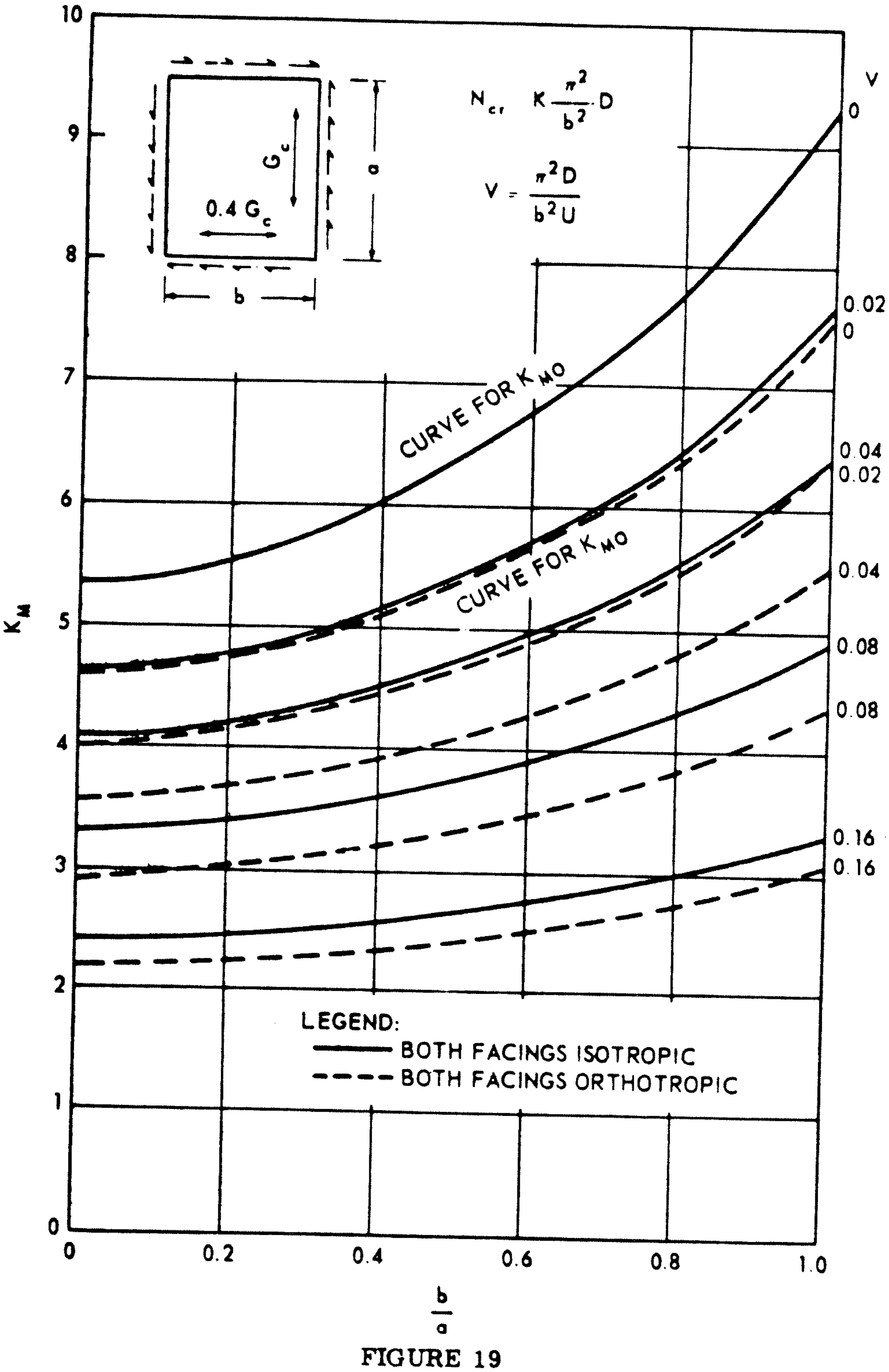


FIGURE 19

K_M for sandwich panel with all edges simply supported, and orthotropic core. ($G_{Cb} = 0.4\ G_{Ca}$).

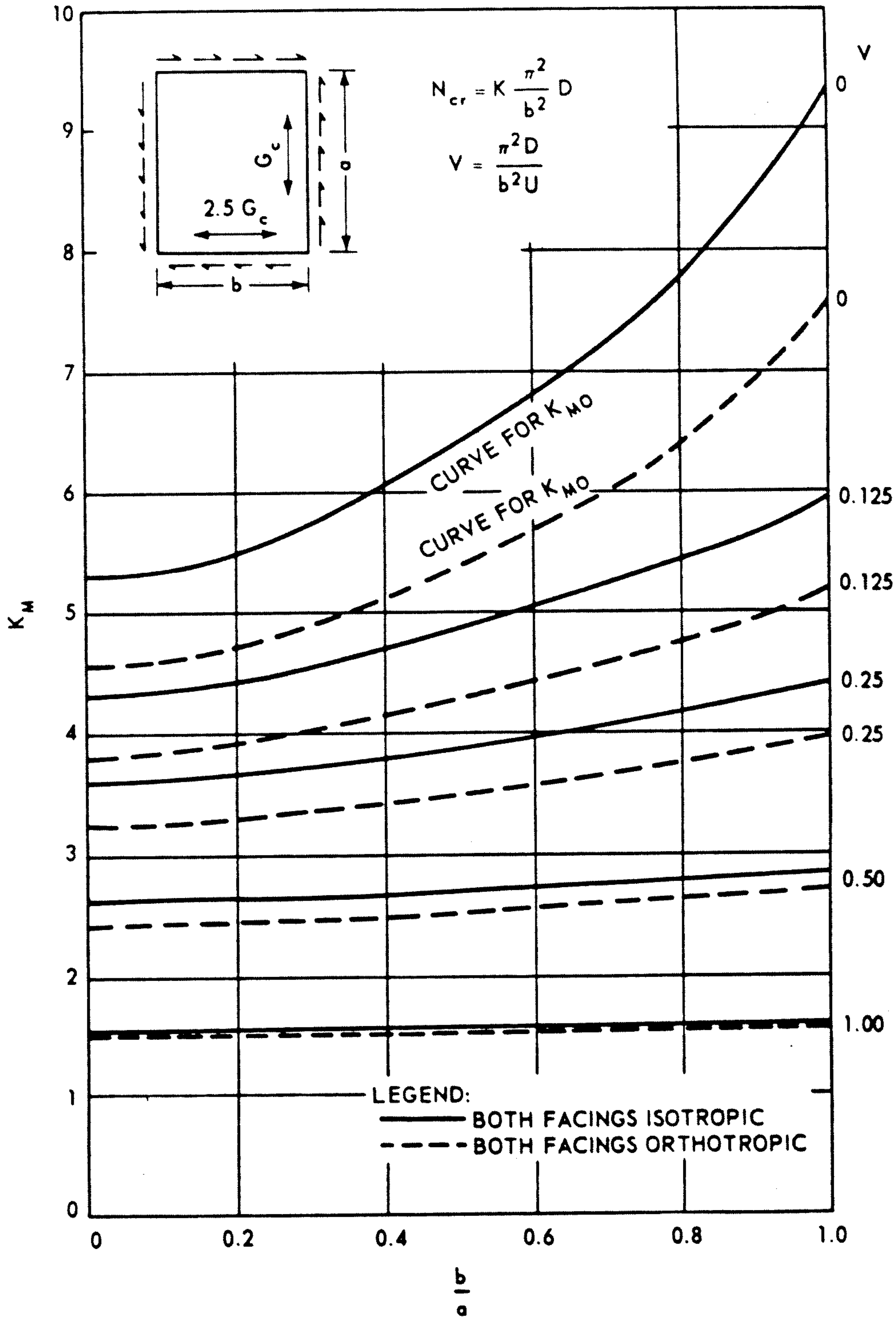


FIGURE 20

K_M for sandwich panel with all edges simply supported, and orthotropic core. (G_{Cb} = 2.5 G_{Ca}).

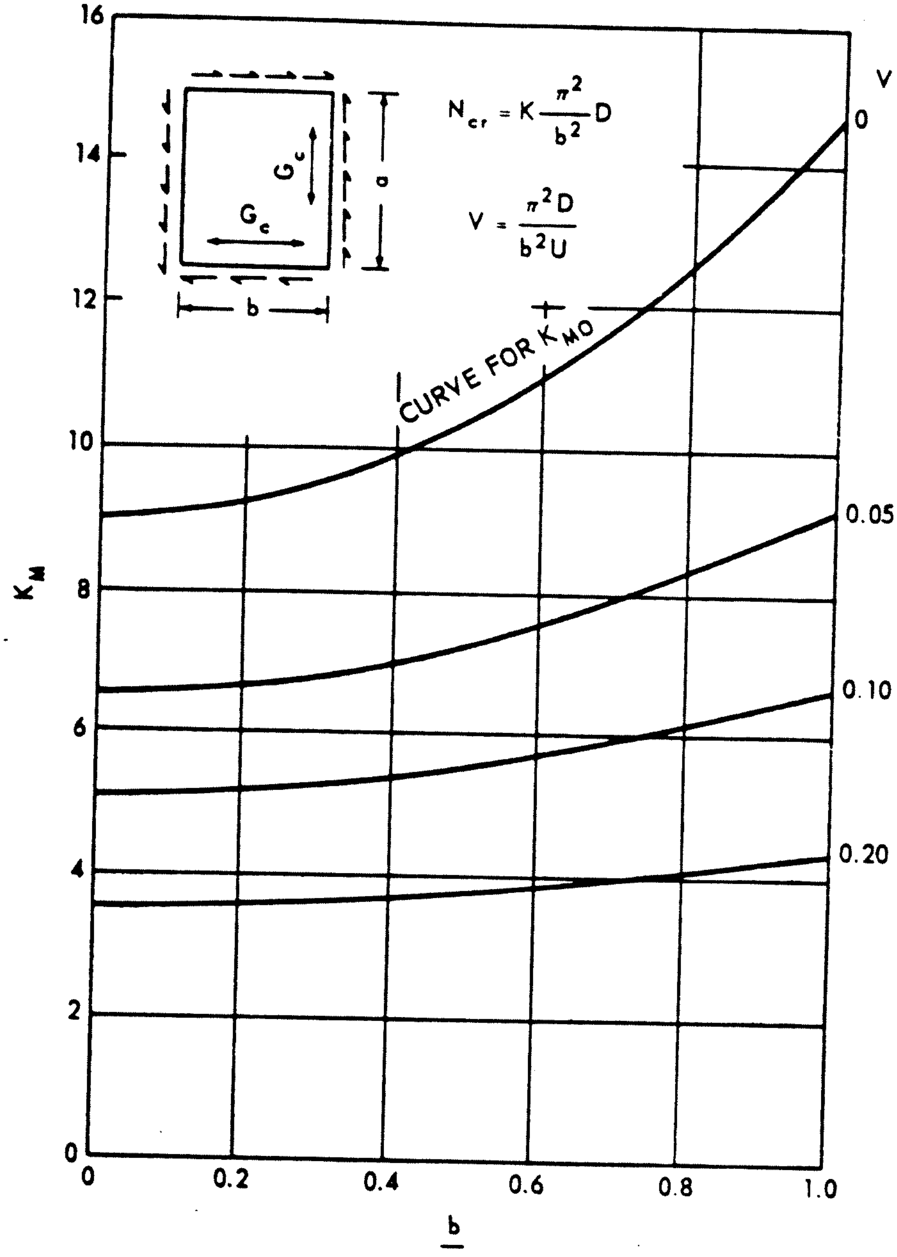


FIGURE 21

K_M for sandwich panel with all edges clamped, isotropic facings and isotropic core.

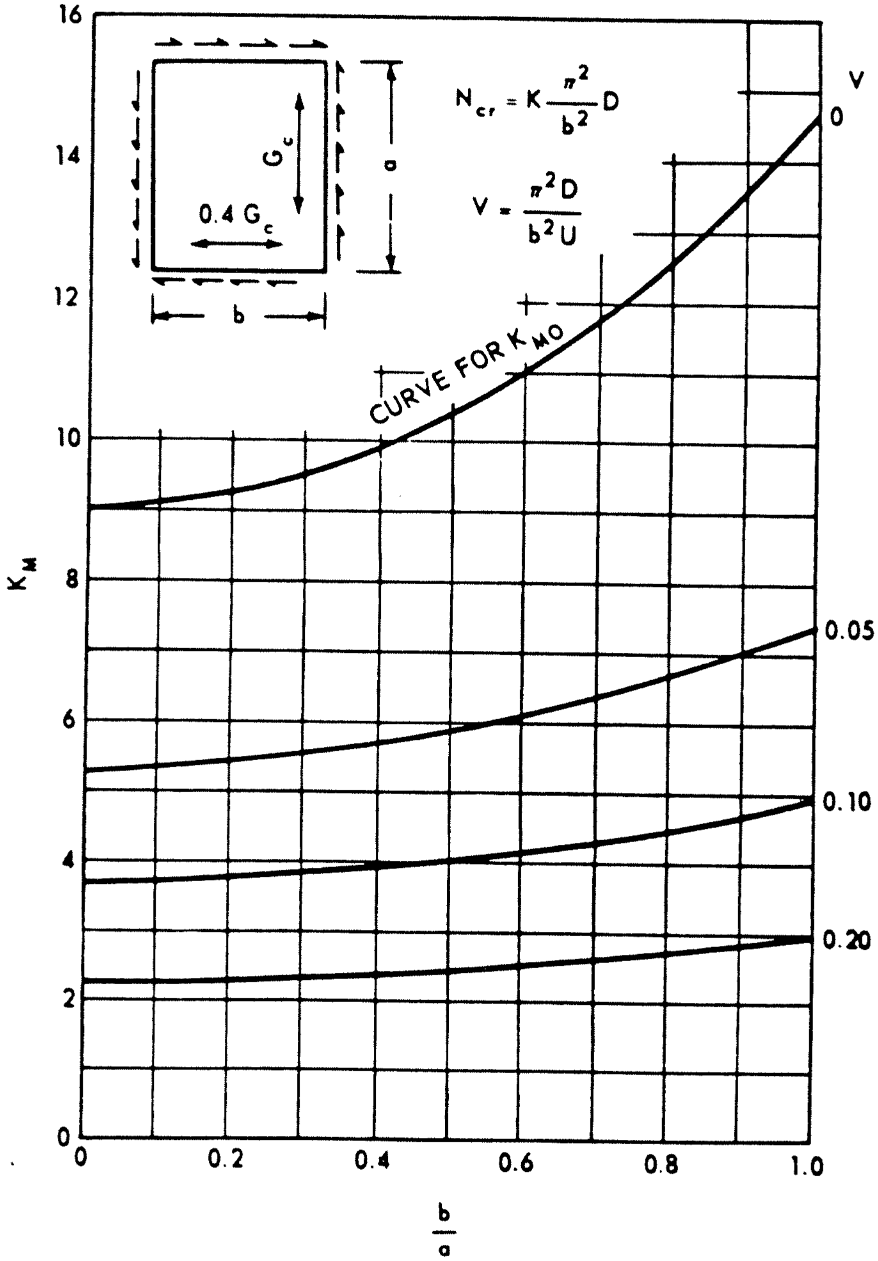


FIGURE 22

K_M for sandwich panel with all edges clamped, isotropic facings and orthotropic core. ($G_{Cb} = 0.4\ G_{Ca}$).

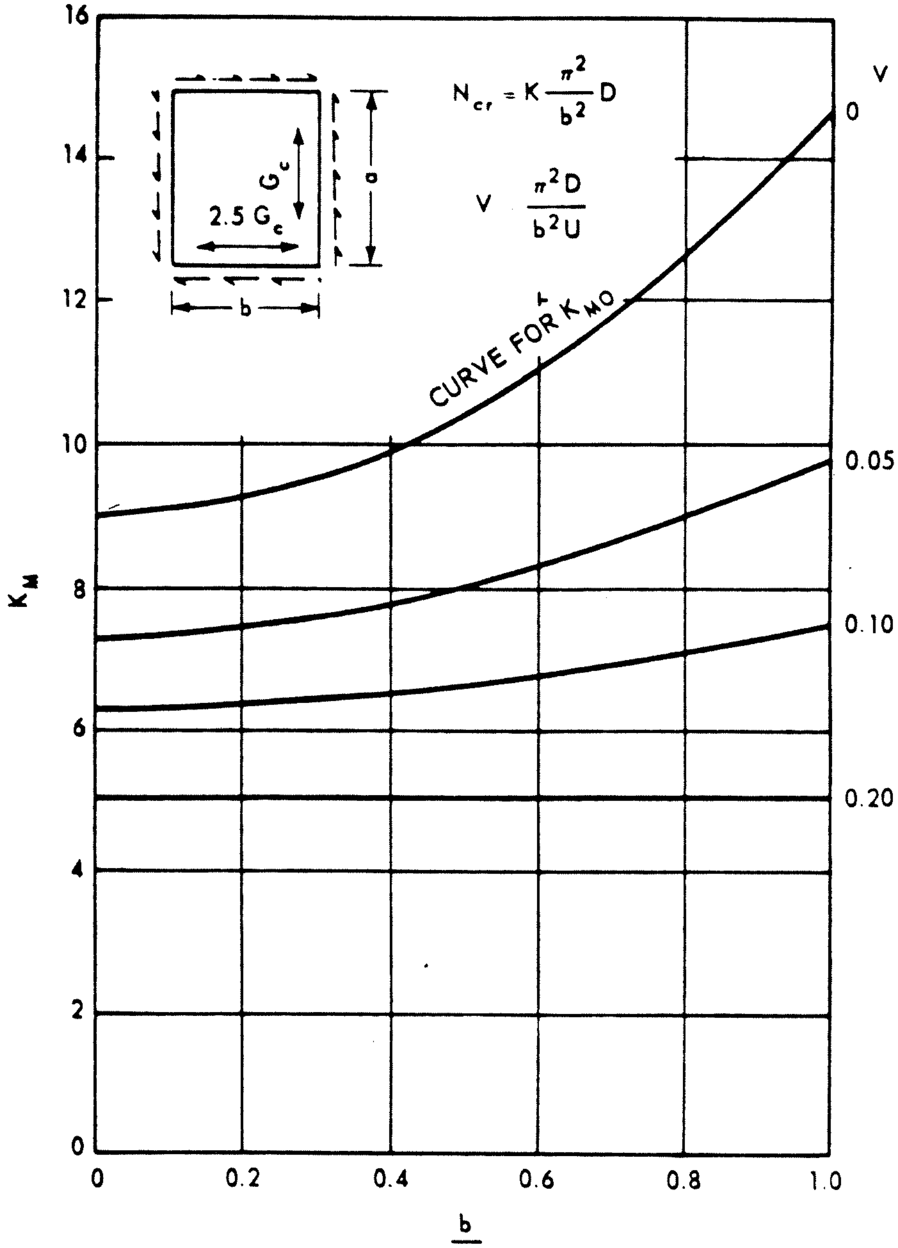


FIGURE 23

K_M for sandwich panel with all edges clamped, isotropic facings and orthotropic core. (G_{Cb} = 2.5 G_{Ca}).

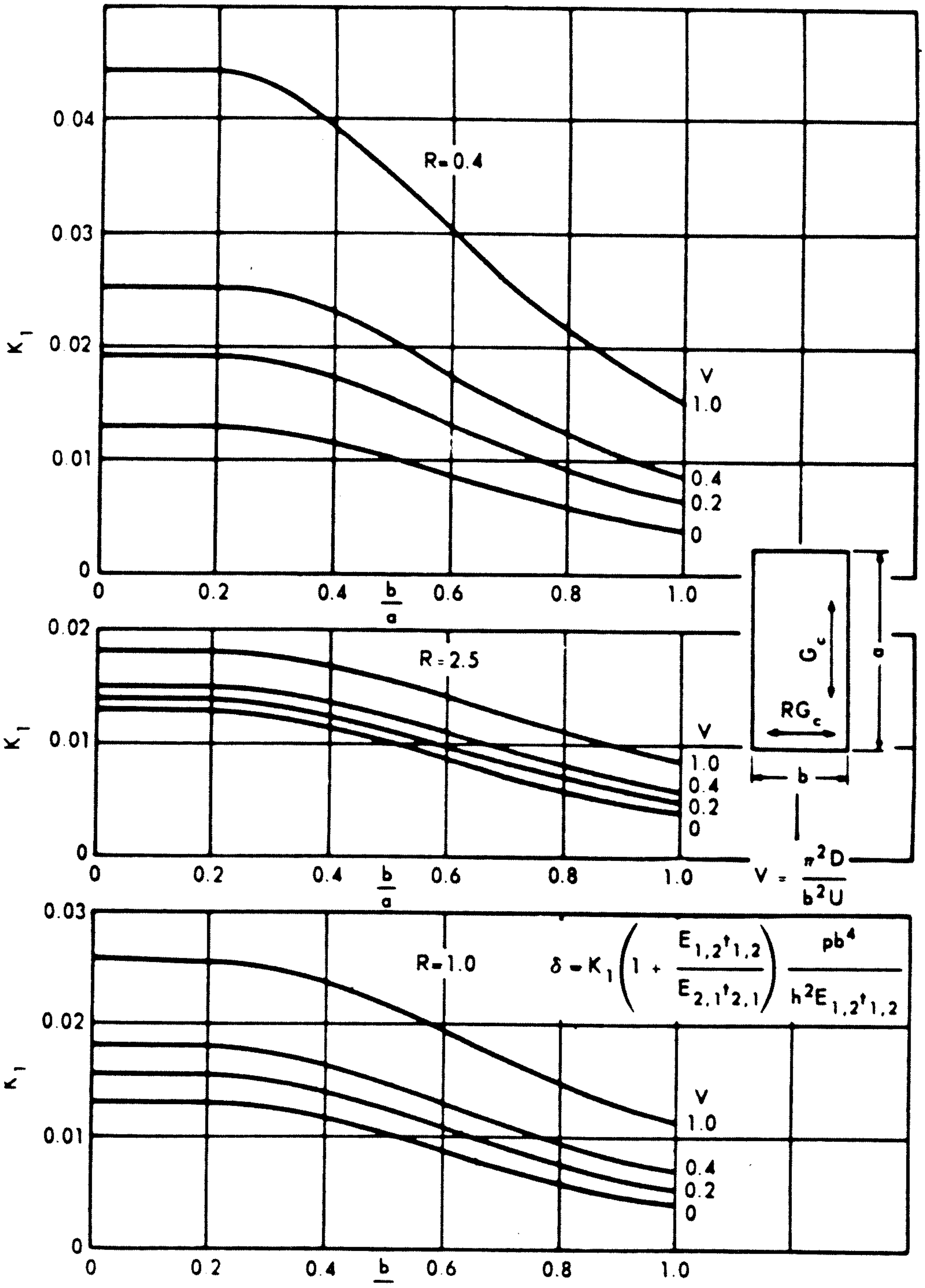


FIGURE 24

K_1, for determining maximum deflection, δ, of flat rectangular sandwich panels with isotropic facings and isotropic or orthotropic core (see sketch) under uniformly distributed normal load.

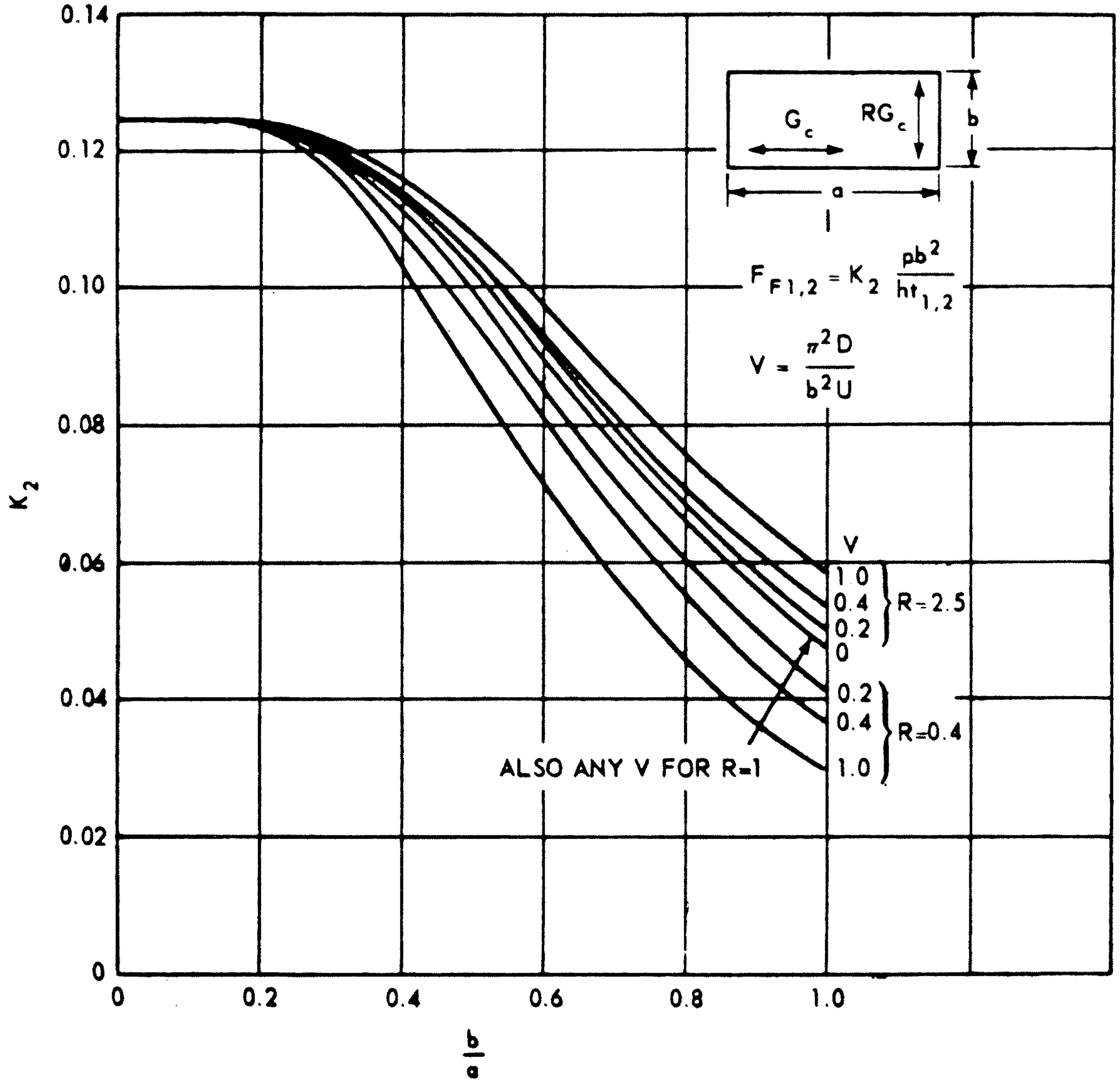
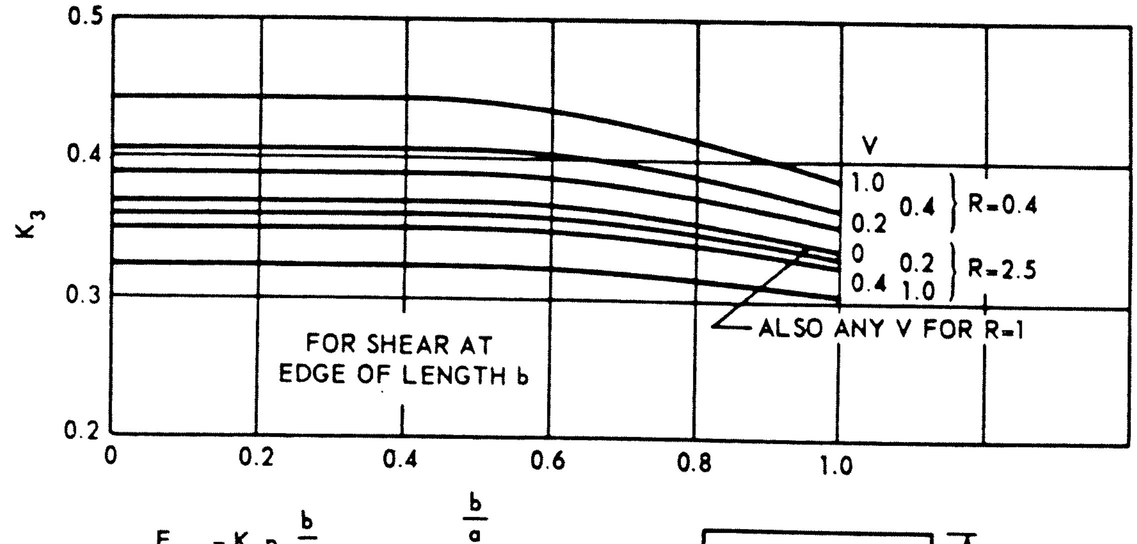


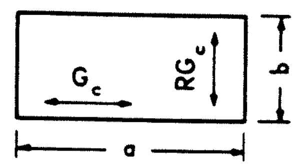
FIGURE 25

K_2, for determining facing stress, F_F, of flat rectangular sandwich panels with isotropic facings and isotropic or orthotropic core (see sketch) under uniformly distributed normal load.



$$F_{Cs} = K_3 p \frac{b}{h}$$

$$V = \frac{\pi^2 D}{b^2 U}$$



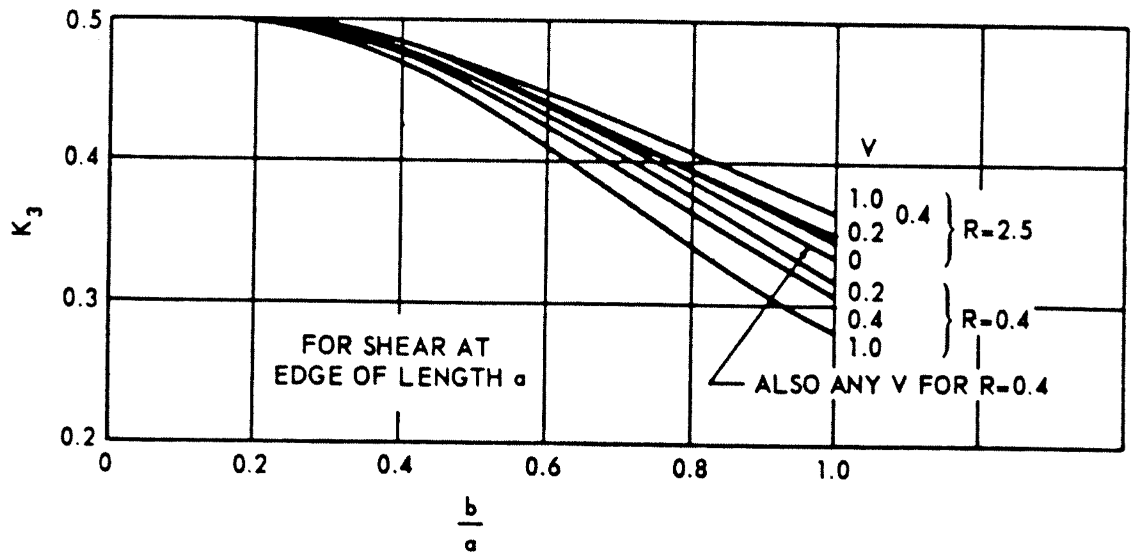


FIGURE 26

K_3, for determining maximum core shear stress, F_{Cs}, for flat rectangular sandwich panels with isotropic facings and isotropic or orthotropic core (see sketch) under uniformly distributed normal load.

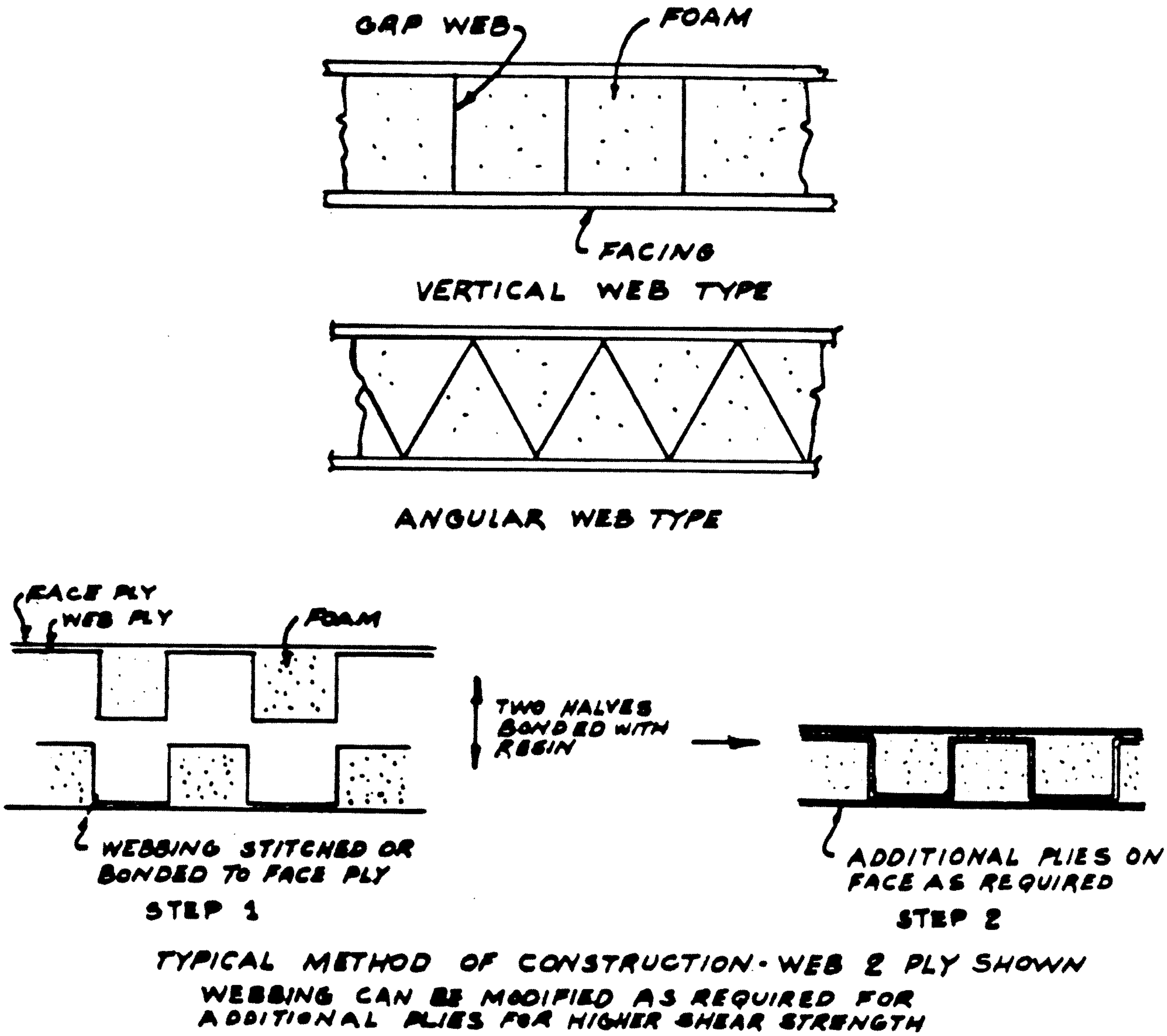


FIGURE 27

Foam-GRP web type cores

APPENDIX A

DDS 9110-9

Design examples

Example 1

Given: A rectangular flat sandwich panel dimensioned and loaded as follows:

Panel width, b = 24 inches

Panel length, a = 48 inches

Edges simply supported

GRP faces - Woven roving-polyester laminates

Face thickness, t_F = 1/16 inch

$G_{Fa} = G_{Fb} = 0.45 \times 10^6$ p.s.i.

$E_{Fa} = E_{Fb} = 1.5 \times 10^6$ p.s.i. (compression modulus)

$\mu_a = \mu_b = 0.19$

F_F = 19,000 p.s.i. (compression)

12,000 p.s.i. (shear)

Note here that for the sandwich example the compressive modulus is used in lieu of the flexural modulus used for simple laminates given in Part I of this DDS.

Balsa core

thickness, t_C = 3/8 inch

$G_{Ca} = G_{Cb}$ = 12,300 p.s.i. (6 pounds per cubic foot density)

$E_{Ca} = E_{Cb}$ = 313,000 p.s.i.

F_{Cs} = 180 p.s.i. (shear stress)

Loading

So a comparison can be made with the example of a single skin laminate given in Part I of this DDS, use the same loads; shear stress 1000 p.s.i. and compressive stress on edge b of 800 p.s.i. (Based on 1/2 inch thick.)

Since a sandwich panel and an equally overall thickness simple laminate have different stresses due to the difference in GRP facings carrying the load, relate in terms of load per inch of panel. For the simple laminate 1/2 inch thick, 1000 p.s.i. shear = 500 pounds per inch and 800 p.s.i. compression = 400 pounds per inch of panel.

Therefore, for our problem use:

N_s = 500 pounds per inch of panel width

N_c = 400 pounds per inch of panel width

Normal load, p = 10 p.s.i.

Determine: Factors of safety

Computation:

Step 1 Minimum face thickness

For shear:

$$t_F = \frac{N_s}{2F_F}$$

$$t_F = \frac{500}{2(12,000)} = 0.021 \text{ inches}$$

Shear factor of safety = $\frac{0.0625}{0.021}$ = 2.98 (answer)

For compression:

$$t_F = \frac{400}{2(19,000)} = 0.01 \text{ inches}$$

Compressive factor of safety = $\frac{0.0625}{0.01}$ = 6.25 (answer)

Step 2 Compressive buckling stress

Use formula for equal faces

$$F_{cr} = \frac{\pi^2 K}{4}\left(\frac{h}{b}\right)^2 \frac{E_F}{\lambda_F}$$

where: b = 24 inches   b = 24 inches

$E_F = 1.5 \times 10^6$ p.s.i.

$\lambda_F = 1 - \mu^2 = 1 - 0.19^2 = 0.964$

$K = K_F + K_M$

$$K_F = \frac{t_F^2}{3h^2} K_{MO}$$

$$\frac{a}{b} = \frac{48}{24} = 2$$

Using Figure 4 for K_{MO}:

Since a/b is greater than 1.0 assume that

$K_F = 0$ (see notes in text)

K_M is found from Figure 6 where $G_{Ca} = G_{Cb}$

and using the orthtropic face curves.

$$V = \frac{\pi^2 t_C E_F t_F}{2\lambda_F b^2 G_C}$$

$$V = \frac{\pi^2 (0.375)\, 1.5 \times 10^6\, (0.0625)}{2\,(0.964)\, 24^2\, (12.3 \times 10^6)}$$

V = 0.000025, assume zero

From Figure 6, using $b/a = \frac{24}{48} = 0.5$ and V = 0

$K_M = 3.2$

$$K = K_F + K_M = 0 + 3.2 = 3.2$$

$$F_{cr} = \frac{\pi^2 (3.2)}{4} \left(\frac{0.4375}{24}\right)^2 \frac{1.5 \times 10^6}{0.964}$$

$$F_{cr} = 4082 \text{ p.s.i.}$$

In terms of load per inch of panel:

$N_{cr} = F_{cr} A$ Where $A = 1/8 \text{ inch}^2$

$$N = 4082\left(\frac{1}{8}\right) = 510.25 \text{ pounds per inch}$$

Step 3 Shear buckling stress

K_M and K_{MO} from Figure 18, using $V = 0$

$$K_{MO} = 5.4$$

$$K_M = 5.4$$

$$K_F = \frac{t_F^2}{3h^2} K_{MO}$$

$$K_F = \frac{(0.0625)^2}{3(0.4375)^2} (5.4) = 0.0367$$

$$K = K_F + K_M = 0.0367 + 5.4 = 5.44$$

$$F_{scr} = \frac{\pi^2 (5.44)}{4} \left(\frac{0.4375}{24}\right)^2 \frac{1.5 \times 10^6}{0.964}$$

$$F_{scr} = 6940 \text{ p.s.i.}$$

In terms of load per inch of panel

$$N_{scr} = 6940\left(\frac{1}{8}\right) = 867.5 \text{ pounds per inch}$$

Step 4 Factors of safety on combined buckling strengths

From formula given in Part I of this DDS:

$$\left(\frac{f_s}{F_{scr}}\right)^2 + \frac{f_c}{F_{ccr}} = \frac{1}{\text{Factor of safety}}$$

Formula modified in terms of load:

$$\frac{N_c}{N_{ccr}} + \left(\frac{N_s}{N_{scr}}\right)^2 = \frac{1}{\text{Factor of safety}}$$

where

$$\frac{N_c}{N_{ccr}} = \frac{400}{510.25} = 0.78$$

$$\frac{N_s}{N_{scr}} = \frac{500}{867.5} = 0.576$$

$$0.78 + (0.576)^2 = \frac{1}{\text{Factor of safety}}$$

$$1.11 = \frac{1}{\text{Factor of safety}}$$

Factor of safety on buckling = 0.90 (answer)

Step 5 Wrinkling of facings

$$F_W = Q\left(\frac{E_F E_C G_C}{\lambda_F}\right)^{\frac{1}{3}}$$

where:

$E_F = 1.5 \quad 10^6$ p.s.i.

$E_C = 0.313 \times 10^6$ p.s.i.

$G_C = 0.0123 \times 10^6$ p.s.i.

$\lambda_F = 0.964$

Q is found using Figure 17:

$$q = \frac{t_C}{t_F} G_C \left(\frac{\lambda_F}{E_F E_C G_C}\right)^{\frac{1}{3}}$$

$$q = \frac{0.375}{0.0625} (0.0123 \times 10^6) \left(\frac{0.964}{1.5 \times 0.313 \times 0.0123 \times 10^{18}}\right)^{\frac{1}{3}}$$

$q = 0.406$

Note that an estimate of the value K is not required for q less 0.6 (see text).

From Figure 93, Q = 0.20

$$F_W = 0.20 \left(\frac{1.5 \times 0.313 \times 0.0123 \times 10^{18}}{0.964}\right)^{\frac{1}{3}}$$

$F_W = 0.0362 \times 10^6 = 36{,}200$ p.s.i.

$N_W = 36{,}200\ (1/8) = 4525$ pounds per inch of panel.

N_W is less then 19,000 p.s.i. allowable;

therefore, wrinkling of face will not occur.

Step 6 Bending strength

Normal load = 10 p.s.i.

Face bending strength

$$F_F = K_2 \frac{pb^2}{ht_F}$$

$p = 10$ p.s.i.

$b^2 = 24^2 = 576$

$h = 0.4375$ inches

$t_F = 0.0625$ inches

$$\frac{b}{a} = \frac{24}{48} = 0.5$$

From Figure 25 using V = 0 previously calculated, and R = 1.0:

$K_2 = 0.102$

$$F_F = 0.102 \left[\frac{(10)(576)}{(0.4375)(0.0625)}\right] = 21{,}486 \text{ p.s.i.}$$

$$\text{Factor of safety} = \frac{F_{Fc}}{F_F} = \frac{19{,}000}{21{,}486} = \underline{\underline{0.88}} \text{ (answer)}$$

Note that the compressive stress, 19,000 p.s.i., of the material is used which is minimum. Tensile strength of woven roving is 33,000 p.s.i.

<u>Shear stress of core:</u>

$$F_{Cs} = K_3\, p\left(\frac{b}{h}\right)$$

From figure 26, K_3 = 0.46 for shear at edge a.

$$F_{Cs} = 0.46\,(10)\,\frac{24}{0.4375}$$

F_{Cs} = 252 p.s.i. Allowable shear strength = 180 p.s.i.

$$\text{Factor of safety} = \frac{180}{252} = \underline{\underline{0.71}} \text{ (answer)}$$

Step 7 <u>Panel deflection</u>

$$\delta = 2\,\frac{K_1}{K_2}\left(\frac{\lambda\, F_{F_F}}{E_F}\right)\left(\frac{b}{h}\right)^2$$

K_2 = 0.102 previously calculated

K_1 from figure 24 = 0.01

$$\delta = 2\left(\frac{0.01}{0.102}\right)\left(\frac{0.964 \times 21486}{1.5 \times 10^6}\right)\left(\frac{24}{0.4375}\right)^2$$

δ = 8.15"

$$\text{Assuming allowable } \delta = \frac{L}{200} = \frac{24}{200} = 0.12$$

Factor of safety on deflection;

$$\frac{0.12}{8.15} = \underline{\underline{0.0147}} \text{ (answer)}$$

The panel used as an example is therefore totally unsatisfactory. Factors of safety on buckling and bending and deflection are below the recommended values.

Example 2

Given: A helicopter landing platform. Existing solid wood decking to be replaced with a sandwich panel 2 1/2 inch maximum thickness. Maximum existing spacing of steel longitudinal supports = 18 inches.

Helicopter wheel load = 369 pounds per square inch

Assume infinitely long panel, designed as a simple beam.

Determine: Core configuration and face thickness. Design to ultimate strength of material.

Computation:

Core compression under normal load

From the helicopter load, the core must be suitable to withstand a compressive load of 369 pounds per square inch under the tire.

From test data on various core materials the following cores could be used:

4 pound density balsa wood - compressive strength 430 p.s.i.

10 pound density polyvinal chloride (PVC) foam, 380 p.s.i.

12 pound density urethane foam, 390 p.s.i.

Another core is the GRP web type (Figure 27). Assuming woven roving 2-ply web and a web compressive strength of 19,000 p.s.i. and a thickness per ply of 0.035 inches:

Allowable load per web per inch:

19,000 (2) (.035) = 1330 pounds per inch

For local compressive stress all the cores appear to be satisfactory and all within a reasonable weight limit. Assuming 2 pound foam in the foam-web type core the weight is approximately 5 pounds per cubic foot which compares with the balsa.

Bending and shear strength

Using the wood deck data in DDS 9110-6 (reference f) bending moment and shear are calculated. In using the DDS complete data on the particular helicopter is required. This data is not given here to simplify the calculation.

Resulting computation:

Bending moment = 7510 inch pounds

Shear load = 1220 pounds

Face thickness

Section modulus required:

$$Z = \frac{M}{F} = \frac{7510}{19{,}000} = 0.395 \text{ inches}^3$$

Using the 2.5 inch depth the section modulus for various face thicknesses are calculated:

For 1/8 inch faces, $Z = 0.28$ inches^3

For 3/16 inch faces, $Z = 0.40$ inches^3

Therefore, 3/16 inch minimum woven roving facings are required.

Core configuration

$$\text{Shear stress in the core} = \frac{P}{A} = \frac{1220}{2.5} = 488 \text{ p.s.i. required}$$

Checking data on core materials the following densities are required for shear:

Balsa - approximately 15 pounds per cubic foot

PVC - approximately 15 pounds per cubic foot

Urethane foam - approximately 20 pounds per cubic foot

It can be seen that much heavier cores are required for shear than for the local compression.

Check using the foam GRP web-type of core.

Two webs shear area = 0.035 (2)(2.5) = 0.175 square inches

f = P/A = 1220/0.175 = 6971 p.s.i.

Allowable shear stress of woven roving = 12,000 p.s.i.

Therefore the two-ply web-foam type panel is satisfactory, and with an estimated core weight of 5 pounds per cubic foot is approximately 1/3 the weight of other core materials.

Where design computation indicates a low shear requirement the other cores would compete, especially on a cost basis.