DDS 112-1

DESIGN DATA SHEET

SUBMARINE MAIN BALLAST TANK STRUCTURAL DESIGN

DEPARTMENT OF THE NAVY
NAVAL SEA SYSTEMS COMMAND
WASHINGTON, DC 20362-5101

REVISION DATE: NOVEMBER 18, 1987

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SUBMARINE MAIN BALLAST TANK

STRUCTURAL DESIGN

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CODE SEA55Y2

NAVAL SEA SYSTEMS COMMAND

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NOMENCLATURE

		UNITS
a'	Length of long side of panel	in
a_{ij}	Area of "i"th space in "j"th flood hole	in^2
A	Effective area of flood hole opening of the tank	ft^2
A'	Deflection coefficient	in^2
A'_H	Effective area of typical frame, pressure hull	in^2
A'_O	Effective area of outer hull frame	in^2
A_{fH}	Area of pressure hull frame (frame only)	in^2
A_{fO}	Area of outer hull frame (frame only)	in^2
A_s	Cross-sectional area of the strut	in^2
A_{TH}	Total area of pressure hull frame-shell combination computed using the effective length of the shell plating, $L_{e,H}$	in^2
A_{TL}	Total cross-sectional area of the longitudinal stiffener plus the effective width of the plating	in^2
A_{TO}	Total area of the outer hull frame-shell combination computed using the effective length of the shell plating, $L_{e,O}$	in^2
b'	Length of short side of panel	in
b'_B	Radial width of wing bulkhead panel, measured perpendicular to outer hull plating	in
b_H	Thickness of pressure hull frame web	in
b_O	Thickness of outer hull frame web	in
B	Maximum beam of hull	ft
B'	Deflection coefficient	in^3
C	Center-of-panel moment coefficient for a flat plate	non-dimensional
C_D	Flood hole discharge coefficient	non-dimensional

NOMENCLATURE

		UNITS
C_h	Athwartship dimension of cut-in-shell	in
$C_{H\Delta p}$	Shell stress parameter relating to blow pressure, pressure hull	non-dimensional
C_{Hp}	Shell stress parameter relating to hydrostatic pressure, pressure hull	non-dimensional
$C_{O\Delta p}$	Shell stress parameter relating to blow pressure, outer hull	non-dimensional
C_{Op}	Shell stress parameter relating to hydrostatic pressure, outer hull	non-dimensional
d_H	Depth of frame web and flange for pressure hull	in
d_O	Depth of frame web and flange for outer hull	in
$D_{H\Delta p}$	Deflection of inner hull at strut due to blow pressure	in
D_{Hp}	Deflection of inner hull at strut due to hydrostatic pressure	in
D_{Hs}	Stiffness of inner hull structure at strut-hull junction	in/lb
$D_{O\Delta p}$	Deflection of outer hull at strut due to blow pressure	in
D_{Op}	Deflection of outer hull at strut due to hydrostatic pressure	in
D_{Os}	Stiffness of outer hull structure at strut-hull junction	in/lb
E	Modulus of elasticity	psi
E_B	Modulus of elasticity of wing bulkhead plating	psi
E_H	Modulus of elasticity of pressure hull structure	psi
E_O	Modulus of elasticity of outer hull structure	psi
E_S	Modulus of elasticity of strut	psi
F	Deflection coefficient	in^3
$F_{1,ij}$	Stress function ($i = O,H$; $j = p, \Delta p$); same as F_1	non-dimensional

NOMENCLATURE

		UNITS
$F_{2,ij}$	Stress function (i = O,H; j = p, Δp); same as F_2	non-dimensional
F_3	Stress function	non-dimensional
$F_{4,ij}$	Stress function (i = O,H; j = p, Δp); same as F_4	non-dimensional
g	Number of spaces between vanes of "j"th flood hole	non-dimensional
$G_{H\Delta p}$	Shell stress parameter relating to blow pressure, pressure hull	non-dimensional
G_{Hp}	Shell stress parameter relating to hydrostatic pressure, pressure hull	non-dimensional
$G_{O\Delta p}$	Shell stress parameter relating to blow pressure, outer hull	non-dimensional
G_{Op}	Shell stress parameter relating to hydrostatic pressure, outer hull	non-dimensional
h	Number of flood holes in tank	non-dimensional
h_A	Vertical distance from tank top to water surface in tank	ft
h_B	Depth of residual water, baseline to top of flood hole	ft
h_f	Flange thickness	in
h_T	Blowable depth of tank, from tank top to flood hole	ft
h_w	Blowable depth of water, from water surface to flood hole	ft
H_M	Shell stiffness coefficient, pressure hull	non-dimensional
H'_M	Shell stiffness coefficient, outer hull	non-dimensional
I_H	Moment of inertia of pressure hull frame-shell combination, including effective length of shell, $L_{e,H}$	in^4
I_O	Moment of inertia of outer hull circumferential frame-shell combination, including effective length of shell, $L_{e,O}$	in^4
J	Parameter for force and moment calculation	in^{-4}

NOMENCLATURE

		UNITS
γ_{Hp}	Shell stress parameter relating to hydrostatic pressure, pressure hull	non-dimensional
$\gamma_{O\Delta p}$	Shell stress parameter relating to blow pressure, outer hull	non-dimensional
γ_{Op}	Shell stress parameter relating to hydrostatic pressure, outer hull	non-dimensional
Γ	Average normal (hoop) force coefficient	non-dimensional
δp	Pressure differential at the flood hole	psi
δ_x	Deflection coefficient at any position x	non-dimensional
δ_ξ	Deflection at any position ξ	in
Δp	Differential pressure across ballast tank structure, due to blowing or expansion	psi
Δp_1	Portion of the differential pressure Δp related to plate bending in cantilevered end tanks	psi
Δp_2	Portion of the differential pressure Δp related to membrane action in cantilevered end tanks	psi
Δp_t	Differential pressure loading on a secondary transverse stiffener in an end tank	psi
ϵ	Deflection coefficient for a uniformly-loaded rectangular plate having all edges fixed	non-dimensional
$\eta_{1,H\Delta p}$	Shell stress parameter relating to blow pressure, pressure hull	non-dimensional
$\eta_{2,H\Delta p}$	Shell stress parameter relating to blow pressure, pressure hull	non-dimensional
$\eta_{1,Hp}$	Shell stress parameter relating to hydrostatic pressure, pressure hull	non-dimensional
$\eta_{2,Hp}$	Shell stress parameter relating to hydrostatic pressure, pressure hull	non-dimensional

NOMENCLATURE

		UNITS
$\eta_{1,O\Delta p}$	Shell stress parameter relating to blow pressure, outer hull	non-dimensional
$\eta_{2,O\Delta p}$	Shell stress parameter relating to blow pressure, outer hull	non-dimensional
$\eta_{1,Op}$	Shell stress parameter relating to hydrostatic pressure, outer hull	non-dimensional
$\eta_{2,Op}$	Shell stress parameter relating to hydrostatic pressure, outer hull	non-dimensional
θ_H	Stiffness coefficient, pressure hull	non-dimensional
θ_O	Stiffness coefficient, outer hull	non-dimensional
θ^*	Pitch angle of ship	(degrees or radians)
θ_t	Angle between two adjacent primary longitudinal stiffeners	degrees
ν	Poisson's ratio	non-dimensional
ξ_x	Moment coefficient at any position x	non-dimensional
ρ	Radius factor	non-dimensional
σ_a	Allowable design stress in the bulkhead plating	psi
σ_{end}	Maximum stress in the secondary transverse stiffener of an end tank	psi
σ_f	Stress in the flange of the longitudinal stiffener	psi
σ_h	Shell plating hoop stress	psi
σ_p	Stress in the outer hull plating at any point along the edge of a longitudinal stiffener	psi
σ_s	Axial stress in strut	psi
σ_t	Maximum stress in the primary transverse stiffener on an end tank	psi
σ_y	Yield strength of the material	psi

NOMENCLATURE

		UNITS
σ_{BSP}	Maximum stress in the primary bulkhead stiffener	psi
σ_{BSS}	Maximum stress in the secondary bulkhead stiffener	psi
$\sigma_{\frac{L}{2}}$	Longitudinal stress in the outer hull plating, at midbay	psi
σ_{LB}	Longitudinal stress in outer hull plating, at edge of wing bulkhead	psi
σ_{LOHP}	Maximum combined stress in the longitudinally stiffened outer hull plating	psi
$\sigma_{Lp}(x)$	Total longitudinal stress in the outer hull plating, at edge of frame, for any position x	psi
$\sigma_{L}(x)$	Longitudinal stress in the outer hull plating at the frame for any circumferential position x	psi
σ_{ϕ}	Circumferential stress in the outer hull plating at midbay	psi
$\sigma_{\phi f}(x)$	Circumferential stress in the flange of the outer hull frame, for any circumferential position x	psi
$\sigma_{\phi p}(x)$	Circumferential stress in the outer hull shell plating at the frame, for any circumferential position x	psi
ϕ	Stress coefficient for a uniformly loaded rectangular plate, all edges fixed	non-dimensional

ABSTRACT

This Design Data Sheet (DDS) provides equations and design criteria for designing submarine ballast tank structure under the concurrent effects of differential blow pressure and hydrostatic pressure. Load and stress equations apply to longitudinally and transversely stiffened end tanks and to circular and noncircular strut-supported tanks. Tables summarizing the design limits for these stresses under emergency blow and normal surfacing conditions are presented. Recent work on updating the strut-frame equations has been incorporated. The finite element method is presented as an alternative to the closed-form hand calculations for stress determination. Theoretical development and background information are presented in the Commentary, Appendix B. Example calculations are given in Appendix C.

ADMINISTRATIVE INFORMATION

This Design Data Sheet (DDS) was developed by NAVSEA 55Y2. This issue supersedes Reference (1).

SECTION 1 - INTRODUCTION

1.1 GENERAL

This Design Data Sheet (DDS) provides formulae and design criteria for designing submarine ballast tank structure, including the main ballast tank (MBT) bulkheads, under the concurrent effects of differential blow pressure and hydrostatic pressure. The design of bow and stern structure under loadings other than these are not addressed in this document.

1.2 APPLICABILITY

The formulae presented herein are based on elastic analysis and apply to longitudinally and transversely stiffened end tanks and to circular and noncircular strut-supported tanks (see Figures 1 through 4). Description of these ballast tanks is given in the Commentary, Appendix B.

1.3 DESIGN LIMITS

Tables I and II in Appendix A summarize the mandatory design limits for the allowable stresses throughout the MBT structure for the emergency blow condition and the normal surfacing condition.

1.4 SUMMARY OF MAJOR REVISIONS

This DDS has been derived from NAVSEC Report NS 0929-003-3010 (Reference 1), incorporating recent work performed by Schwalbe (Reference 2) and NAVSEA on updating the strut load equations. Portions of the General Dynamics/Electric Boat Report on differential pressure prediction (Reference 3) have also been included to illustrate the calculation methods and procedures. The main text of this DDS contains only criteria and formulae for designing the ballast tank structure, with the background and theoretical derivation of these criteria and formulae presented separately in the Commentary, Appendix B.

1.5 CORROSION ALLOWANCE

A corrosion allowance is not included in the determination of ballast tank scantlings. The amount of the corrosion allowance must be determined based on applicable Ship Specifications.

1.6 ARRANGEMENT OF DESIGN DATA SHEET

Section 2 provides methods for estimating ballast tank differential or blow pressure which must be determined before calculations of ballast tank loads and stresses can proceed. Section 3 contains the load equations and Section 4 the stress equations for circular or noncircular, strut-supported ballast tanks. Sections 5 and 6 contain the load and stress equations for longitudinally or transversely stiffened end tanks. Section 7 contains the stress equations for ballast tank bulkheads and is applicable to either the strut-supported tanks or the stiffened end tanks. Section 8 presents the finite element method and steps to verify a particular program for Navy acceptance as a design/analysis tool. Appendix A provides the tables and figures as they apply to the main text, Sections 1 through 8. Appendix B is as described in Section 1.4 with the numbering of paragraphs in Appendix B corresponding to the respective paragraph numbering in the main text sections for cross-reference. Appendix C contains illustrative examples for the design equations presented in the text.

SECTION 2 - DESIGN PRESSURE CONSIDERATIONS

2.1 GENERAL

This section presents methods for determining the pressure loadings to be used in designing MBT structure. Hydrostatic pressure requirements shall be in accordance with applicable Ship Specifications.

2.2 DESIGN CRITERIA

a. Differential Pressure for Emergency Blow Condition -

1. At surface - The MBT structure shall withstand a differential pressure loading, Δp, without exceeding the design stress limits in Table I. The differential pressure shall be the larger of the steady-state differential pressure, Δp, or 30 psi.

2. At collapse depth - For strut-supported MBTs only (see Figure 1), the structure shall withstand the effects of hydrostatic pressure, p, on the inner hull plus the effects of the hull contraction on the outer hull and struts. Stresses shall not exceed the design limits given in Table I.

b. Differential Pressure for Normal Surfacing Condition - The ballast tank structure shall withstand a differential pressure equal to the hydrostatic pressure created by a column of water the height of the ballast tank plus a 5 psi surcharge pressure. The stresses in the tank structure shall not exceed the design limits of Table II.

SECTION 3 - OUTER HULL AND STRUT LOAD EQUATIONS FOR STRUT-SUPPORTED TANKS

3.1 GENERAL

The following load equations are for circular strut-supported ballast tanks located in the double hull portion of the submarine (see Figure 1) and are provided as guidance. Finite element analysis, as discussed in Section 8, may be used for design of strut-supported tanks. The design loadings and design limits shall be as given in Section 2.2 and Tables I and II.

3.2 STRUT LOAD

The strut load, W, may be obtained by equating the deflections of the outer and inner hull structure with the strut elongation:

$$W = \frac{D_{O\Delta p} + D_{Op} - D_{H\Delta p} - D_{Hp}}{\dfrac{L_s}{A_s\ E_s} + D_{Os} - D_{Hs}}, \text{ lb} \qquad (3\text{-}1)$$

where the subscripts O, H, and s stand for outer hull, pressure hull, and strut, respectively; p and Δp are hydrostatic and differential pressure, respectively, and:

D_{is} = stiffness of the strut at hull i, in/lb (i = O,H);

D_{ij} = deflection at hull i, due to pressure j, inches, (i = O,H and j = p,Δp)

A_s = cross-sectional area of the strut, in^2;

L_s = length of strut, from outer frame flange to inner frame flange, inches; and

E_s = modulus of elasticity of strut, psi.

The above terms are further defined as follows:

The above terms are further defined as follows:

$$D_{Os} = \frac{R_O}{2E_O}\left[\frac{2R_O^2}{I_O}\left[\delta_x\right]_{\frac{x}{a}=1.0} + \frac{\Gamma}{A_{TO}}\right]; \tag{3-2}$$

$$D_{Hs} = \frac{R_H}{2E_H}\left[\frac{2R_H^2}{I_H}\left[\delta_x\right]_{\frac{x}{a}=1.0} + \frac{\Gamma}{A_{TH}}\right]; \tag{3-3}$$

$$D_{O\Delta p} = \Delta p\left[1 - \frac{K_2\nu}{2}\right]\frac{R_O^2}{E_O t_O}\Big\{1 - C_{O\Delta p}\left[(\cosh\eta_{1,O\Delta p}\theta_O)(\cos\eta_{2,O\Delta p}\theta_O)\right.$$

$$\left. + G_{O\Delta p}(\sinh\eta_{1,O\Delta p}\theta_O)(\sin\eta_{2,O\Delta p}\theta_O)\right]\Big\}; \tag{3-4}$$

$$D_{Op} = \frac{\nu K_4 p}{2}\left[\frac{R_O^2}{E_O t_O}\right]\Big\{1 - C_{Op}\left[(\cosh\eta_{1,Op}\theta_O)(\cos\eta_{2,Op}\theta_O)\right.$$

$$\left. + G_{Op}(\sinh\eta_{1,Op}\theta_O)(\sin\eta_{2,Op}\theta_O)\right]\Big\}; \tag{3-5}$$

$$D_{H\Delta p} = \Delta p\left[1 + \frac{K_1\nu}{2}\right]\frac{R_H^2}{E_H t_H}\Big\{1 - C_{H\Delta p}\left[(\cosh\eta_{1,H\Delta p}\theta_H)(\cos\eta_{2,H\Delta p}\theta_H)\right.$$

$$\left. + G_{H\Delta p}(\sinh\eta_{1,H\Delta p}\theta_H)(\sin\eta_{2,H\Delta p}\theta_H)\right]\Big\}; \tag{3-6}$$

$$D_{Hp} = p\left[1 - \frac{K_3\nu}{2}\right]\frac{R_H^2}{E_H t_H}\Big\{1 - C_{Hp}\left[(\cosh\eta_{1,Hp}\theta_H)(\cos\eta_{2,Hp}\theta_H)\right.$$

$$\left. + G_{Hp}(\sinh\eta_{1,Hp}\theta_H)(\sin\eta_{2,Hp}\theta_H)\right]\Big\}; \tag{3-7}$$

where:

$$\left[\delta_x\right]_{\frac{x}{a}=1.0} = \frac{1}{4}\left[\frac{a}{\sin^2 a} + \cot a - \frac{2}{a}\right]; \tag{3-8a}$$

where:

α = half-angle between struts, radians; and

x = angular distance along the frame from midpoint between struts, radians. $x = \alpha$ at strut; $x = 0$ midway between struts;

$$\Gamma = \frac{1}{2}\left[\frac{1}{\sin\alpha} + \cot\alpha\right]; \qquad (3\text{-}8b)$$

$$K_1 = \left[\frac{R_O^2 - R_H^2}{R_O t_O + R_H t_H}\right]\frac{t_H}{R_H}; \qquad (3\text{-}8c)$$

$$K_2 = \left[\frac{R_O^2 - R_H^2}{R_O t_O + R_H t_H}\right]\frac{t_O}{R_O}; \qquad (3\text{-}8d)$$

$$K_3 = \frac{R_H t_H}{R_O t_O + R_H t_H}; \qquad (3\text{-}8e)$$

$$K_4 = \left[\frac{R_H^2}{R_O t_O + R_H t_H}\right]\frac{t_O}{R_O}; \qquad (3\text{-}8f)$$

$$C_{ij} = \frac{\alpha_i F_{2,ij}}{\alpha_i + \beta_i + (1-\beta_i)\,F_{1,ij}};\ (i = O,H;\ j = \Delta p, p); \qquad (3\text{-}8g)$$

$$\alpha_i = \frac{A'_i}{L_i t_i};\quad (i = O,H); \qquad (3\text{-}8h)$$

$$\beta_i = \frac{b_i}{L_i};\quad (i = O,H); \qquad (3\text{-}8i)$$

$$A'_O = (A_{fO}) \frac{R_O}{R'_{Ocg}} ; \qquad (3\text{-}8j\text{-}1)$$

$$A'_H = (A_{fH}) \left[\frac{R_H}{R'_{Hcg}} \right]^2 ; \qquad (3\text{-}8j\text{-}2)$$

$$\theta_i = \sqrt[4]{3(1 - \nu^2)} \left[\frac{L_i - b_i}{R_i t_i} \right]; \quad (i = O,H) ; \qquad (3\text{-}8k)$$

$$\gamma_{i\Delta p} = \frac{\Delta p}{2E} \left[\frac{R_i}{t_i} \right]^2 \sqrt{3(1-\nu^2)} ; \quad (i = O,H) ; \qquad (3\text{-}8l)$$

$$\gamma_{ip} = \frac{p}{2E} \left[\frac{R_i}{t_i} \right]^2 \sqrt{3(1-\nu^2)} ; \quad (i = O,H) ; \qquad (3\text{-}8m)$$

$$\eta_{1,ij} = \frac{1}{2} \sqrt{1 - \gamma_{ij}} ; \quad (i = O,H ; j = \Delta p,p) ; \qquad (3\text{-}8n)$$

$$\eta_{2,ij} = \frac{1}{2} \sqrt{1 + \gamma_{ij}} ; \quad (i = O,H ; j = \Delta p,p) ; \qquad (3\text{-}8o)$$

$$F_{1,ij} = \left[\frac{4}{\theta_i} \right] \frac{\cosh^2 \eta_{1,ij}\theta_i - \cos^2 \eta_{2,ij}\theta_i}{\dfrac{(\cosh \eta_{1,ij}\theta_i) \sinh \eta_{1,ij}\theta_i}{\eta_{1,ij}} + \dfrac{(\cos \eta_{2,ij}\theta_i) \sin \eta_{2,ij}\theta_i}{\eta_{2,ij}}} ; \qquad (3\text{-}8p)$$

$$F_{2,ij} = \frac{\dfrac{(\cosh \eta_{1,ij}\theta_i) \sin \eta_{2,ij}\theta_i}{\eta_{2,ij}} + \dfrac{(\sinh \eta_{1,ij}\theta_i) \cos \eta_{2,ij}\theta_i}{\eta_{1,ij}}}{\dfrac{(\cosh \eta_{1,ij}\theta_i) \sinh \eta_{1,ij}\theta_i}{\eta_{1,ij}} + \dfrac{(\cos \eta_{2,ij}\theta_i) \sin \eta_{2,ij}\theta_i}{\eta_{2,ij}}} ; \qquad (3\text{-}8q)$$

$$F_{4,ij} = \sqrt{\frac{3}{1-\nu^2}} \; \frac{\dfrac{(\cosh\eta_{1,ij}\theta_i)\sin\eta_{2,ij}\theta_i}{\eta_{2,ij}} - \dfrac{(\sinh\eta_{1,ij}\theta_i)\cos\eta_{2,ij}\theta_i}{\eta_{1,ij}}}{\dfrac{(\cosh\eta_{1,ij}\theta_i)\sinh\eta_{1,ij}\theta_i}{\eta_{1,ij}} + \dfrac{(\cos\eta_{2,ij}\theta_i)\sin\eta_{2,ij}\theta_i}{\eta_{2,ij}}} \; ; \quad (3\text{-}8r)$$

(NOTE: For equations (3-8p,q,r) $i = O,H$; $j = \Delta p, p$.)

$$G_{ij} = \frac{\sqrt{\dfrac{1-\nu^2}{3}} \left[\dfrac{F_{4,ij}}{F_{2,ij}}\right] + \gamma_{ij}}{4\,\eta_{1,ij}\,\eta_{2,ij}} \qquad (i = O,H;\; j = \Delta p,p)\;; \qquad (3\text{-}8s)$$

with,

R_i = radius to mid-thickness of hull i, inches ($i = O,H$);

I_i = moment of inertia of frame-shell combination at hull i, in^4 ($i = O,H$); and

A_{Ti} = area of frame-shell combination at hull i, in^2 ($i = O,H$);

and the effective shell width for inertial and area computations is:

$$L_{e,i} = F_{1,ij}\,(L_i - b_i) + b_i\,; \qquad (3\text{-}8t)$$

where the subscript $i = O,H$ for outer hull and pressure hull, respectively; and $j = p$ for hydrostatic pressure condition for equation (3-8t). Other parameters are as defined in the Nomenclature Section.

The stress functions $F_{1,ij}$, $F_{2,ij}$, and $F_{4,ij}$ have been plotted as functions of θ and η in Figures 13, 14, and 16 respectively.

The calculation of all parameters, especially D_{Os} and D_{Hs}, in the preceeding formulae requires retaining significant figures of 10 digits behind the decimal point. This is necessary in order to obtain accurate strut load values for the subsequent stress calculations. For this reason, it is recommended that a computer be used to perform these calculations, rather than a hand calculator.

3.3 HOOP LOAD IN THE OUTER HULL FRAME

With the axial load (W) in the strut determined, the circumferential hoop load in the outer hull frame at any position x along the frame may be determined from:

$$T(x) = \left[P_c R_o - W(\gamma_x)\right]\left\{\frac{A_{fo}}{A_{TO}}\left[\frac{a - x}{a}\right] + \frac{x}{a}\right\},\ \text{lb} \qquad (3\text{-}9)$$

where:

$$\gamma_x = \frac{1}{2}\left[\frac{\cos x}{\sin a}\right] = \text{normal (hoop) force coefficient at position x;} \qquad (3\text{-}9a)$$

$$P_c = L_{e,o}\,(\Delta p) = \text{circumferential linear load on outer hull frame, lb/in.} \qquad (3\text{-}9b)$$

3.4 MOMENT IN THE OUTER HULL FRAME

The moment in the outer hull frame may also be determined as a function of the strut load (W), radius (R_o), and non-dimensional moment coefficient (ξ_x), at any position "x" as:

$$M_f(x) = W\,R_o\,(\xi_x),\ \text{in-lb} \qquad (3\text{-}10)$$

where:

$$\xi_x = \gamma_x - \frac{1}{2a} = \text{moment coefficient at position x.} \qquad (3\text{-}10a)$$

SECTION 4 - STRESSES FOR STRUT-SUPPORTED TANKS

4.1 GENERAL

Using the strut load, hoop load, and moment in the outer hull structure determined from Section 3, the stresses in a strut-supported ballast tank can be calculated by using the equations in this section.

The equations in this section are provided for guidance only, as answers are approximate. Finite element methods, discussed in Section 8, may be used as an alternative method of stress prediction.

4.2 LONGITUDINAL STRESS IN THE OUTER HULL PLATING AT THE FRAME

The longitudinal stress in the outer hull plating at the frame for the inside and outside surfaces (+ for inside, - for outside) for any circumferential position "x" with respect to the struts is:

$$\sigma_L(x) = \pm \frac{1.734K'}{(2V'N' - K'^2)(1-\nu^2)^{1/2}} \left\{ \frac{WR_O^2}{I_O}\left[\delta_x\right] + \frac{W}{A_{TO}}\gamma_x + \frac{(\Delta p)R_O}{A_{TO}}\left[1 - \frac{\nu}{6}\rho\right]\left[\frac{A_{TO}}{t_O} + H'_M \frac{A_{fO}}{t_O} - L_{e,O}\right] + \frac{p'R_H^2E_O}{A_{TH}R_OE_H}\left[1 - \frac{\nu}{2}\right]\left[\frac{A_{TH}}{t_H} + \frac{H_MA_{fH}}{t_H} - L_{e,H}\right]\right\} + \frac{(\Delta p)R_O\rho}{6t_O} \text{ ; psi} \quad (4\text{-}1)$$

where:

$$V' = \frac{\cosh\theta_O + \cos\theta_O}{\sinh\theta_O + \sin\theta_O}; \quad (4\text{-}1a)$$

$$K' = \frac{\sinh \theta_O - \sin \theta_O}{\sinh \theta_O + \sin \theta_O} ; \qquad (4\text{-}1b)$$

$$\delta_x = \frac{1}{4}\left[\frac{(a\cos a)(\cos x)}{\sin^2 a} + \frac{\cos x}{\sin a} - \frac{2}{a} + \frac{x\sin x}{\sin a}\right]; \qquad (4\text{-}1c)$$

$$N' = \frac{\cosh \theta_O - \cos \theta_O}{\sinh \theta_O + \sin \theta_O} ; \qquad (4\text{-}1d)$$

$$H'_M = -2\left[\frac{\sinh \frac{\theta_O}{2} \cos \frac{\theta_O}{2} + \cosh \frac{\theta_O}{2} \sin \frac{\theta_O}{2}}{\sinh\theta_O + \sin\theta_O}\right]; \qquad (4\text{-}1e)$$

$$H_M = -2\left[\frac{\sinh \frac{\theta_H}{2} \cos \frac{\theta_H}{2} + \cosh \frac{\theta_H}{2} \sin \frac{\theta_H}{2}}{\sinh \theta_H + \sin \theta_H}\right]; \qquad (4\text{-}1f)$$

$$\rho = 2 - \frac{R_H}{R_O} - \left[\frac{R_H}{R_O}\right]^2 ; \qquad (4\text{-}1g)$$

$$p' = p + \Delta p . \qquad (4\text{-}1h)$$

4.3 CIRCUMFERENTIAL STRESS IN THE FRAME FLANGE

The stress in the free flange of the outer hull circumferential frame may be calculated at any position "x" (see Figures 10 and 11), by substitution of the corresponding axial (hoop) component and moment (calculated by equations (3-9) and (3-10), respectively) to obtain:

$$\sigma_{\phi f}(x) = \frac{T(x)}{A_{TO}} - \frac{M_f(x)}{Z_f} , \text{ psi} \qquad (4\text{-}2)$$

where:

Z_f = section modulus of the frame-shell combination with respect to frame flange, in^3 (see Figure 11).

4.4 CIRCUMFERENTIAL STRESS IN THE PLATING AT THE FRAME

The stress in the plating flange of the outer hull circumferential frame is similarly calculated by appropriate substitution in the following equation:

$$\sigma_{\phi p}(x) = \frac{T(x)}{A_{TO}} + \frac{M_f(x)}{Z_p} + \nu \, \sigma_L(x) \text{ , psi} \qquad (4\text{-}3)$$

where:

Z_p = section modulus of the frame-shell combination with respect to plating flange, in^3 (see Figure 11).

4.5 TOTAL LONGITUDINAL OUTER HULL STRESS AT THE FRAME

The total longitudinal stress in the outer hull plating for any circumferential orientation along the edge of the frame is:

$$\sigma_{Lp}(x) = \sigma_L(x) + \nu \left[\frac{T(x)}{A_{TO}} + \frac{M_f(x)}{Z_p} \right] \text{, psi.} \qquad (4\text{-}4)$$

4.6 STRUT STRESS

The axial stress in the strut is simply the strut load divided by the cross-sectional area of the strut:

$$\sigma_s = \frac{W}{A_s} \text{ , psi.} \qquad (4\text{-}5)$$

4.7 LONGITUDINAL STRESS AT THE WING BULKHEAD

The longitudinal stress in the outer hull plating for the inside and outside surfaces (+ for inside, - for outside) at the intersection with the wing bulkhead (see Figure 10) is:

$$\sigma_{LB} = \pm \frac{1.734K'(1-\nu^2)^{-1/2}}{2V'N' - (K')^2} \Bigg\{ \left[\frac{(\Delta p)R_O}{t_O}\right] \cdot \left[(1+H'_M) + \left(\frac{L_{e,O}t_O}{A_{TO}}\right) \frac{b'_B t_O^3}{b'_B t_O^3 + L_{e,O} t_B^3}\right]\left[1 - \frac{\nu}{6}\rho\right] + \frac{p'R_H^2 E_O}{A_{TH}R_O E_H}\left[1 - \frac{\nu}{2}\right]\left[\frac{A_{TH}}{t_H} + \frac{A_{fH}H_M}{t_H} - L_{e,H}\right]\Bigg\} + \frac{(\Delta p)R_O \rho}{6t_O}, \text{ psi} \quad (4\text{-}6)$$

where:

b'_B = radial width of wing bulkhead panel, measured perpendicular to outer hull plating, inches; and

t_B = thickness of bulkhead flat plating, inches.

4.8 STRESSES AT MIDBAY

The effective length of the outer hull plating interacting with the outer hull frame, as computed by equation (3-8t), is normally less than the spacing between frames. As a consequence, the effect of submergence pressure on the outer hull plating at midbay is negligible; therefore, it is necessary to calculate midbay stress only for the case of maximum differential pressure (ship at surface).

4.8.1 Stress States When the Ship Broaches

The stresses in the outer hull plating at midbay are at a maximum when the ship broaches and are as follows:

(A) for the circumferential stress at midbay

$$\sigma_{\phi} = \frac{(\Delta p)\, R_O}{t_O} + \nu \frac{(\Delta p)\, R_O \rho}{6t_O} \text{, psi, and} \qquad (4\text{-}7)$$

(B) for the longitudinal stress at midbay

$$\sigma_{\frac{L}{2}} = \frac{(\Delta p)\, R_O \rho}{6t_O} + \nu \frac{(\Delta p)\, R_O}{t_O} \text{, psi.} \qquad (4\text{-}8)$$

4.8.2 Stress States When the Ship is Submerged

It should be noted that equations (4-7) and (4-8) do not include the effects of hydrostatic pressure and that, at a submerged depth, the outer hull plating is designed for the maximum stress at the frame rather than at midbay under a combined loading of differential and hydrostatic pressure. Consequently, the equations in Sections 4.2, 4.4, and 4.5 should be used when designing outer hull plating at a submerged depth.

4.9 STRESSES FOR NONUNIFORM AND NONCIRCULAR SECTIONS

In analyzing a strut-supported noncircular ballast tank, or a tank having struts of unequal spacing or stiffness, the following guidance is provided:

a. Tank structure, in way of each strut, may be analyzed separately using the instantaneous radius to the point of intersection of the outer hull plating and the neutral axis of the strut, and the average angle between adjacent struts. For example, in analyzing the structure at point "B" in Figure 2, the load in strut "B" would be calculated by inserting in equations (3-1) through (3-8) the radius R_B, and the average angle α_B. The same procedure would be followed in calculating the load in struts "A", "C", and "D". The angle α' shown in Figure 2 should be limited to a maximum value of 5° in order to minimize the load component normal to the strut at its connection to the pressure hull.

b. After obtaining the load in strut "B", the procedures outlined in the preceding sections are then followed to obtain the stress levels in each structural element, using the corresponding radius, R_B, in this case, in lieu of R_O. Thus, separate calculations are required for each change of radius.

SECTION 5 - LOADS FOR STIFFENED END TANKS

5.1 GENERAL

The stiffened end tanks are located at the bow and stern of the submarine and are cantilevered off the submarine end closures. Load criteria for these tanks, in addition to typical internal pressures, must include the various externally applied loads for all bow and stern structure.

5.2 DESIGN LOADS

5.2.1 Internal Design Loads

The stiffened end tanks shall be designed using the conditions in Section 2.2 and Tables I and II. The allowable stress limits shall be as given in Tables I and II.

5.2.2 External Design Loads

External load criteria for the stern end tank is found in Reference (6) and Detailed Ship Specifications. In addition, the design of the stern end tank may be dependent on design deflection limits established by the alignment tolerances of the control surface connecting rods, propulsion shafting, and associated bearings, or by vibration considerations.

The bow end tank also must withstand the external loads due to sea slap, maneuvering, and other operational pressures as given in the Ship Specifications. The progression toward larger submarines and higher

where:

$$M_T = -3.919 \left[\frac{(\Delta p)\ L_O\ R_T^2\ J}{Q} \right]; \qquad (6\text{-}3a)$$

$$P_T = -(\Delta p)\ L_O\ R_T + 14.346 \left[\frac{(\Delta p)\ L_O\ R_T\ J}{Q} \right]; \qquad (6\text{-}3b)$$

$$Q = .174 \left[\frac{R_T^4}{I_O^2} \right] + 18.614 \left[\frac{R_T^2}{A_{TO} I_O} \right] + \frac{18.44}{A_{TO}^2}; \qquad (6\text{-}3c)$$

$$J = \frac{R_T^2}{A_{TO} I_O} + \frac{1}{A_{TO}^2}; \qquad (6\text{-}3d)$$

Z_{min} = minimum section modulus, Z_p or Z_f, whichever is the minimum value, in^3, (see Figure 11); and

R_T = radius to the neutral axis of the circumferential frame, inches.

The moment, M_T, and the axial force, P_T, given above, are taken from the derivation in Appendix D using the virtual work method.

6.3.2 Secondary Transverse Stiffeners

Transverse stiffeners used in conjunction with primary longitudinal stiffeners subdivide the outer hull plating into plate panels and provide intermediate support to the primary longitudinal stiffeners. Since these secondary transverse stiffeners are continuous and rigidly attached to the primary longitudinal stiffeners, they may be designed as fixed-ended beams under an average uniformly distributed load equivalent to the average vertical component of the differential pressure multiplied by the width of the outer hull plating interacting with the stiffener (see Figure 12). The hoop force is combined with the average uniformly distributed load acting on the beam in order to determine the stress in the secondary transverse stiffener:

$$\sigma_{end} = \frac{M_{ST}}{Z_{min}} + \frac{(\Delta p) L_O R_T}{A_{TO}} \text{, psi} \qquad (6\text{-}4)$$

where:

$$M_{ST} = \frac{(\Delta p_t) L_O l_T^2}{12}$$ = bending moment at the end of the secondary stiffener (in-lb); (6-4a)

$$\Delta p_t = \frac{\Delta p}{2} \left(1 + \cos \frac{\theta_t}{2}\right) \text{, psi (see Figure 12);} \qquad (6\text{-}4b)$$

l_T = length of transverse beam element, inches; and

θ_t = angle between two adjacent primary longitudinal stiffeners, degrees (see Figure 12).

6.4 OUTER HULL PLATING

The maximum stresses in the outer hull plating due to the uniformly distributed differential pressure, Δp, are presented in the following sections for (a) the longitudinally stiffened end tank, and (b) the transversely stiffened cantilevered end tank.

6.4.1 Longitudinally Stiffened Outer Hull Plating

The maximum combined stress in the outer hull plating of a longitudinally stiffened end tank occurs in the panel edge at the midpoint of the long side and is obtained by adding together the flat plate and membrane solution stresses from References (7) and (8) as follows:

$$\sigma_{LOHP} = \phi \frac{\Delta p_1 b'^2}{t_O^2} + n'_2 \left[E \left(\frac{\Delta p_2 a'}{t_O} \right)^2 \right]^{1/3} \text{, psi} \qquad (6\text{-}5)$$

where:

$\Delta p_1 = \dfrac{Z\, Et_O^{3}}{\epsilon\, b'^{4}}$, portion of the differential pressure Δp related to plate bending, psi, where $\Delta p = \Delta p_1 + \Delta p_2$; (6-5a)

$\Delta p_2 = \dfrac{Z^3\, Et_O}{(n'_1)^3 a'^4}$, portion of the differential pressure Δp related to membrane action, psi, where $\Delta p = \Delta p_1 + \Delta p_2$; (6-5b)

$Z = \sqrt[3]{-\dfrac{B'}{2} + F} + \sqrt[3]{-\dfrac{B'}{2} - F}$, deflection of outer hull plating for longitudinally stiffened end tanks, inches; (6-5c)

$F = \sqrt{\dfrac{B'^2}{4} + \dfrac{A'^3}{27}}$; (6-5d)

$B' = -\dfrac{\Delta p\,(n'_1)^3\, a'^4}{Et_O}$; (6-5e)

$A' = \dfrac{(n'_1)^3\, t_O^{2}}{\epsilon}\left[\dfrac{a'}{b'}\right]^4$; (6-5f)

E = modulus of elasticity of cantilevered structure, psi;

a' = length of long side of panel, inches;

b' = length of short side of panel, inches;

ϵ = deflection coefficient for a uniformly loaded rectangular plate having all edges fixed (see Table A);

ϕ = stress coefficient for a uniformly loaded rectangular plate having all edges fixed (see Table A);

n'_1 = membrane deflection coefficient (see Table B); and

n'_2 = membrane stress coefficient (see Table B).

TABLE A

DEFLECTION AND STRESS COEFFICIENTS FOR
A UNIFORMLY LOADED RECTANGULAR
PLATE, ALL EDGES FIXED

a'/b'	1.0	1.2	1.4	1.6	1.8	2.0	∞
ϵ	.0138	.0188	.0226	.0251	.0267	.0277	.0284
ϕ	.3078	.3834	.4356	.4680	.4872	.4974	.498

TABLE B

MEMBRANE DEFLECTION AND
STRESS COEFFICIENTS

a'/b'	1.0	1.5	2.0	2.5	3.0	4.0	5.0
n'_1	.318	.228	.160	.125	.100	.068	.052
n'_2	.356	.370	.336	.304	.272	.230	.205

6.4.2 Transversely Stiffened Outer Hull Plating

The outer hull plating for a transversely stiffened cantilevered end tank should be designed using the maximum stresses as computed by the following equations. In equations (6-6), (6-7), (6-8), and (6-9), the upper plus sign in the parenthesis relates to the "outside surface" stress and the lower minus sign relates to the "inside surface" stress.

The circumferential stress at midbay is:

$$\sigma_{\phi} = \sigma_h \; [1 + k\,(-F_2 \pm 0.3\,F_4)] \text{ , psi.} \qquad (6\text{-}6)$$

The longitudinal stress at midbay is:

$$\sigma_{\frac{L}{2}} = \sigma_h \; (0.5 \pm k\,F_4) \text{ , psi.} \qquad (6\text{-}7)$$

The circumferential stress at the frame is:

$$\sigma_{\phi p}(x) = \sigma_h \; [1 + k\,(-1 \pm 0.3F_3)] \text{ , psi.} \qquad (6\text{-}8)$$

The longitudinal stress at the frame is:

$$\sigma_L(x) = \sigma_h \; (0.5 \pm k\,F) \text{ , psi.} \qquad (6\text{-}9)$$

The parameters in the equations above are determined as follows:

$$\sigma_h = \frac{\Delta p \, R_O}{t_O} \text{ , shell plating hoop stress, psi;} \qquad (6\text{-}10a)$$

$$k = \frac{(1 - \nu/2)\alpha_O}{\alpha_O + \beta_O + F_1\,(1-\beta_O)} \; ; \qquad (6\text{-}10b)$$

where:

α_O, β_O and A'_O are as previously defined in Section 3, equations 3-8h, 3-8i and 3-8j-1, respectively.

The following equations which represent the stress functions F_1, F_2, F_3, and F_4 are presented graphically in Figures 13 through 16 of Appendix A:

$F_1 = F_{1,O\Delta p}$ (same as equation 3-8p);

$F_2 = F_{2,0\Delta p}$ (same as equation 3-8q);

$$F_3 = \sqrt{\frac{3}{1-\nu^2}}\;\frac{\dfrac{(\cos\eta_{2,0\Delta p}\theta_O)\sin\eta_{2,0\Delta p}\theta_O}{\eta_{2,0\Delta p}} + \dfrac{(\cosh\eta_{1,0\Delta p}\theta_O)\sinh\eta_{1,0\Delta p}\theta_O}{\eta_{1,0\Delta p}}}{\dfrac{(\cosh\eta_{1,0\Delta p}\theta_O)\sinh\eta_{1,0\Delta p}\theta_O}{\eta_{1,0\Delta p}} + \dfrac{(\cos\eta_{2,0\Delta p}\theta_O)\sin\eta_{2,0\Delta p}\theta_O}{\eta_{2,0\Delta p}}}$$

$F_4 = F_{4,0\Delta p}$ (same as equation 3-8r).

SECTION 7 - BULKHEAD STRESS EQUATIONS

7.1 GENERAL

The structural arrangement of an MBT bulkhead is comprised of a flat plate orthogonally stiffened (see Figure 17a), or a flat plate stiffened by radial and tangential members (see Figure 9c) and may generally be classified in the following two categories:

a. Intermediate Bulkheads - Bulkheads that separate adjacent compartments of an MBT by forming a common boundary, such as a longitudinal centerline bulkhead (see Figure 4). Intermediate bulkheads are also referred to as wing bulkheads when they are used in ballast tanks that are concentric to the pressure hull and strut-supported.

b. End Bulkheads - Bulkheads that are used to close the ends of the MBT. When end bulkheads are used in ballast tanks that are concentric to the pressure hull and strut-supported, they are also referred to as wing bulkheads (see Figure 9a).

7.2 DESIGN LOADING CONDITIONS

The mandatory design limits and design loading conditions for intermediate and end bulkheads are given in Tables I and II.

Figure 17a represents a typical loading distribution on any bulkhead stiffener. The parameter "s" is the spacing between any two stiffeners, and the parameter "L_{BS}" represents the length of any bulkhead stiffener. The trapezoidal loading distribution shown applies to primary as well as secondary stiffeners, with the primary stiffeners extending from one edge of the bulkhead to the other, while secondary stiffeners extend only between primary stiffeners. In addition, the loading distribution is applicable to radial and tangential bulkhead stiffeners.

7.3 DESIGN CRITERIA

The following design criteria are applicable to both intermediate and end bulkheads, whether they are orthogonally stiffened or radially and tangentially stiffened. For the following paragraphs, the radially oriented stiffeners are treated as primary members, while those oriented in the tangential direction, and positioned between radial stiffeners, are treated as secondary members. For orthogonal stiffeners, the large continuous stiffeners are the primary stiffeners, and the smaller stiffeners positioned between the primary stiffeners are the secondary stiffeners. If the secondary and primary stiffeners are approximately the same size, then a computer solution of the grillage is necessary and the following equations cannot be used.

7.3.1 Flat Plating

The thickness of flat plating used in MBT bulkheads is determined from the following equation:

$$t_B = \left[\frac{6C\,(\Delta p)\,(b')^2}{\sigma_a} \right]^{1/2}, \text{ inches} \qquad (7\text{-}1)$$

where:

$C = \dfrac{0.125}{3 + 4\left[\dfrac{b'}{a'}\right]^4}$ = center-of-panel moment coefficient for a flat plate;

Δp = maximum differential pressure, psi;

σ_a = allowable design stress in the bulkhead plating, psi; and

a',b' = length of long and short side of the panel, respectively, inches.

7.3.2 Primary Stiffeners

The primary stiffeners shall be designed by assuming that both ends of the stiffener are simply supported. The load on each stiffener is a trapezoidal load equal to the product of the maximum differential pressure, Δp, and the primary stiffener spacing, s_b, (see Figure 17b).

The following equation represents the maximum stress, σ_{BSP}, on the primary bulkhead stiffeners at $L_{BSP/2}$, (see Reference 13):

$$\sigma_{BSP} = \pm \frac{\Delta p \; s_b \; (L_{BSP})^2}{24 \; Z_B} \left[3 - \left(\frac{s_b}{L_{BSP}} \right)^2 \right], \text{ psi} \qquad (7\text{-}2)$$

where:

L_{BSP} = length of the primary bulkhead stiffener, inches, (for wing bulkhead primary radial stiffeners, $L_{BSP} = R_O - R_H$);

s_b = spacing of the bulkhead primary stiffeners, inches, (for wing bulkheads, s_b, is the average tangential distance between radial stiffeners at the mid-height position of each stiffener), (see Figure 9c);

Z_B = section modulus of the free flange and web of the primary bulkhead stiffener plus the effective width of the bulkhead plating, w_B, in^3; and

w_B = $2t_B\sqrt{E_B/\sigma_y}$ or s_b, whichever is less, inches. $2t_B\sqrt{E_B/\sigma_y}$ is the effective width of the bulkhead plating.

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TABLE I EMERGENCY BLOW DESIGN LIMITS

STRUCTURAL COMPONENTS	TYPE OF TANKS	DESIGN CONDITION	DESIGN LOAD	DESIGN LIMIT (1)
GROUP A				
1. OUTER HULL PLATING 2. PRIMARY LONGITUDINAL STIFFENERS 3. PRIMARY CIRCUMFERENTIAL FRAMES 4. SECONDARY STIFFENERS 5. END BULKHEAD PLATES 6. END BULKHEAD STIFFENERS	END TANKS	SHIP BROACHING	DIFFERENTIAL PRESSURE - Δp GREATER OF Δp OR 30 psi	$1.25\sigma_Y$
GROUP B		DESIGN ALL STRUCTURAL COMPONENTS (1-6) FOR BOTH DESIGN CONDITIONS BELOW		
1. OUTER HULL PLATING 2. PRIMARY LONGITUDINAL STIFFENERS 3. PRIMARY CIRCUMFERENTIAL FRAMES 4. SECONDARY STIFFENERS 5. END BULKHEAD PLATING 6. END BULKHEAD STIFFENERS	STRUT-SUPPORTED TANKS	1. SHIP BROACHING	DIFFERENTIAL PRESSURE - Δp GREATER OF Δp OR 30 psi	$1.25\sigma_Y$
		2. COLLAPSE DEPTH	HYDROSTATIC PRESSURE - P	σ_Y
GROUP C				
1. INTERMEDIATE BULKHEAD (2) PLATING AND STIFFENERS (INCLUDES CENTERLINE BULKHEADS)	END TANKS AND STRUT-SUPPORTED TANKS	NONE	INTERMEDIATE BULKHEAD PLATING AND STIFFENERS ARE NOT DESIGNED FOR EMERGENCY BLOW CONDITIONS (SEE NORMAL SURFACING CONDITION - TABLE II)	N/A (SEE TABLE II)

(1) SHEAR STRESS FOR ALL DESIGN CONDITIONS IS $0.60\sigma_y$.
(2) SEE SECTION 7 FOR DESCRIPTION OF BULKHEAD

TABLE II NORMAL SURFACING CONDITION DESIGN LIMITS

STRUCTURAL COMPONENTS	TYPE OF TANKS	DESIGN CONDITION	DESIGN LOAD	DESIGN LIMIT(1)
GROUP A				
1. OUTER HULL PLATING 2. END BULKHEAD PLATING 3. INTERMEDIATE BULKHEAD PLATING (INCLUDES CENTERLINE BULKHEAD PLATING)	END TANKS AND STRUT-SUPPORTED TANKS	ANY SUBMERGED DEPTH, BUT NOT GREATER THAN OPERATING DEPTH	EQUIVALENT DIFFERENTIAL PRESSURE-WHICH IS THE HYDROSTATIC PRESSURE CREATED BY A COLUMN OF WATER THE HEIGHT OF THE BALLAST TANK PLUS 5 PSI SURCHARGE	σ_Y
GROUP B				
1. PRIMARY LONGITUDINAL STIFFENERS 2. PRIMARY CIRCUMFERENTIAL FRAMES 3. SECONDARY STIFFENERS 4. END BULKHEAD STIFFENERS 5. INTERMEDIATE BULKHEAD(2) STIFFENERS (INCLUDES CENTERLINE BULKHEADS)	END TANKS AND STRUT-SUPPORTED TANKS	ANY SUBMERGED DEPTH, BUT NOT GREATER THAN OPERATING DEPTH	EQUIVALENT DIFFERENTIAL PRESSURE-WHICH IS THE HYDROSTATIC PRESSURE CREATED BY A COLUMN OF WATER THE HEIGHT OF THE BALLAST TANK PLUS 5 PSI SURCHARGE	$.67\sigma_Y$

(1) SHEAR STRESS FOR ALL DESIGN CONDITIONS IS $0.60\sigma_y$.
(2) SEE SECTION 7 FOR DESCRIPTION OF BULKHEAD

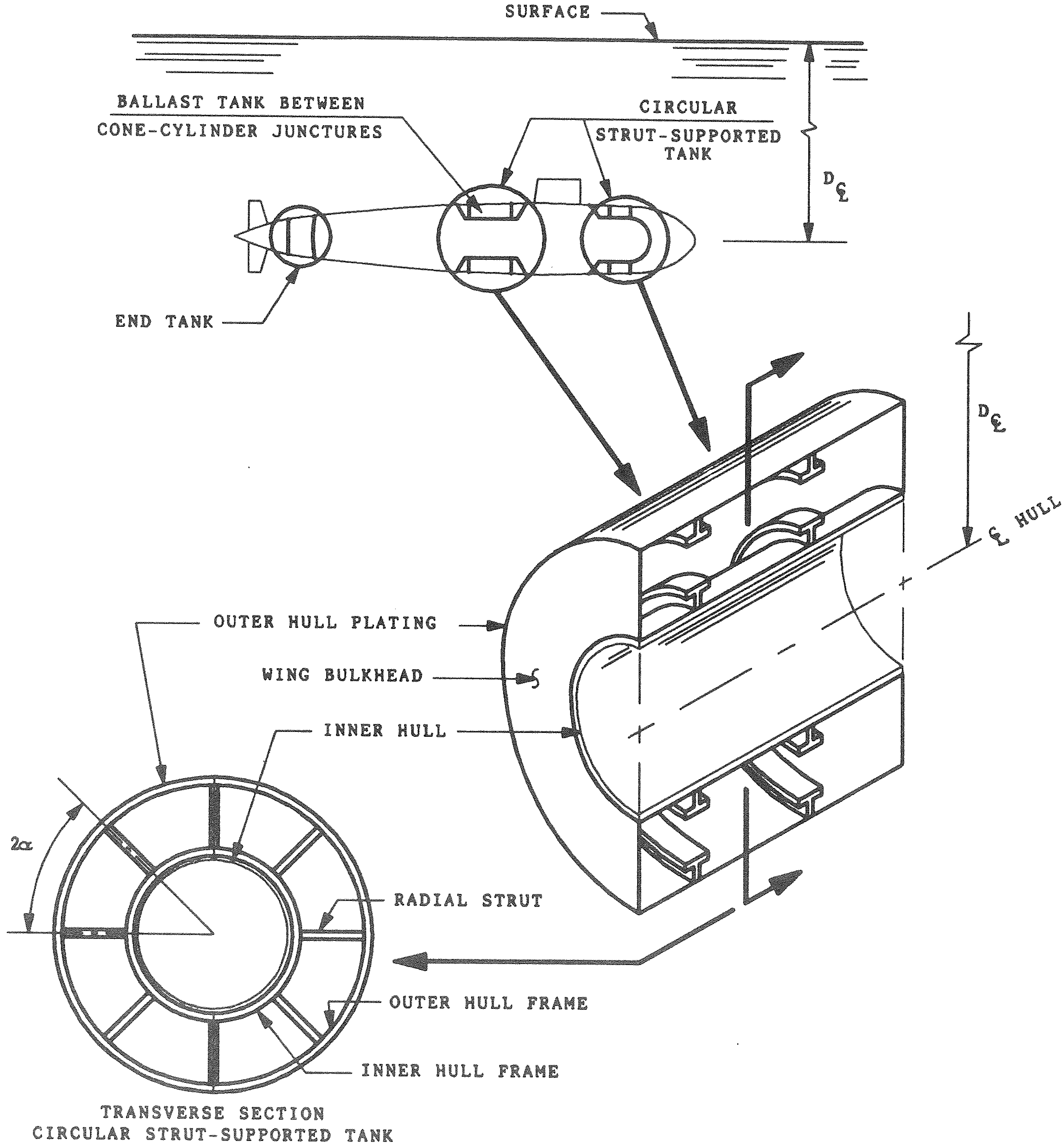


FIGURE 1. STRUCTURAL ARRANGEMENT FOR CIRCULAR MAIN BALLAST TANKS

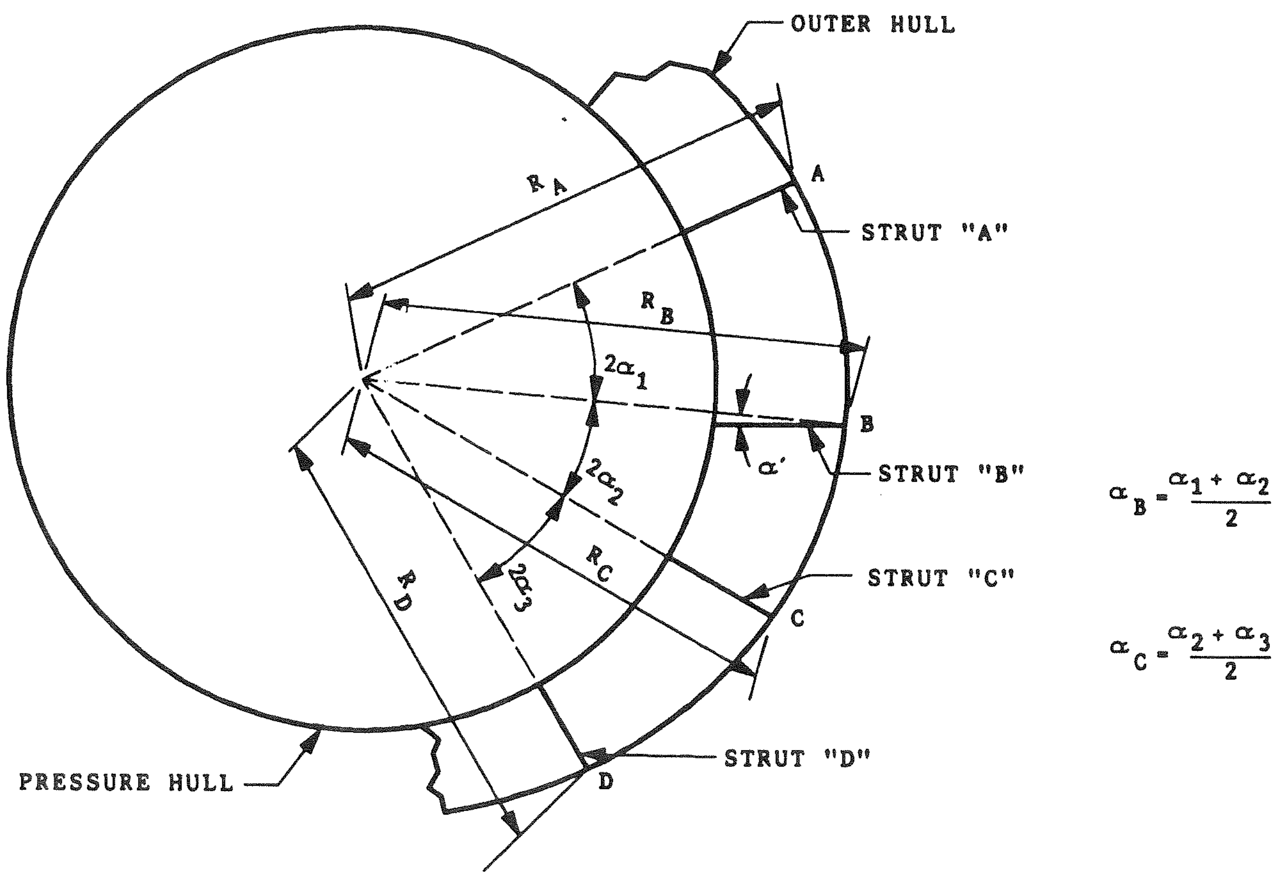
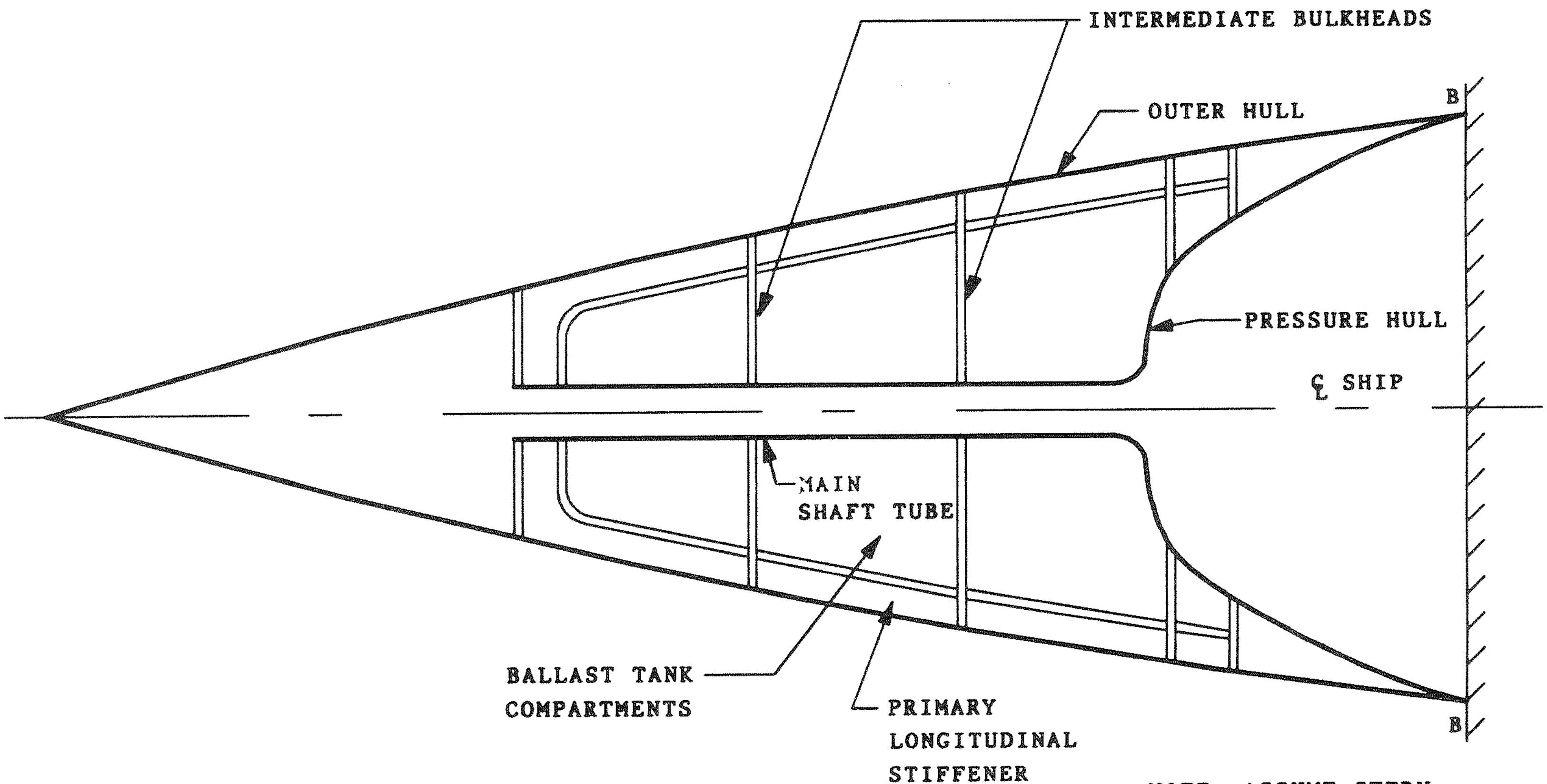
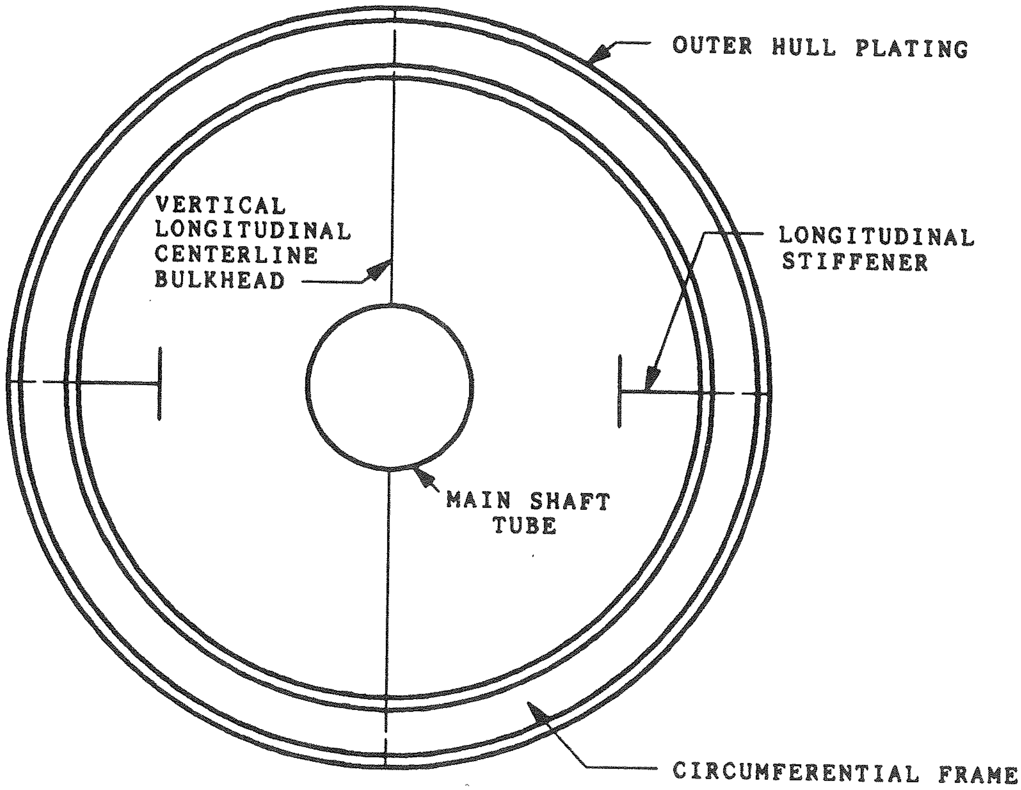


FIGURE 2. STRUCTURAL ARRANGEMENT FOR NONCIRCULAR MAIN BALLAST TANKS

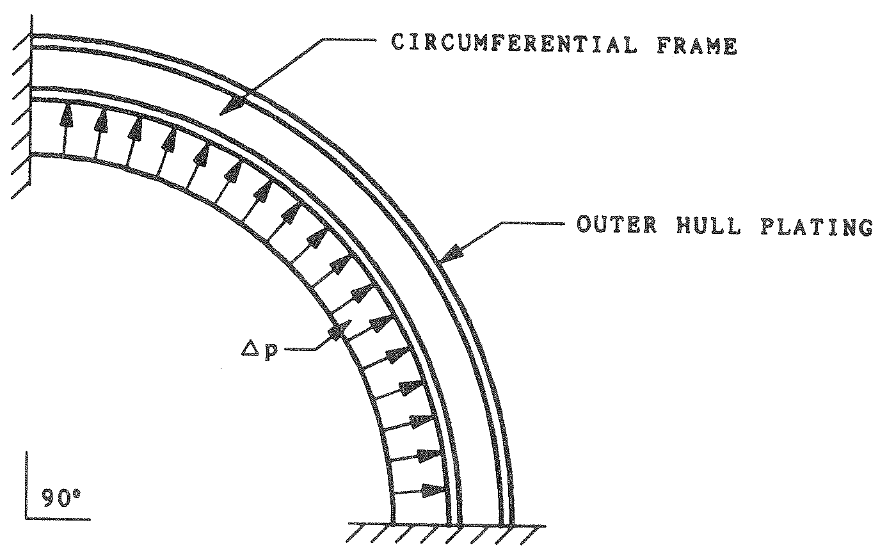


NOTE: ASSUME STERN STRUCTURE CANTILEVERED AT CONNECTION OF OUTER HULL TO PRESSURE HULL B-B

FIGURE 3. CANTILEVERED END TANK - LONGITUDINALLY STIFFENED



a.) <u>CROSS-SECTION OF TRANSVERSELY STIFFENED END BALLAST TANK</u>



b.) <u>TYPICAL SECTION REPRESENTATION FOR ANALYSIS</u>

FIGURE 4. <u>TRANSVERSELY STIFFENED END BALLAST TANK</u>

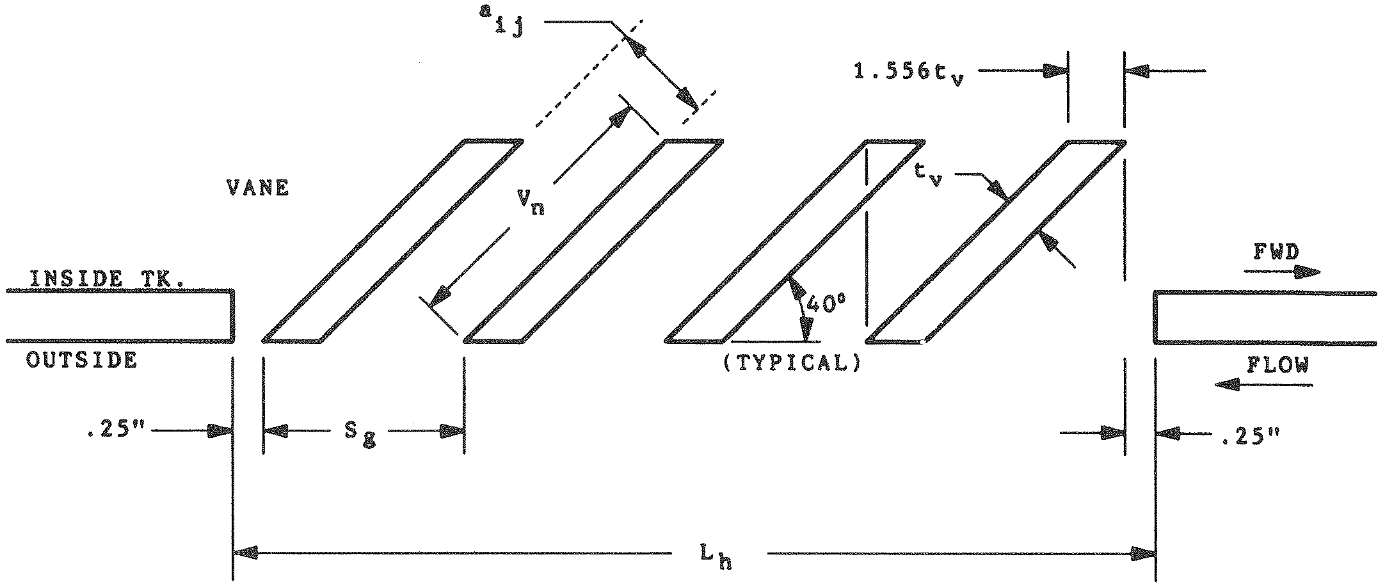


FIGURE 5. FLOOD HOLE PARAMETERS

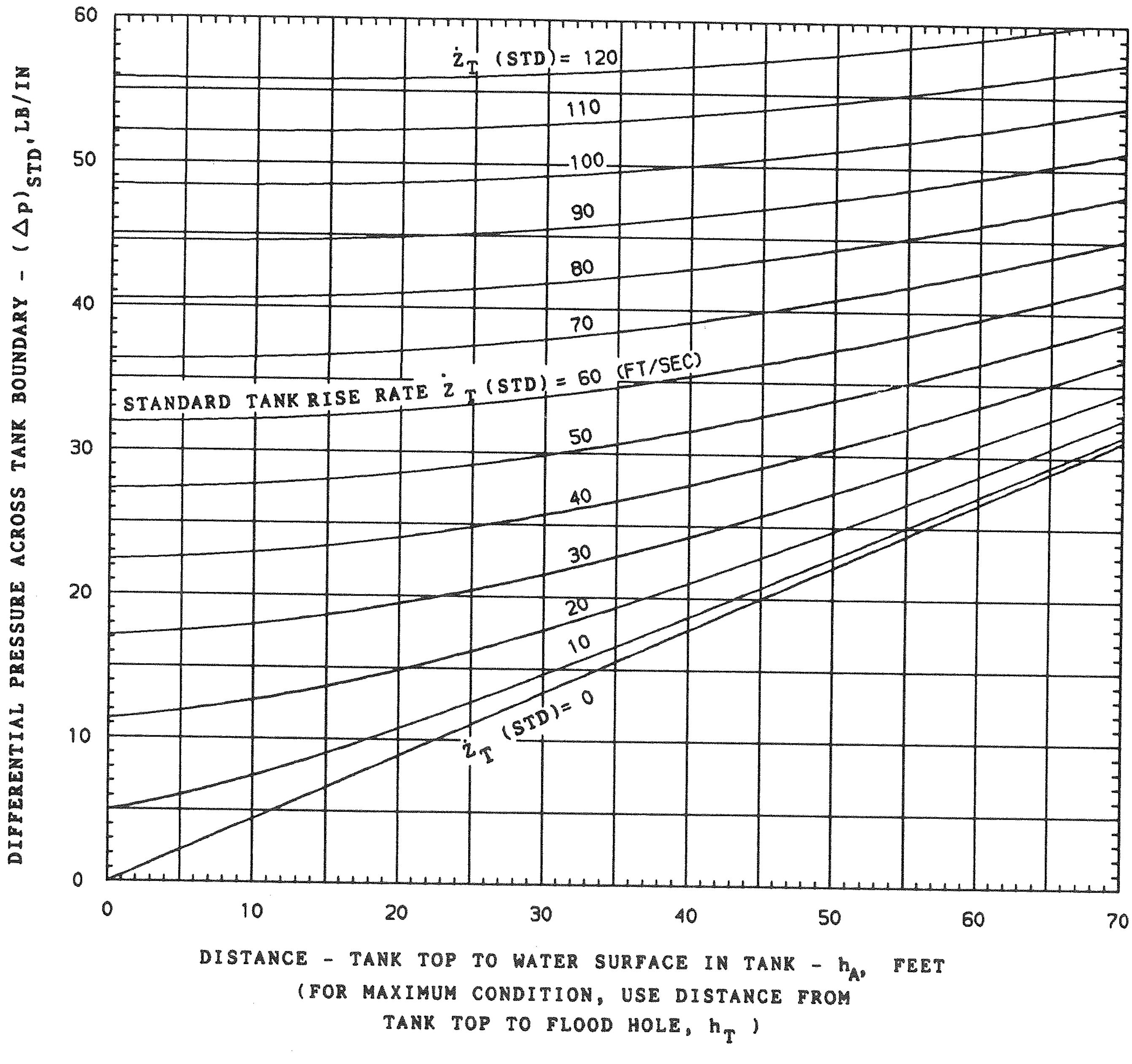
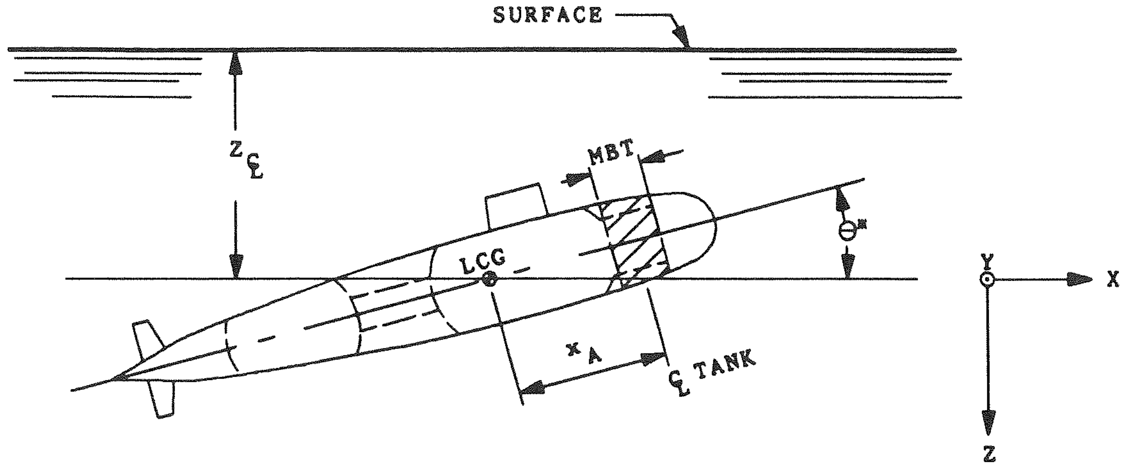
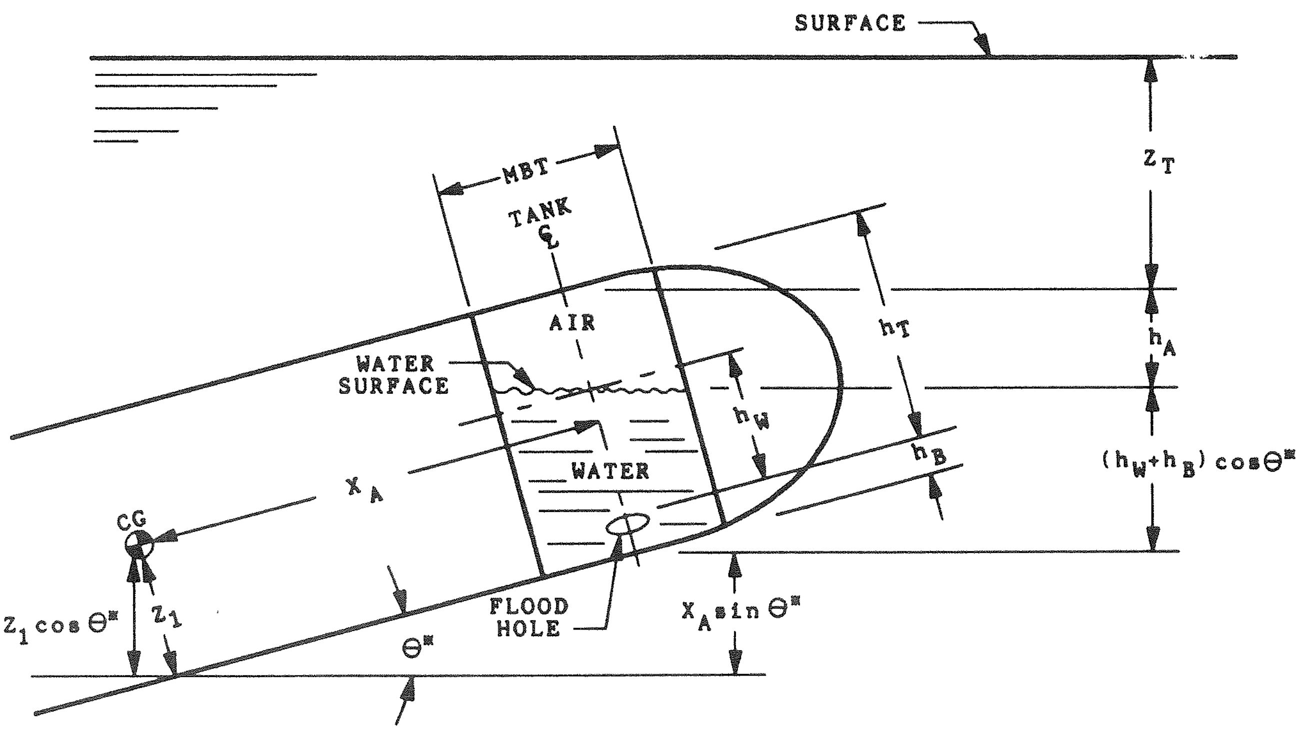


FIGURE 6. STEADY-STATE PRESSURE DIFFERENTIAL AT SURFACE OF BALLAST WATER (FOR "STANDARD" TANK WITH V_T = 100 A)

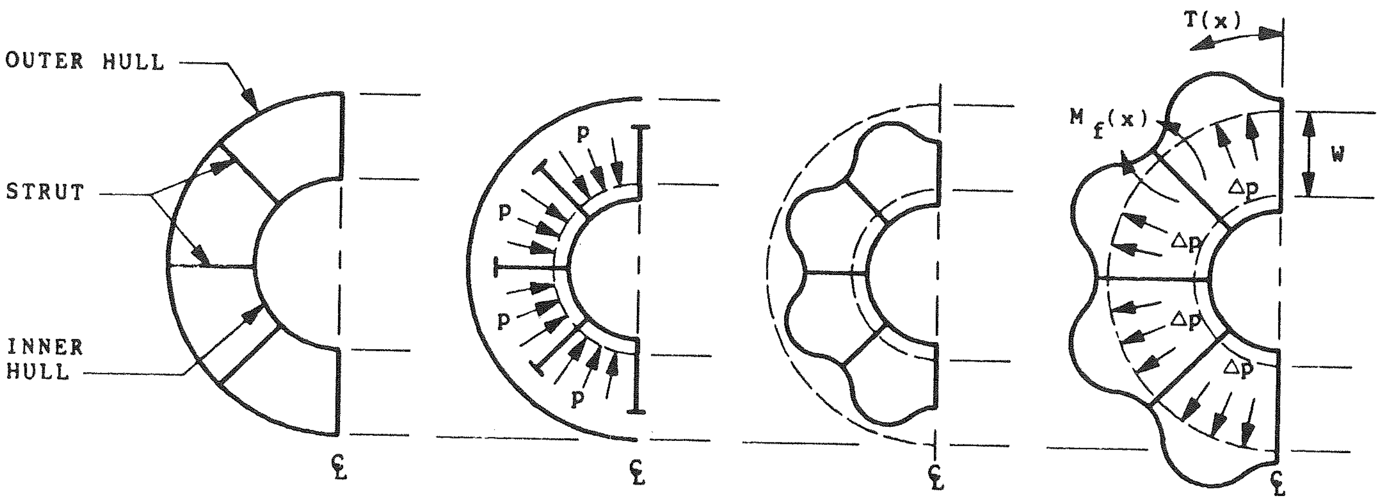


a.) SHIP DEPTH AND PITCH ANGLE



b.) TANK VOLUME PARAMETERS

FIGURE 7. DIMENSIONS FOR TANK VOLUME AND LOCATION



a.) INITIAL CONFIGURATION

b.) INNER HULL CONTRACTS AT DEPTH

c.) STRUTS PULL OUTER HULL INWARD AND OUTER HULL FLEXES

d.) BLOW PRESSURE EXTENDS STRUTS AND EXPANDS OUTER HULL

FIGURE 8. SEQUENCE OF LOAD DEFLECTIONS
(TRANSVERSE SECTION THROUGH MAIN BALLAST TANK)

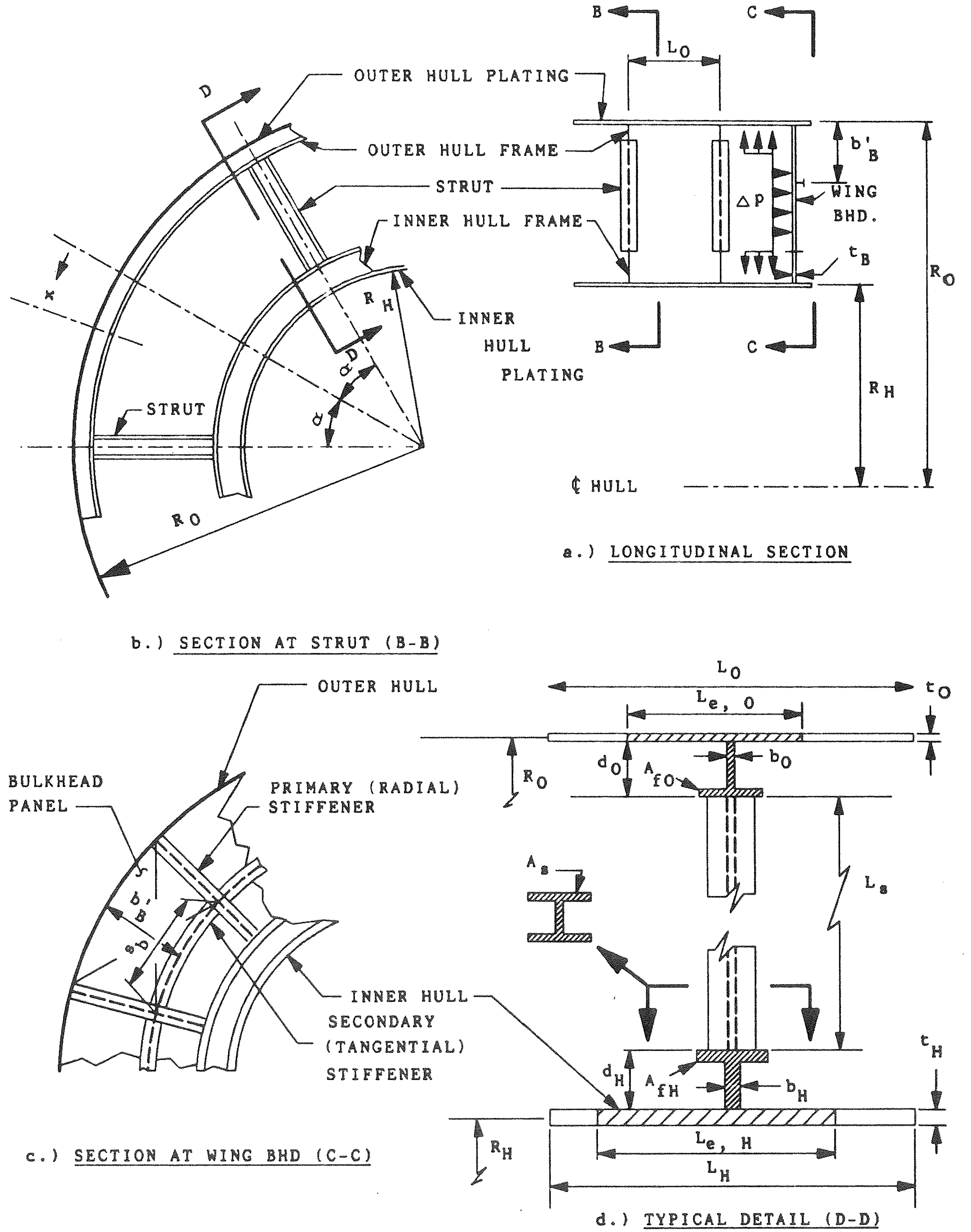


FIGURE 9. DIMENSIONS FOR TANK STRUCTURE (INNER AND OUTER HULLS, AND WING BULKHEAD)

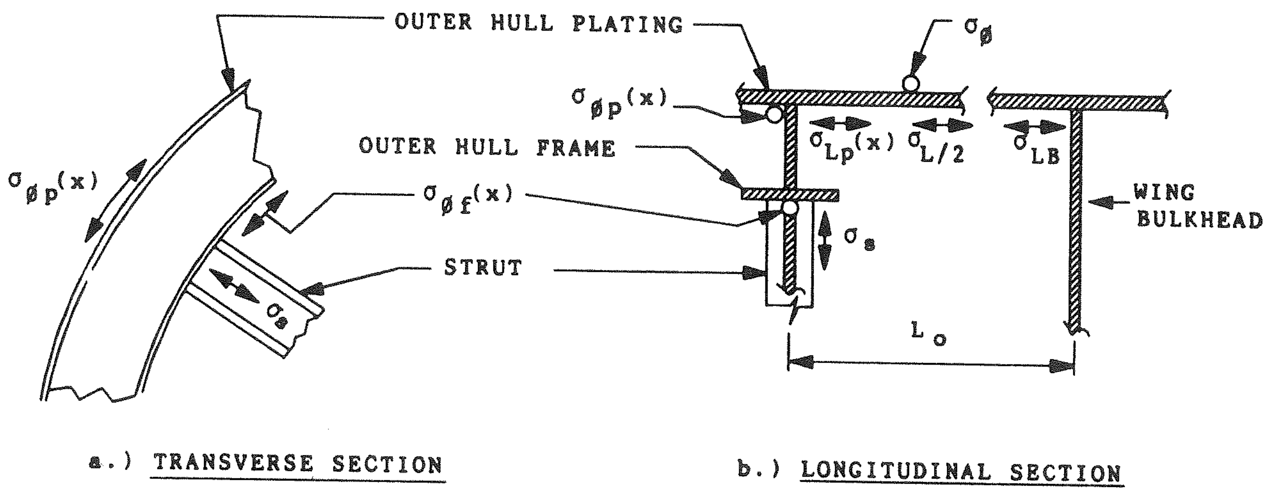
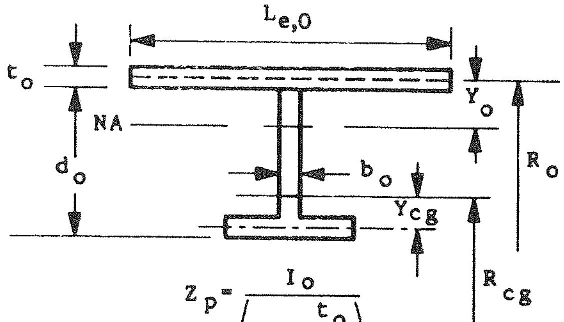


FIGURE 10. OUTER HULL STRUT-SUPPORTED STRUCTURE-
LOCATION AND DIRECTION OF CONTROLLING STRESSES

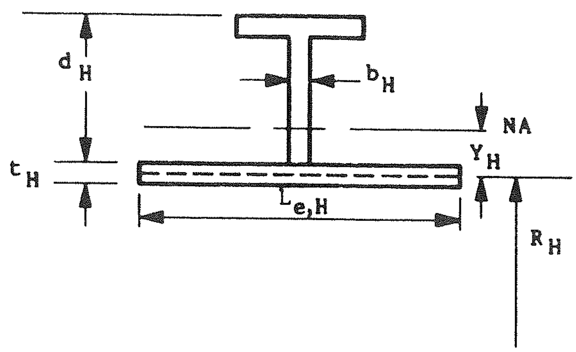
OUTER HULL CIRCUMFERENTIAL FRAME



$$Z_p = \frac{I_o}{\left(Y_o + \frac{t_o}{2}\right)}$$

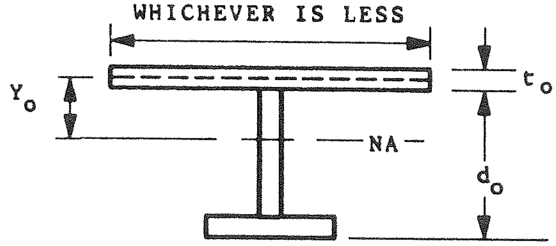
$$Z_f = \frac{I_o}{\left(d_o + \frac{t_o}{2}\right) - Y_o}$$

PRESSURE HULL CIRCUMFERENTIAL FRAME



OUTER HULL LONGITUDINAL FRAME

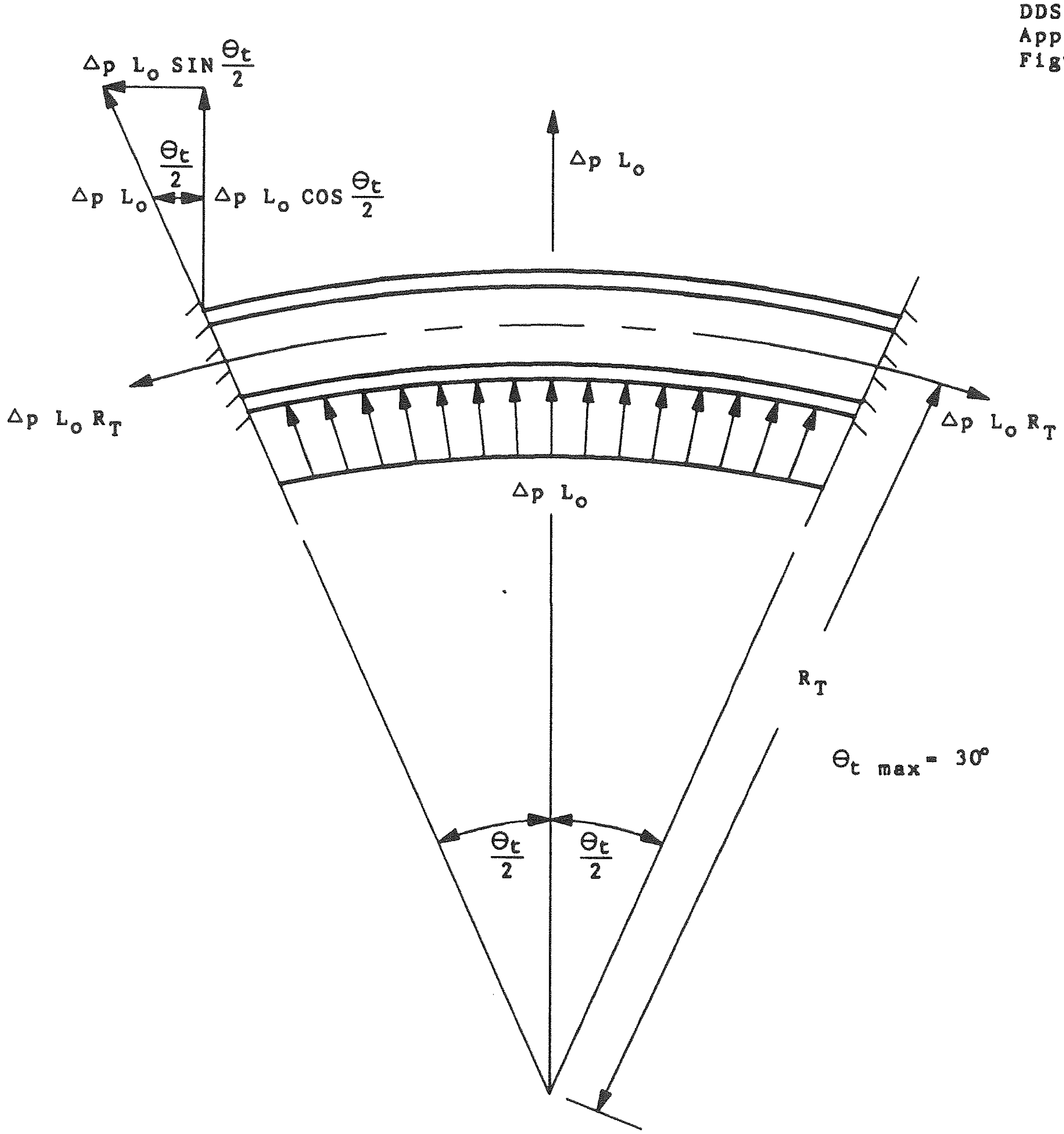
$W_L = 2\ t_o \sqrt{E/\sigma_Y}$
OR FRAME SPACING,
WHICHEVER IS LESS



$$Z_{pL} = \frac{I_o}{\left(Y_o + \frac{t_o}{2}\right)}$$

$$Z_{fL} = \frac{I_o}{\left(d_o + \frac{t_o}{2}\right) - Y_o}$$

FIGURE 11. FRAME GEOMETRIES



NEGLECTING $\Delta p\ L_o\ SIN\ \frac{\Theta_t}{2}$, THE CURVED SECONDARY TRANSVERSE STIFFENER BECOMES A FIXED ENDED BEAM WITH AN AVERAGE UNIFORMLY DISTRIBUTED LOAD $\Delta p_t\ L_o$

WHERE: $\Delta p_t\ L_o = \frac{\Delta p\ L_o \cos\frac{\Theta_t}{2} + \Delta p\ L_o}{2} = \frac{\Delta p\ L_o}{2}\left(1 + \cos\frac{\Theta_t}{2}\right)$

NOTE: AS $\frac{\Theta_t}{2} \longrightarrow 0 \qquad \Delta p_t \longrightarrow \Delta p$

FIGURE 12. SECONDARY TRANSVERSE STIFFENER LOAD DISTRIBUTION

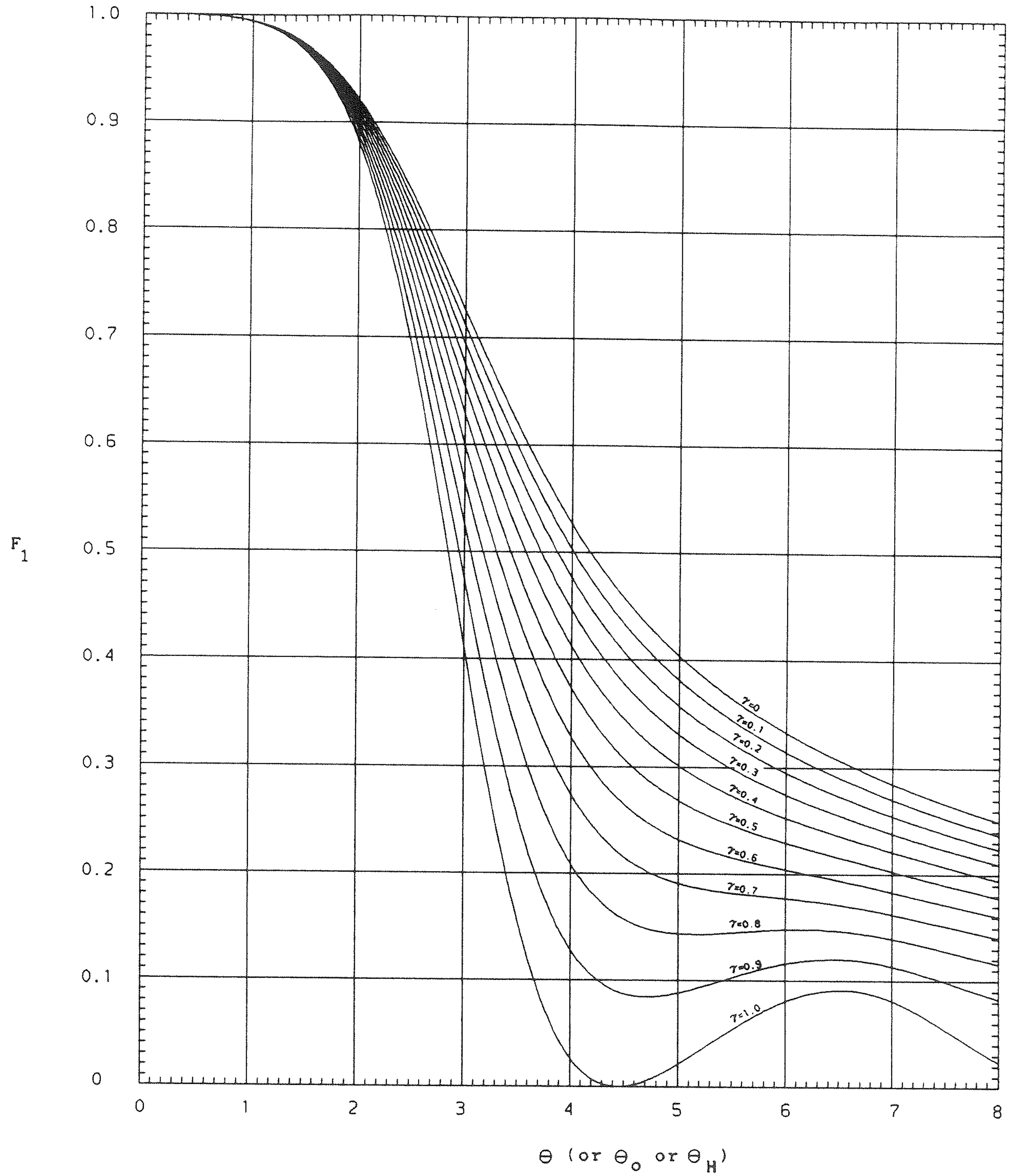


FIGURE 13. STRESS FUNCTION, F_1

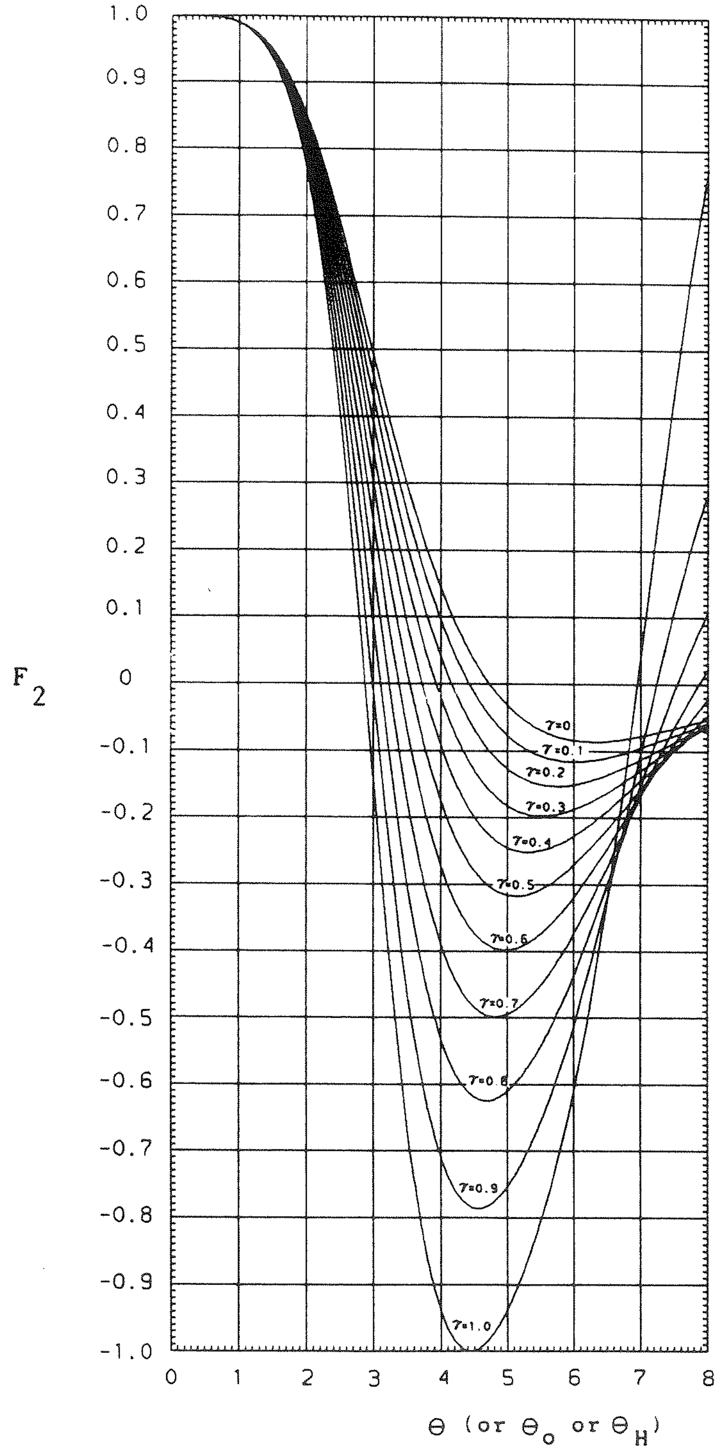


FIGURE 14. STRESS FUNCTION, F_2

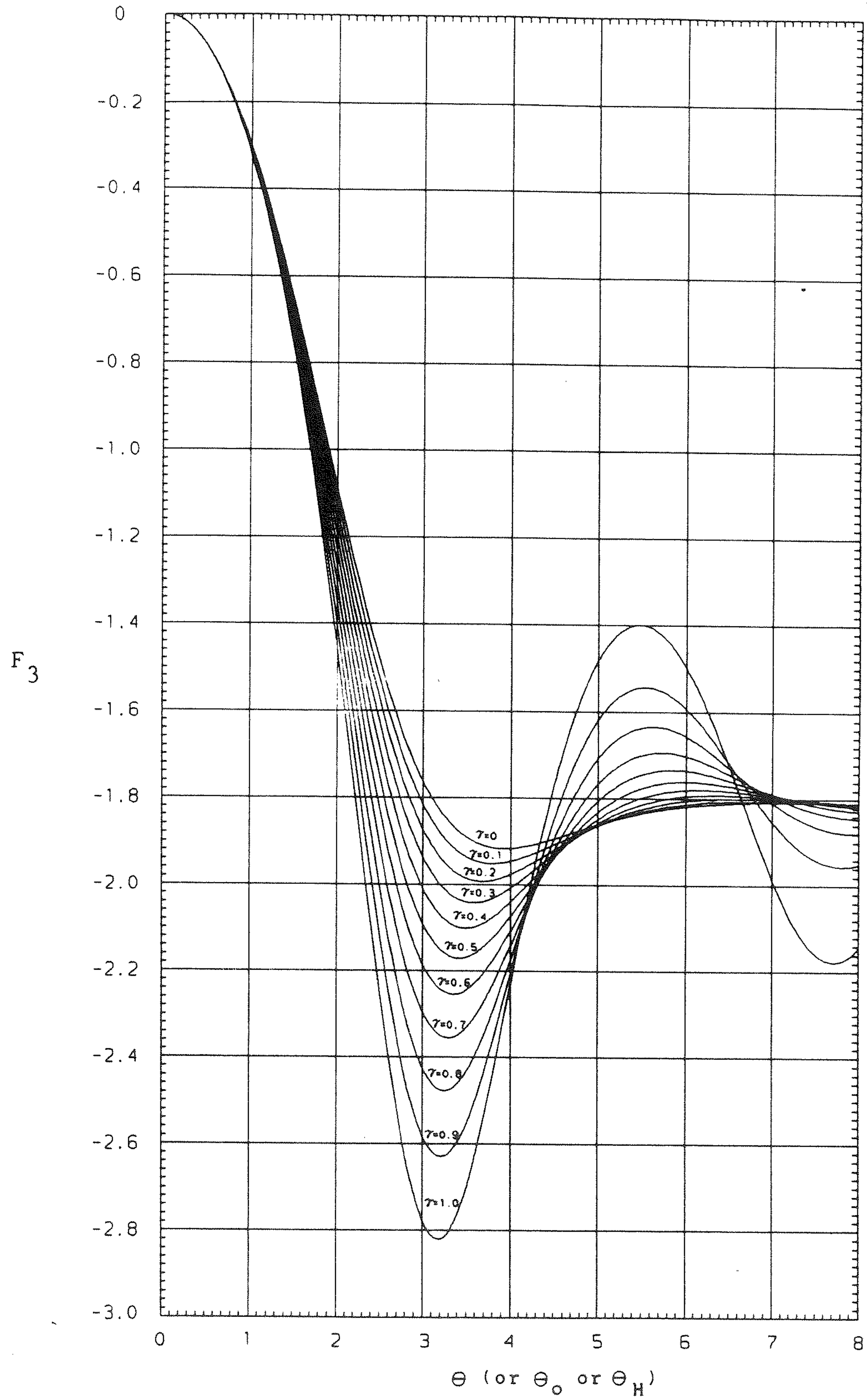


FIGURE 15. STRESS FUNCTION, F_3

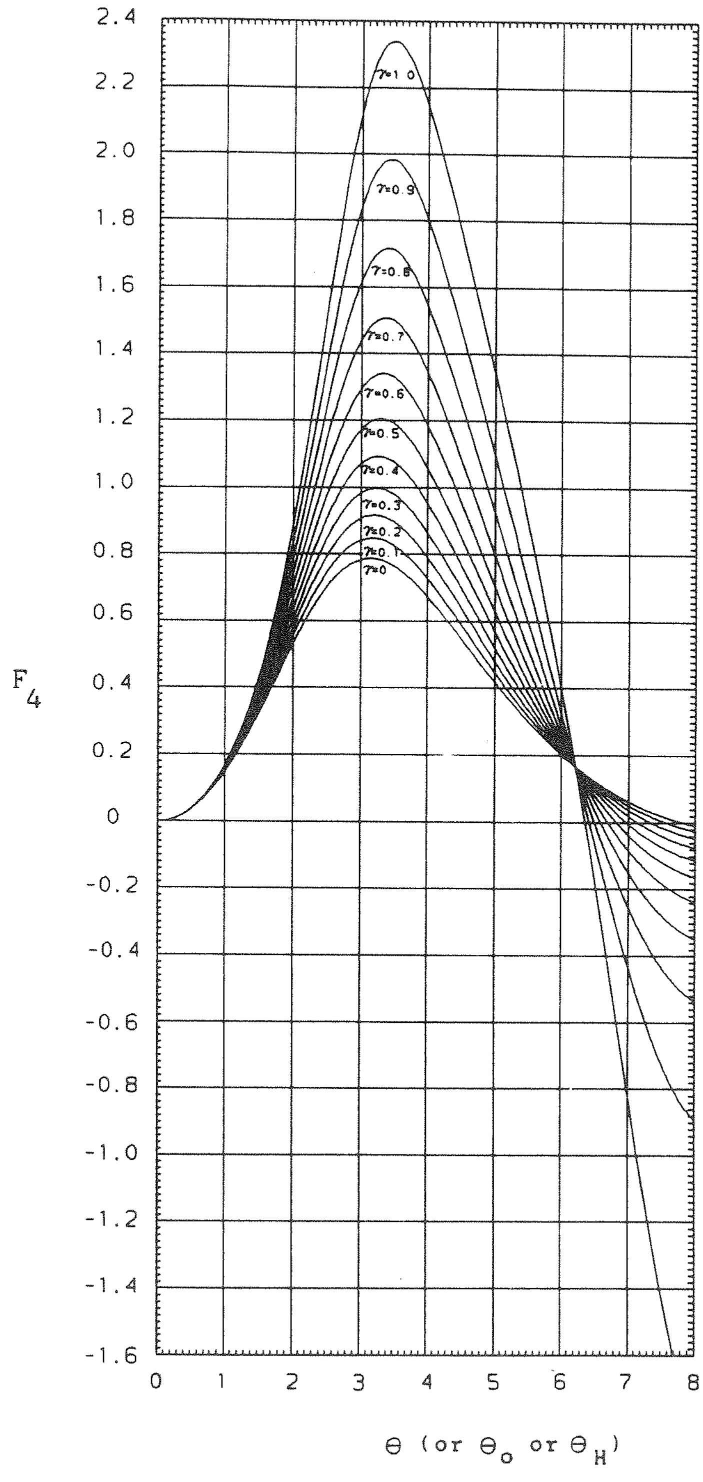
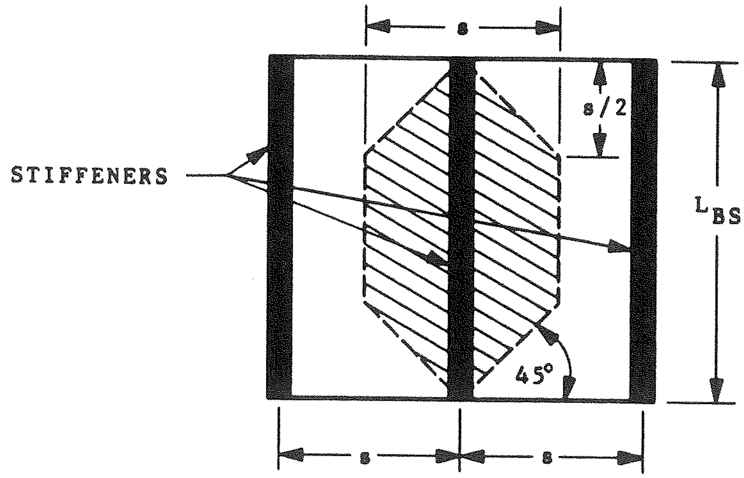
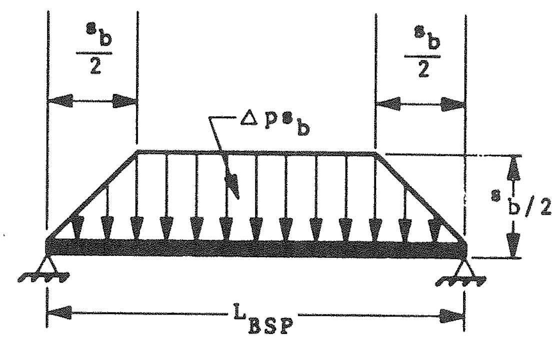


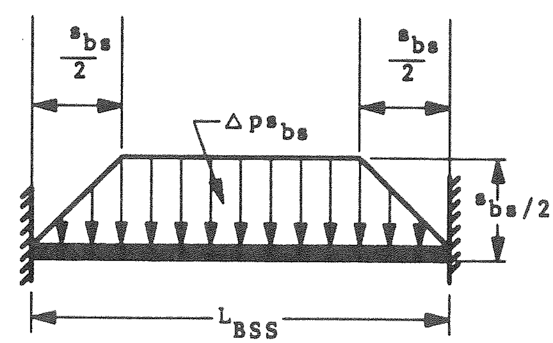
FIGURE 16. STRESS FUNCTION, F_4



a.) GENERAL LOADING DISTRIBUTION FOR BULKHEAD STIFFENERS



b.) LOADING AND END CONDITIONS ON A PRIMARY BULKHEAD STIFFENER



c.) LOADING AND END CONDITIONS ON A SECONDARY BULKHEAD STIFFENER

FIGURE 17. LOADINGS ON BULKHEAD STIFFENERS

APPENDIX B - COMMENTARY

SECTION 1 - INTRODUCTION

1.1 GENERAL

Submarine MBT structure is located outside of the pressure hull and may be generally classified as follows:

a. Circular Strut-Supported Tanks - These tanks have a continuous outer shell which is supported at a constant distance from the circular pressure hull plating by struts joining the two shells at discrete points around the circumference (e.g., concentric hulls). In all cases the radius of the outer hull is greater than the radius of the pressure hull (see Figure 1).

b. Noncircular Strut-Supported Tanks - These tanks have a continuous outer shell plating which is not concentric with the pressure hull. They are supported by struts that are not necessarily radially oriented or equally spaced (see Figure 2).

c. Stiffened End Tanks - These tanks can be circular or noncircular, and are not strut-supported. They are cantilevered off an end closure of the submarine, and can be longitudinally and/or transversely stiffened (see Figures 1, 3, and 4).

Note: The numbering of paragraphs in Appendix B corresponds to the respective paragraph numbering in the main text sections.

1.4 SUMMARY OF MAJOR REVISIONS

The original theoretical derivations leading to the strut-supported ballast tank analyses were presented in Reference (4) and the results have been compared in Reference (5) with experimental data obtained from ship and model tests. The derivations in Reference (4) for the strut-supported ballast tank analyses have been modified in Reference (1). A comparative study of the modified equations in Reference (1) with the equations in Reference (4) has been performed by NAVSEC. The study showed that the modified equations in Reference (1) yield stresses which correlate much closer to the measured data of Reference (5) than the original equations from Reference (4), especially for greater depths. Schwalbe subsequently presented the strut load equations, Reference (2), which were further corrected for this DDS, based on a theoretical analysis considered to be more exact than that of References (1) and (4). In this DDS, the corrected Schwalbe equations are presented for strut load calculation. Other equations presented in Reference (1) are included for the computation of hoop load, moment, and stresses in the outer hull plating and frame.

SECTION 2 - DESIGN PRESSURE CONSIDERATIONS

2.1 GENERAL

The submarine MBT, whether strut-supported or cantilevered, is subject to a positive differential pressure across the tank boundary which causes the tank to bulge outward.

Although this differential pressure is normally very low, it can reach a significant level under two specific conditions. The first is a transient overpressure condition which occurs upon introduction of air into the flooded ballast tank. The second is a steady-state condition which follows the first transient phase and occurs when the submarine rises into water of lower pressure and the flow of ballast water through the flood holes in the ballast tank is insufficient to relieve the internal pressure in the tank.

For the first condition, the transient overpressure condition, the parameters which control the differential pressure are: the submarine's submerged depth; the rate of increase in air flow; the size of the flood holes; the stiffness of the tank boundaries; and the relative size of the water mass being accelerated through the flood holes. Since the outside hydrostatic pressure decreases with decreasing submerged depth, the maximum transient overpressure occurs if air is first introduced into a full tank when the top of the tank is awash.

For the second condition, overpressure due to decrease in submarine depth, the principal parameters affecting this steady-state differential pressure are: ship rise rate; tank volume; flood hole area; ship depth; and the instantaneous volume of air in the tank. Due to the accelerating expansion of the air volume as the ship approaches the surface, the maximum steady-state overpressure occurs when the tank becomes dry coincidentally with the broaching of the tank top. Section 2.3 presents equations for determining the steady-state differential pressure.

2.2 DESIGN CRITERIA

Two operating modes are considered for structural design of the MBT: an emergency blow condition and a normal surfacing condition.

In case of an emergency, rapid surfacing of a submarine is facilitated by quick expulsion of water from the MBTs by means of a high-pressure air system. This is known as an emergency blow condition.

When the submarine is submerged near the surface, the MBTs are blown down to the residual waterline using a low-pressure blower, causing the submarine to surface. This is referred to as the normal surfacing condition.

Design criteria for these two operating conditions differ and are discussed in Section 2.2. The differential pressure used in the design criteria is governed by the steady-state rather than the transient condition. Past experience and measurements of pressure have shown that the steady-state pressure is greater than the transient overpressure and, as a result, the transient overpressure is not considered in MBT design. This is true for standard configuration tanks and flood holes. Designers of nonstandard configurations and covered flood holes should ensure that the peak transient pressure is less than the steady-state pressure.

The transient overpressure is dependent on the submarine's depth, the rate of increase of the air flow, the size of the flood holes, the stiffness of the tank boundaries, and the relative size of the mass to be accelerated. The rate of air-flow build-up is controlled by the characteristics of the air system, of which the valve opening time is the most significant. Theoretical investigation, verified by ship testing, has demonstrated that valve opening times in excess of one-half second will keep transient pressure to very low and acceptable levels.

Table I reflects this change in differential pressure design loading.

2.3 CALCULATIONS AND STEADY-STATE DIFFERENTIAL PRESSURE FOR EMERGENCY BLOW CONDITIONS

The principal parameters affecting the steady-state differential pressure are: ship rise rate, tank volume, flood hole area, ship depth, and the instantaneous volume of air in the tank. The most severe loading condition, due to rise rate, occurs when the tank blows dry at the instant the tank broaches. For this broached condition, the maximum differential pressure occurs at the top of the tank, and decreases with tank depth by the amount of the ambient external hydrostatic pressure.

The effective flood hole area, A, of equation 2-2, is the net blow area perpendicular to the vanes (see Figure 5), and may be expressed by the following equation:

$$A = \sum_{j=1}^{h} \sum_{i=1}^{g} \frac{a_{ij}}{144}, \ ft^2$$

where:

g = number of spaces between vanes of "j"th flood hole; and

h = number of flood holes in tank.

This area of the "i"th space in the "j"th flood hole, a_{ij}, may be obtained by the following equation:

$$a_{ij} = 0.88\ C_h\ (S_g - 1.556\ t_v)\ \sin 40^\circ,\ in^2;$$

where:

a_{ij} = area of "i"th space in the "j"th flood hole, in^2;

C_h = athwartship dimension of cut-in-shell, inches;

S_g = fore-aft spacing of vanes, inches; and

t_v = thickness of flood hole vanes, inches.

Addtionally,

$$S_g = \frac{L_h - 0.5 - 1.556\ t_v}{g},\ \text{inches};$$

where:

L_h = fore-aft dimension of cut-in-shell, inches.

The term a_{ij} represents the net flood hole area. The area occupied by the vanes must be subtracted from the cut-in-shell area. Also, longitudinal webs along the transverse span of the vane must be taken into account for a_{ij}. These longitudinal webs, which stiffen the vanes, reduce the net flow area; therefore, an allowance of about twelve percent of the athwartship dimension of cut-in-shell is made, as seen in the equation for a_{ij}.

It is standard practice that the vanes overlap, i.e., the upper edge of one vane overlaps the lower aft edge of the next vane forward (see Figure 5).

The vane depth is a function of spacing and, in order to maintain reasonable depth, vane spacing is limited to four to six inches. The following empirical formula, developed from standard practice, may be used as guidance:

$$V_n = \frac{S_g}{0.75}$$

where:

V_n = vane depth of "n"th flood hole, inches.

The above equation for S_g takes into account both a clearance between the vanes and the shell, and the thickness of the vanes themselves in determining fore-aft vane spacing.

If not yet defined, the effective flood hole area, A, may be assumed to be one-half the cut-in-shell area.

Steady-state treatment of the air entering and water exiting the ballast tank results in a cubic equation for the pressure, the derivation of which may be seen in Reference (3). The cubic equation was solved for a "standard" tank having a tank volume to flood hole area ratio of 100 (ft), and a discharge coefficient C_D = 0.6, for the broached loading condition. The results of this solution are plotted in Figure 6. These curves require that the total rise rate, $\dot{Z}_T$, be calculated, and that the blowable depth of the tank in feet, h_T, be known.

It should be noted that the cubic pressure equation was derived for tank depths measured in absolute vertical, while h_T is measured relative to the ship's vertical axis (see Figure 7). The difference between h_A and h_T when the tank is fully blown is the cosine of the pitch angle θ^*. Use of small angle theory allows the approximation $h_A \simeq h_T$. This approximation occurs on the side of conservatism, as the slightly higher value of h_A leads to a corresponding increase in Δp.

Another application for the equations is approximating required flood hole area. For a given design steady-state differential pressure, Δp, and tank depth, h_T, Figure 6 may be used to obtain $\dot{Z}_T(STD)$. For a tank of a given volume, V_T, the equations may be re-arranged to obtain:

$$A = \frac{V_T}{100} \left[\frac{\dot{Z}_T}{\dot{Z}_T(STD)} \right], \; ft^2 \; .$$

As $\dot{Z}_T$ is independent of tank parameters, required flood hole area may be obtained.

SECTION 3 - OUTER HULL AND STRUT LOAD EQUATIONS FOR STRUT-SUPPORTED TANKS

3.1 GENERAL

An understanding of the deflected shape of the structure under load is essential to a prediction of the distribution and magnitude of stress. As illustrated in Figure 8, the sequence in which the ballast tank structure deflects can be described as follows:

The inner hull (pressure hull) structure contracts uniformly under the external hydrostatic pressure, p, and the attached radial struts are pulled inward (see Figure 8b). This inward movement of the struts is resisted by the outer hull, resulting in strut elongation and outer hull bending (see Figure 8c). Blowing of the ballast tanks then produces an internal pressure, Δp, inside the ballast tank (see Figure 8d), which causes the following to occur: (1) the struts elongate further until the axial strut load, W, is reached; and (2) the outer hull structure then expands under the hoop force, T(x), and bulges outward under the moment, $M_f(x)$, applied to the outer hull frames.

The load equations given in Section 3 are applicable for the strut-supported ballast tank located between cone-cylinder junctures and for a ballast tank located at the end of the pressure hull where the pressure hull necks down (see Figure 1).

3.2 STRUT LOAD

Theoretical equations describing the structural response of the strut-supported ballast tank, due to the loadings of internal blow pressure and external hydrostatic pressure, were first developed by Pohler, et.al., based on an elastic analysis, (see Reference 4). These equations allowed stresses to be determined in the outer hull, the pressure hull, and the radial struts. Bernstein (see Reference 1) later modified the strut load equations of Reference (4). In Bernstein's derivation of the strut load, the wing bulkhead at the end of the neck-down pressure hull was assumed to be rigid and not deflect in the longitudinal direction. This assumption was attributed to the rigidity of the longitudinal centerline bulkhead which connects the pressure hull to the outer hull, and thus maintains the same relative axial deflection for both hulls. Consequently, in Bernstein's derivation of the strut load, the longitudinal stresses in the outer hull are not relieved. To further improve the strut load equations derived by Bernstein, Schwalbe (see Reference 2) has taken into account the non-linear beam column effect. Additionally, Schwalbe also considered the secondary bending of the inner hull structure which Bernstein neglected in his derivation. Schwalbe's work was further modified by NAVSEA, with the major correction being in the sign convention for displacements used in the strut load calculation. The resulting strut load equations are adopted in the present DDS. Other load and stress equations presented here are taken from Reference (1).

SECTION 4 - STRESSES FOR STRUT-SUPPORTED TANKS

4.1 GENERAL

For nonroutine operations such as an emergency blow condition, slight yielding of the ballast tank plating is allowed; therefore, the stress limit in the MBT when the ship broaches under an emergency blow condition is defined as 1.25 times the yield strength of the material. For routine operations such as a normal surfacing condition, the stresses in the tank plating must be within the yield strength of the material, and the stresses in the frames and stiffeners must be within 67% of the yield strength. Limiting the stresses in the frames and stiffeners to 67% of the yield strength of the material is equivalent to past design practices in which the differential pressure on the frames and stiffeners was taken as 1.5 times the differential pressure acting on the outer hull plating.

Dadley had conducted parametric studies (see Reference 5) which showed that the stresses on the outer hull at operating depth, due to the contraction of the inner hull under hydrostatic pressure, can be greater than the outer hull stresses produced by a pressure differential when the ship broaches; therefore, all MBTs, which are concentric to the pressure hull and strut-supported, should also be designed for emergency blow conditions to withstand the effects of hydrostatic pressure on the inner hull at collapse depth so that the stresses in the outer hull plating, circumferential framing, and struts do not exceed the yield strength of the material. Tables I and II in Appendix A provide the mandatory design limits for both the emergency blow and normal surfacing conditions.

4.9 STRESSES FOR NONUNIFORM AND NONCIRCULAR SECTIONS

It should be noted that only the radial load component is used to determine the stress in the strut for the nonuniform and noncircular ballast tank configurations; however, when the spacings of the struts are unequal and/or the inertia of the outer hull frame varies, an unbalanced moment may result in the strut, and should be accounted for in the design.

SECTION 5 - LOADS FOR STIFFENED END TANKS

5.1 GENERAL

Because the stern end ballast tank is part of the stern structure, its design may be dependent on the design deflection limits established by the alignment tolerances of the control surface connecting rods, propulsion shafting, and associated bearings of the stern structure. Furthermore, if the foundations of the sonar sphere and bow dome permit relative deflections, then the clearance around the sonar sphere may have to be deflection limited. Should this be the case, then the general approach used to design the stern structure in Reference (6) could be used to design the bow structure, keeping in mind that some of the externally applied loads will be different.

5.2 DESIGN LOADS

5.2.1 Internal Design Loads

The internal loads on a stiffened end tank are the buoyancy forces and the tank differential pressure; however, the MBT buoyancy forces are small in comparison to the design differential pressure and therefore are not considered in the design of the ballast tank structure. Therefore, the differential pressure, as described in Section 2, is to be used as the design load for determining the stresses in the cantilevered end tank.

SECTION 6 - STRESSES FOR STIFFENED END TANKS

6.1 GENERAL

Stiffened end tanks shall be designed so that the stresses in the tank structure when the ship broaches do not exceed 1.25 times the yield strength of the material under an emergency blow condition. In addition, the tanks are designed to preclude development of differential pressures which would cause stresses in the outer hull plating to exceed the yield strength of the material and stresses in the frames and stiffeners to exceed 67% of the yield strength of the material for normal surfacing conditions, ballast blowing tests, training exercises, and proof tests. Tables I and II in Appendix A provide the mandatory design limits for both the emergency blow and normal surfacing conditions.

6.2 PRIMARY LONGITUDINAL STIFFENERS

Longitudinal stiffeners provide an increase in the cross-sectional inertia and longitudinal stability of a cantilevered end tank and are only used as primary stiffeners (see Figure 3). Once the cantilevered structure has been designed to meet the governing stern design criteria in Reference (7), the bending stresses in the longitudinal stiffeners are analyzed by the methods described in Section 6.2.

In using the equations in this section, the primary longitudinal stiffeners are sized without accounting for the existence of the transverse stiffeners. However, in an orthogonally stiffened cylinder, the transverse ring frames will support the primary longitudinals, even if the section properties of the ring frames are less than those of the primary longitudinals. This is due to the inherent strength of ring stiffened cylinders. In cases where weight is a primary concern or a better first estimate of scantlings is desired for use in a finite element program, the stiffness of the transverse ring frames may be accounted for when designing the longitudinals of an orthogonally stiffened ballast tank structure.

A suggested analysis method is to 1) determine the required scantlings for the ring frames when subject to tank pressures, without considering the stiffness of the primary longitudinals, and 2) determine the required scantlings for the primary longitudinals by considering the stiffness of the transverse ring frames. If a centerline bulkhead is present, the transverse frame may be designed by analyzing a 180° arc with ends fixed at the centerline bulkhead. After an appropriate section is chosen for the ring frame, a spring rate is calculated for the ring frame and is applied along the length of the primary longitudinal at the frame locations. The longitudinal can then be analyzed for the required load conditions and appropriate end conditions by using either the moment distribution method or a continuous beam program.

It should be noted that this approach merely provides a better first-cut approximation for scantlings, and engineering judgement should be used to determine the trade-off between accuracy and effort in the primary design stage.

6.3 TRANSVERSE STIFFENERS

6.3.1 Primary Transverse Stiffeners

As discussed in Section 6, in the stress analysis of the primary transverse stiffener, the subject is represented as a 90° circular arch, fixed at both ends, with a uniformly distributed load, Δp, acting on it. This representation is, however, an indeterminate structure. To solve for the reactions at the fixed ends of the arch, the method of virtual work has been used and is presented in Appendix D.

6.4 OUTER HULL PLATING

6.4.1 Longitudinally Stiffened Outer Hull Plating

The outer hull plating is to be designed as a two-dimensional plate, all edges fixed, which undergoes large deflections and thereby supports the differential pressure, Δp, partly by its bending resistance and partly by membrane action. The reasoning for this design practice is as follows:

Thick plating whose bending stiffness is great relative to the loads applied and therefore has small deflections, is designed satisfactorily by plate bending theory. On the other hand, very thin plating under lateral loads great enough to cause large deflections is treated as a membrane whose bending stiffness is ignored. As it happens, the most efficient plating designs generally fall between these two extremes, Reference (7). If the designer is to take advantage of the presence of the primary longitudinal stiffeners and the secondary transverse stiffeners, it is not necessary to make the outer hull plating so heavy as to behave like a pure plate or so thin as to necessitate supporting all of the differential pressure by stretching and developing membrane stresses.

6.4.2 Transversely Stiffened Outer Hull Plating

The stress equations for designing the outer hull plating for a transversely stiffened end tank, as given in Section 6.4.2, represent the graphical solution of Krenzke and Short (see References 9 and 10) as applied to the work of von Sanden and Gunther (see Reference 11). That work was further extended by Salerno and Pulos (see Reference 12) to include the beam-column combined action of axial load and radial deflection and the "Viterbo" deformation of the frame due to axial load. By using this method it is assumed that the discontinuity stresses which occur in the outer hull plating at the intersection of the longitudinal centerline bulkhead and the two longitudinal stiffeners are small, and that expansion of the outer hull plating due to a differential pressure loading is axisymmetric.

SECTION 7 - BULKHEAD STRESS EQUATIONS

7.3 DESIGN CRITERIA

The intermediate bulkheads are not subjected to loads under an emergency blow condition and, therefore, will only be designed for the normal surfacing condition using an equivalent differential pressure defined as a hydrostatic pressure created by a column of water the height of the ballast tank plus a 5 psi surcharge. For this loading the stresses in the plating shall not exceed the yield strength of the material and the stresses in the stiffeners shall not exceed 67% of the yield strength of the material. Limiting the stresses in the stiffeners to 67% of the yield strength of the material is equivalent to past design practices in which differential pressure on the stiffeners was taken as 1.5 times the differential pressure acting on the plating.

The same design limits specified in the preceding paragraph for intermediate bulkheads will be used for end bulkheads under normal surfacing conditions; however, end bulkheads must also be designed for an emergency blow condition. The stresses in the plating and stiffeners of an end bulkhead shall not exceed 1.25 times the yield strength of the material for the maximum differential tank pressure, Δp, (not to be less than 30 psi) under an emergency blow condition. In addition, end bulkheads that are used to close the end of a strut-supported tank (wing bulkheads) shall be designed to withstand the effects of contraction of the pressure hull under hydrostatic pressure at collapse depth in addition to peak transient differential pressure, so that the stresses in the end bulkheads shall not exceed the yield strength of the material.

The mandatory design limits and loadings for intermediate and end bulkheads are given in Tables I and II of Appendix A.

APPENDIX C - EXAMPLES

C-1 GENERAL

Appendix C provides examples illustrating the use of the equations presented in Sections 1 through 7 in the main text of the DDS. Section C-2 provides an example of a calculation to determine the steady-state differential pressure for a typical ballast tank. Section C-3 presents calculations of loads and stresses induced in the structure of a strut-supported ballast tank model, due to 1 psi blow pressure. Similar calculations of loads and stresses induced in an end-stiffened ballast tank are provided in the last example, Section C-4.

C-2 STEADY-STATE DIFFERENTIAL PRESSURE CALCULATION

Assume the ship broaches as the tank is fully blown, i.e., $h_A \simeq h_T$. The submarine is assumed to have a pitch angle, θ^*, of 20°. Given the maximum beam, B, of 32 feet, a reserve buoyancy factor, r, of 0.167, and a broaching velocity of 10 knots (16.89 ft/sec), determine the maximum steady-state pressure.

The rise rate, $\dot{Z}_T$, is the sum of the two vertical components, $\dot{Z}_V$ and $\dot{Z}_B$.

$$\dot{Z}_V = \dot{u}\sin\theta^* = 16.89 \ (\sin 20^\circ)$$
$$= 5.78 \text{ ft/sec}$$

$$\dot{Z}_B = 8.42\sqrt{rB} = 8.42\sqrt{(0.167)(32)}$$
$$= 19.46 \text{ ft/sec}$$

Thus:

$$\dot{Z}_T = 5.78 + 19.46 = 25.24 \text{ ft/sec}$$

This rise rate must be converted to a standard tank rise rate in order to use the standardized curves found in Figure 6. Assuming the tank in question has a volume, V_T, of 800 ft^3, and an effective flood hole area, A, of 12 ft^2, then the conversion is as follows:

$$\dot{Z}_T(\text{STD}) = \left[\frac{V_T}{100A}\right] \dot{Z}_T$$

$$\dot{Z}_T(\text{STD}) = \left[\frac{800}{100(12)}\right] (25.24)$$

$$= 16.83 \text{ ft/sec}$$

For the maximum condition, $h_A \simeq h_T$, and assuming h_T is 15 feet, then the curves from Figure 6 may be used to obtain $\Delta p(\text{STD}) = 12$ psi which is less than the 30 psi minimum. Therefore design would be to 30 psi, (per Section 2.2a).

C-3 CALCULATION OF LOADS AND STRESSES FOR MAIN BALLAST TANK STRUCTURE

C-3.1 Strut-Supported Ballast Tank

The structural responses of the Model FA1.8 (see Table C-1) due to a blow pressure of 1.0 psi at the surface are illustrated in the following paragraphs.

C-3.1.1 Structural Dimensions

The structural dimensions of the MBT Model FA1.8 are given in Table C-1.

C-3.1.2 Calculation Procedures

C-3.1.2.1 Calculation of Parameters

The calculation of all parameters, especially D_{Os} and D_{Hs}, requires retaining significant figures of up to 10 digits behind the decimal point. This is necessary in order to obtain accurate strut load values for the subsequent stress calculations. For this reason, it is recommended that a digital computer be used in these calculations, rather than using a hand calculator. In some cases, the computed value used in the example represents the answer calculated by digital computer rather than the hand-computed values. These values are indicated by an asterisk (*).

From Table C-1, $2\alpha = 22^\circ$

therefore, $\alpha = 11^\circ = 11 \times \frac{\pi}{180} = 0.192$ rad.

$\sin\alpha = 0.191 \qquad \sin^2\alpha = 0.0364$

$\cot\alpha = 5.145 \qquad \cos\alpha = 0.982$

Equation (3-8a) gives

$$\left[\delta_x\right]_{\frac{x}{\alpha} = 1.0} = \frac{1}{4}\left[\frac{0.192}{0.0364} + 5.145 - \frac{2}{0.192}\right] = 0.00007918^*$$

TABLE C-1
MBT MODEL FA1.8 STRUCTURAL DIMENSIONS

Tank Depth, h_T	0 ft
Differential Pressure, Δp	1 psi
Submergence Pressure, P	0 psi
Poisson's Ratio, ν	0.3
Frame Spacing, $L_H = L_O$	27.200 in
Radial Struts:	
Angle Between Struts, 2α	22°
Cross-Sectional Area, A_S	1.915 in^2
Young's Modulus, E_S	30 x 10^6 psi
Outer Hull:	
Shell Thickness, t_O	0.375 in
Radius (To Outside Surface), $R_O + t_O/2$	147.600 in
Young's Modulus, E_O	30 x 10^6 psi
T-Frame Depth (Web Plus Flange), d_O	4.000 in
T-Frame Web Thickness, b_O	0.230 in
T-Frame Flange Width, $W_{f,O}$	4.000 in
T-Frame Flange Thickness, $h_{f,O}$	0.254 in
Pressure Hull:	
Shell Thickness, t_H	0.900 in
Radius (To Outside Surface), $R_H + t_H/2$	102.000 in
Young's Modulus, E_H	30 x 10^6 psi
T-Frame Depth (Web Plus Flange), d_H	8.000 in
T-Frame Web Thickness, b_H	0.600 in
T-Frame Flange Width, $W_{f,H}$	6.400 in
T-Frame Flange Thickness, $h_{f,H}$	0.900 in
Wing Bulkhead:	
Radial Panel Width, b'_B	16.000 in
Plating Thickness, t_B	0.500 in

Equation (3-8b) gives:

$$\Gamma = \frac{1}{2}\left[\frac{1}{0.191} + 5.145\right] = 5.192699^{*}$$

Equation (3-9a) gives

$$\gamma_x = \frac{1}{2}\frac{\cos x}{\sin a}$$

Thus,

$$\gamma_0 = \frac{1}{2}\frac{\cos(0)}{0.191} = 2.620^{*} \text{ for midbay location}$$

$$\gamma_a = \frac{1}{2}\cot a = \frac{1}{2}(5.145) = 2.572^{*} \text{ for "at strut" location}$$

Equation (3-10a) gives

$$\xi_x = \gamma_x - \frac{1}{2a}$$

Thus,

$$\xi_0 = 2.620 - \frac{1}{2(0.192)} = 0.0158 \text{ using face value for midbay location}$$

$$\xi_a = 2.572 - \frac{1}{2(0.192)} = -0.0322 \text{ using face value for "at strut" location}$$

From Table C-1, we have:

$$R_0 = 147.6 - \frac{0.375}{2} = 147.412 \text{ in}$$

$$R_H = 102.0 - \frac{0.9}{2} = 101.55 \text{ in}$$

$$L_s = (R_O - R_H) - d_O - d_H - \frac{t_O}{2} - \frac{t_H}{2}$$

$$= (147.412 - 101.55) - 4.0 - 8.0 - 0.188 - 0.45 = 33.224 \text{ in.}$$

o Calculate K_1 thru K_4 from equations (3-8c) through (3-8f):

$$K_1 = \left[\frac{(147.412)^2 - (101.55)^2}{(147.412)(0.375)+(101.55)(0.9)}\right] \frac{0.9}{101.55}$$

$$= 77.845 \left[\frac{0.9}{101.55}\right] = 0.690$$

$$K_2 = 77.845 \left[\frac{0.375}{147.412}\right] = 0.198$$

$$K_3 = \frac{(101.55)(0.9)}{(147.412)(0.375)+(101.55)(0.9)} = 0.623$$

$$K_4 = \frac{(101.55)^2}{(147.412)(0.375)+(101.55)(0.9)} \frac{0.375}{147.412} = 0.179$$

o Calculate θ, γ, η_1, and η_2 from equations (3-8k) through (3-8o):

$$\theta_O = \sqrt[4]{3\,[1-(0.3)^2]}\left[\frac{27.2 - 0.230}{\sqrt{(147.412)(0.375)}}\right]$$

$$= 1.285\ [3.627] = 4.663^*$$

$$\theta_H = \sqrt[4]{3\,[1-(0.3)^2]}\left[\frac{27.2 - 0.600}{\sqrt{(101.55)(0.9)}}\right]$$

$$= 1.285\ [2.782] = 3.577^*$$

$$\gamma_{O\Delta p} = \frac{1}{2(30X10^6)} \left[\frac{147.412}{0.375}\right]^2 \sqrt{3\,[1-(0.3)^2]} = 0.00426$$

$$\gamma_{H\Delta p} = \frac{1}{2(30X10^6)} \left[\frac{101.55}{0.9}\right]^2 \sqrt{3\ [1-(0.3)^2]} = 0.000351$$

$\gamma_{Op} = \gamma_{Hp} = 0$ since hydrostatic pressure = 0 in this case.

$$\eta_{1,O\Delta p} = \frac{1}{2} \sqrt{1 - \gamma_{O\Delta p}}$$

$$= \frac{1}{2} \sqrt{1 - 0.00426} = 0.499$$

$$\eta_{1,H\Delta p} = \frac{1}{2} \sqrt{1 - \gamma_{H\Delta p}}$$

$$= \frac{1}{2} \sqrt{1 - 0.000351} = 0.500$$

$$\eta_{1,Op} = \eta_{1,Hp} = \frac{1}{2} \sqrt{1 - 0} = 0.500$$

$$\eta_{2,O\Delta p} = \frac{1}{2} \sqrt{1 + \gamma_{O\Delta p}}$$

$$= \frac{1}{2} \sqrt{1 + 0.00426} = 0.501$$

$$\eta_{2,H\Delta p} = \frac{1}{2} \sqrt{1 + \gamma_{H\Delta p}}$$

$$= \frac{1}{2} \sqrt{1 + 0.000351} = 0.500$$

$$\eta_{2,Op} = \eta_{2,Hp} = \frac{1}{2} \sqrt{1 + 0} = 0.500$$

o Calculate F_1, F_2 and F_4 from equations (3-8p) through (3-8r):

$$\eta_{1,O}\Delta p^{\theta}_{O} = 0.499\ (4.663) = 2.327$$

$$\eta_{1,H}\Delta p^{\theta}_{H} = 0.500\ (3.577) = 1.789$$

$$\eta_{2,O}\Delta p^{\theta}_{O} = 0.501\ (4.663) = 2.336$$

$$\eta_{2,H}\Delta p^{\theta}_{H} = 0.500\ (3.577) = 1.789$$

$$\eta_{1,O}p^{\theta}_{O} = 0.500\ (4.663) = 2.332$$

$$\eta_{2,H}p^{\theta}_{H} = 0.500\ (3.577) = 1.789$$

$$\eta_{2,O}p^{\theta}_{O} = 0.500\ (4.663) = 2.332$$

$$\eta_{2,H}p^{\theta}_{H} = 0.500\ (3.577) = 1.789$$

$$\cosh\ \eta_{1,O}\Delta p^{\theta}_{O} = \cosh\ (2.327) = 5.169^{*}$$

$$\cosh\ \eta_{1,O}p^{\theta}_{O} = \cosh\ (2.332) = 5.195^{*}$$

$$\sinh\ \eta_{1,O}\Delta p^{\theta}_{O} = \sinh\ (2.327) = 5.072^{*}$$

$$\sinh\ \eta_{1,O}p^{\theta}_{O} = \sinh\ (2.332) = 5.097^{*}$$

$$\cosh\ \eta_{1,H}\Delta p^{\theta}_{H} = \cosh\ (1.789) = 3.072^{*}$$

$$\cosh\ \eta_{1,H}p^{\theta}_{H} = \cosh\ (1.789) = 3.073^{*}$$

$$\sinh\ \eta_{1,H}\Delta p^{\theta}_{H} = \sinh\ (1.789) = 2.905^{*}$$

$$\sinh\ \eta_{1,H}p^{\theta}_{H} = \sinh\ (1.789) = 2.906^{*}$$

$$\cos\ \eta_{2,O}\Delta p^{\theta}_{O} = \cos\ (2.336) = -0.693^{*}$$

$$\sin\ \eta_{2,O}\Delta p^{\theta}_{O} = \sin\ (2.336) = 0.721^{*}$$

$$\cos\ \eta_{2,O}p^{\theta}_{O} = \cos\ (2.336) = -0.689^{*}$$

$$\sin\ \eta_{2,O}p^{\theta}_{O} = \sin\ (2.336) = 0.724^{*}$$

$$\cos\ \eta_{2,H}\Delta p^{\theta}_{H} = \cos\ (1.789) = -0.216^{*}$$

$$\sin\ \eta_{2,H}\Delta p^{\theta}_{H} = \sin\ (1.789) = 0.976^{*}$$

$$\cos\ \eta_{2,H}p^{\theta}_{H} = \cos\ (1.789) = -0.216^{*}$$

$$\sin\ \eta_{2,H}p^{\theta}_{H} = \sin\ (1.789) = 0.976^{*}$$

$$F_{1,O\Delta p} = \left[\frac{4}{\theta_O}\right] \frac{\cosh^2\eta_{1,O\Delta p}\theta_O - \cos^2\eta_{2,O\Delta p}\theta_O}{\dfrac{(\cosh\eta_{1,O\Delta p}\theta_O)\sinh\eta_{1,O\Delta p}\theta_O}{\eta_{1,O\Delta p}} + \dfrac{(\cos\eta_{2,O\Delta p}\theta_O)\sin\eta_{2,O\Delta p}\theta_O}{\eta_{2,O\Delta p}}}$$

$$= \left[\frac{4}{4.663}\right] \frac{(5.169)^2 - (-0.693)^2}{\dfrac{(5.169)(5.072)}{0.499} + \dfrac{(-0.693)(0.721)}{0.501}}$$

$$= 0.437^*$$

This value can also be obtained by using the curves of Figure 13. In Figure 13, for $\theta_O = 4.663$, the curve for $\gamma = 0$, gives $F_1 \simeq 0.44$.

A similar calculation arrives at $F_{1,H\Delta p} = 0.601^*$,

$F_{1,Op} = 0.4377^*$,

and $F_{1,Hp} = 0.6027^*$.

$$F_{2,O\Delta p} = \frac{\dfrac{(\cosh\eta_{1,O\Delta p}\theta_O)\sin\eta_{2,O\Delta p}\theta_O}{\eta_{2,O\Delta p}} + \dfrac{(\sinh\eta_{1,O\Delta p}\theta_O)\cos\eta_{2,O\Delta p}\theta_O}{\eta_{1,O\Delta p}}}{\dfrac{(\cosh\eta_{1,O\Delta p}\theta_O)\sinh\eta_{1,O\Delta p}\theta_O}{\eta_{1,O\Delta p}} + \dfrac{(\cos\eta_{2,O\Delta p}\theta_O)\sin\eta_{2,O\Delta p}\theta_O}{\eta_{2,O\Delta p}}}$$

$$= \frac{\dfrac{(5.169)(0.721)}{0.501} + \dfrac{(5.072)(-0.693)}{0.499}}{\dfrac{(5.169)(5.072)}{0.499} + \dfrac{(-0.693)(0.721)}{0.501}}$$

$$= 0.00767^*$$

Also, from Figure 14, for $\theta_O = 4.663$, the curve for $\gamma = 0.0$ gives $F_2 \simeq 0.01$. The above calculated value is therefore checked.

A similar calculation will arrive at $F_{2,H\Delta p} = 0.2721^*$.

$$F_{4,O\Delta p} = \sqrt{\frac{3}{1-\nu^2}}\ \frac{\dfrac{(\cosh\eta_{1,O\Delta p}\theta_O)\sin\eta_{2,O\Delta p}\theta_O}{\eta_{2,O\Delta p}} - \dfrac{(\sinh\eta_{1,O\Delta p}\theta_O)\cos\eta_{2,O\Delta p}\theta_O}{\eta_{1,O\Delta p}}}{\dfrac{(\cosh\eta_{1,O\Delta p}\theta_O)\sinh\eta_{1,O\Delta p}\theta_O}{\eta_{1,O\Delta p}} + \dfrac{(\cos\eta_{2,O\Delta p}\theta_O)\sin\eta_{2,O\Delta p}\theta_O}{\eta_{2,O\Delta p}}}$$

$$= \sqrt{\frac{3}{1-(0.3)^2}}\ \frac{\dfrac{(5.169)(0.721)}{0.501} - \dfrac{(5.072)(-0.693)}{0.499}}{\dfrac{(5.169)(5.072)}{0.499} + \dfrac{(-0.693)(0.721)}{0.501}}$$

$$= 0.510^*$$

From Figure 16, the value of F_4, for $\gamma = 0.00426$ and $\theta_O = 4.663$, is 0.50, approximately, thus checks the above calculated value.

A similar calculation will get $F_{4,H\Delta p} = 0.7556.^*$

Now that F_1, F_2, and F_4 have been calculated, we proceed to compute the hull plating inertias, I_O and I_H, and areas, A_{TO} and A_{TH}, using the effective length, L_e, as follows:

$$L_{e,O} = F_{1,Op}(L_O - b_O) + b_O$$
$$= (0.4377)(27.2 - 0.230) + 0.230 = 12.035 \text{ in}$$

$$L_{e,H} = F_{1,Hp}(L_H - b_H) + b_H$$
$$= (0.6027)(27.2 - 0.600) + 0.600 = 16.632 \text{ in}$$

- Area calculations

Shell area: $A_{sh,O} = L_{e,O}(t_O) = 12.035\ (0.375) = 4.513\ in^2$

$A_{sh,H} = L_{e,H}(t_H) = 16.632\ (0.9) = 14.969\ in^2$

Web area: $A_{w,O} = (d_O - h_{f,O})\ b_O = (4.0 - 0.254)\ 0.230 = 0.862\ in^2$

$A_{w,H} = (d_H - h_{f,H})\ b_H = (8.0 - 0.90)\ 0.60 = 4.26\ in^2$

Flange area: $A_{flg,O} = W_{f,O}\ (h_{f,O}) = 4.0\ (0.254) = 1.016\ in^2$

$A_{flg,H} = W_{f,H}\ (h_{f,H}) = 6.4\ (0.90) = 5.76\ in^2$

Frame area: $A_{f,O} = A_{w,O} + A_{flg,O} = 0.862 + 1.016 = 1.878\ in^2$

$A_{f,H} = A_{w,H} + A_{flg,H} = 4.26 + 5.76 = 10.02\ in^2$

Total area: $A_{TO} = A_{f,O} + A_{sh,O} = 1.878 + 4.513\ in^2 = 6.391\ in^2$

$A_{TH} = A_{f,H} + A_{sh,H} = 10.02 + 14.969 = 24.989\ in^2$

- Inertia calculations

Shell inertia: $$I_{sh,O} = \frac{(L_{e,O})t_O^3}{12} = \frac{12.035\ (0.375)^3}{12} = 0.053\ in^4$$

$$I_{sh,H} = \frac{(L_{e,H})\ t_H^3}{12} = \frac{16.632\ (0.9)^3}{12} = 1.010\ in^4$$

Web inertia: $$I_{w,O} = \frac{b_O(d_O-h_{f,O})^3}{12} = \frac{0.230(3.746)^3}{12} = 1.008\ in^4$$

$$I_{w,H} = \frac{b_H(d_H-h_{f,H})^3}{12} = \frac{0.60(7.1)^3}{12} = 17.896\ in^4$$

Flange inertia: $$I_{flg,O} = \frac{(W_{f,O})h_{f,O}^3}{12} = \frac{4.0(0.254)^3}{12} = 0.005\ in^4$$

$$I_{flg,H} = \frac{(W_{f,H})h_{f,H}^3}{12} = \frac{6.4\ (0.90)^3}{12} = 0.389\ in^4$$

o Compute neutral axis location of frame-shell combination:

$$\overline{NA}_O = \frac{A_{sh,O}\left(\frac{t_O}{2}\right) + A_{w,O}\left(t_O + \frac{d_O - h_{f,O}}{2}\right) + A_{flg,O}\left(t_O + d_O - \frac{h_{f,O}}{2}\right)}{A_{TO}}$$

$$= \frac{4.513(0.188) + 0.862(0.375 + 1.873) + 1.016(0.375 + 4.0 - 0.127)}{6.391}$$

$= 1.111$ in. from outside surface of outer hull

Thus, $Y_O = 1.111 - 0.188 = 0.923$ in. from mid-thickness of outer hull plating.

Similarly,

$$\overline{NA}_H = \frac{14.969(0.45) + 4.26(0.90 + 3.55) + 5.76(0.90 + 8.0 - 0.45)}{24.989}$$

$= 2.976$ in. from inside surface of pressure hull

Thus, $Y_H = 2.976 - 0.45 = 2.526$ in. from mid-thickness of pressure hull plating.

o Calculate the total inertia about the neutral axis:

$$I_O = I_{sh,O} + A_{sh,O}(Y_O)^2 + I_{w,O} + A_{w,O}\left(t_O + \frac{d_O - h_{f,O}}{2} - \overline{NA}_O\right)^2 + I_{flg,O}$$

$$+ A_{flg,O}\left(t_O + d_O - \frac{h_{f,O}}{2} - \overline{NA}_O\right)^2$$

$$= 0.053 + (4.513)(0.923)^2 + 1.008 + 0.862(2.248-1.111)^2 + 0.005$$

$$+ 1.016(4.248 - 1.111)^2$$

$$= 16.023 \text{ in}^4$$

Similarly,

$$I_H = 1.010 + 14.969(2.526)^2 + 17.896 + 4.26(4.45 - 2.976)^2 + 0.389$$

$$+ 5.76(8.45 - 2.976)^2$$

$$= 296.659 \text{ in}^4$$

o Calculate frames' centroid location:

$$\overline{CG}_O = \frac{A_{w,O}\left(\frac{d_O - h_{f,O}}{2}\right) + A_{flg,O}\left(d_O - \frac{h_{f,O}}{2}\right)}{A_{w,O} + A_{flg,O}}$$

$$= \frac{0.862(1.873) + 1.016(3.873)}{0.862 + 1.016}$$

$= 2.955$ in. from web/shell junction

Thus,

$$R'_{Ocg} = R_O - \left(\frac{t_O}{2} + \overline{CG}_O\right)$$

$$= 147.412 - (0.188 + 2.955)$$

$$= 144.269 \text{ in.}$$

Similarly,

$$\overline{CG}_H = \frac{4.26(3.55) + 5.76(7.55)}{4.26 + 5.76} = 5.849 \text{ in. from web/shell junction}$$

$$R'_{Hcg} = R_H + \frac{t_H}{2} + \overline{CG}_H$$

$$= 101.55 + (0.45 + 5.849) = 107.849 \text{ in.}$$

o Calculate α_i, β_i, A'_i, C_{ij} and G_{ij} from equations (3-8h) through (3-8j), (3-8g), and (3-8s):

$$A'_O = A_{fO}\left[\frac{R_O}{R'_{Ocg}}\right] = 1.878 \left[\frac{147.412}{144.269}\right] = 1.919 \text{ in}^2$$

$$A'_H = A_{fH}\left[\frac{R_H}{R'_{Hcg}}\right]^2 = 10.02 \left[\frac{101.550}{107.849}\right]^2 = 8.884 \text{ in}^2$$

$$a_O = \frac{A'_O}{L_O t_O} = \frac{1.919}{(27.2)(0.375)} = 0.188$$

$$a_H = \frac{A'_H}{L_H t_H} = \frac{8.884}{(27.2)(0.90)} = 0.363$$

$$\beta_O = \frac{b_O}{L_O} = \frac{0.230}{27.2} = 0.00846$$

$$\beta_H = \frac{b_H}{L_H} = \frac{0.600}{27.2} = 0.0221$$

$$C_{O\Delta p} = \frac{a_O F_{2,O\Delta p}}{a_O + \beta_O \div (1-\beta_O) F_{1,O\Delta p}} = 0.00229$$

$$C_{H\Delta p} = \frac{a_H F_{2,H\Delta p}}{a_H + \beta_H + (1-\beta_H) F_{1,H\Delta p}} = 0.1015$$

$$G_{O\Delta p} = \frac{\sqrt{\dfrac{1-\nu^2}{3}} \left[\dfrac{F_{4,O\Delta p}}{F_{2,O\Delta p}}\right] + \gamma_{O\Delta p}}{4 \quad (\eta_{1,O\Delta p}) \; \eta_{2,O\Delta p}} = 36.645$$

$$G_{H\Delta p} = \frac{\sqrt{\dfrac{1-\nu^2}{3}} \left[\dfrac{F_{4,H\Delta p}}{F_{2,H\Delta p}}\right] + \gamma_{H\Delta p}}{4 \quad (\eta_{1,H\Delta p}) \; \eta_{2,H\Delta p}} = 1.5282$$

o Calculate parameters V', K', δx, N', H'_M, H_M and ρ by equations (4-1a) through (4-1g):

since: $\theta_O = 4.663^*$ and $\theta_H = 3.577^*$ as computed previously,

then: $\cosh \theta_O = 52.981, \quad \sinh \theta_O = 52.971,$

$\cos \theta_O = -0.0494, \quad \sin \theta_O = -0.9988,$

$\cosh \frac{\theta_O}{2} = 5.195, \quad \sinh \frac{\theta_O}{2} = 5.098,$

$\cos \frac{\theta_O}{2} = -0.689, \quad \sin \frac{\theta_O}{2} = 0.724,$

$\cosh \theta_H = 17.897, \quad \sinh \theta_H = 17.869,$

$\cos \theta_H = -0.907, \quad \sin \theta_H = -0.422,$

$\cosh \frac{\theta_H}{2} = 3.074, \quad \sinh \frac{\theta_H}{2} = 2.907,$

$\cos \frac{\theta_H}{2} = -0.216, \quad \sin \frac{\theta_H}{2} = 0.976$

Therefore:

$$V' = \frac{52.981 + (-0.0494)}{52.971 + (-0.9988)} = 1.018$$

$$K' = \frac{52.971 - (-0.9988)}{52.971 + (-0.9988)} = 1.038$$

$$\delta_{x=0} = \frac{1}{4}\left[\frac{0.192(0.982)(1)}{0.0364} + \frac{1}{0.191} - \frac{2}{0.192} + 0\right] = -0.000075^*$$

$\delta_{x=a} = 0.00007918^*$ as calculated previously

$$H'_M = (-2)\left[\frac{(5.098)(-0.689) + (5.195)(0.724)}{52.971 + (-0.9988)}\right] = -0.009569$$

$$H_M = (-2)\left[\frac{(2.907)(-0.216) + (3.704)(0.976)}{17.869 + (-0.422)}\right] = -0.342430$$

$$N' = \frac{52.981 - (-0.0494)}{52.971 + (-0.9988)} = 1.020$$

$$\rho = 2 - \frac{101.55}{147.412} - \left[\frac{101.55}{147.412}\right]^2 = 0.837$$

C-3.1.2.2 Calculation of Loads

With the above parameters calculated, the deflection and stiffness terms in equations (3-2) through (3-7) can be computed as follows:

$$D_{Os} = \left[\frac{R_O}{2E_O} \quad \frac{2\,R_O^2}{I_O} \left[\delta_x\right]_{\frac{x}{a} = 1.0} + \frac{\Gamma}{A_{TO}}\right]$$

$$= \frac{147.412}{2(30x10^6)}\left[\frac{2\,(147.412)^2}{16.023}\;(0.00007918) + \frac{5.192699}{6.391}\right]$$

$$= 2.524 \times 10^{-6} \text{ in/lb}^*$$

$$D_{HS} = \frac{R_H}{2E_H}\left[\frac{2\,R_H^2}{I_H} \left[\delta_x\right]_{\frac{x}{a} = 1.0} + \frac{\Gamma}{A_{TH}}\right].$$

$$= \frac{101.55}{2(30x10^6)}\left[\frac{2(101.55)^2}{296.659}\;(0.00007918) + \frac{5.192699}{24.989}\right]$$

$$= 0.3610 \times 10^{-6} \text{ in/lb}^*$$

$$\frac{L_s}{A_s E_s} = \frac{33.224}{1.915(30 \times 10^6)} = 0.578 \times 10^{-6} \quad \text{in/lb}$$

$$D_{O\Delta p} = \Delta p \left[1 - \frac{K_2 \nu}{2}\right] \frac{R_O^2}{E_O t_O} \left\{ 1 - C_{O\Delta p} \left[(\cosh \eta_{1,O\Delta p} \theta_O)(\cos \eta_{2,O\Delta p} \theta_O) \right.\right.$$

$$\left.\left. + G_{O\Delta p} (\sinh \eta_{1,O\Delta p} \theta_O) \sin \eta_{2,O\Delta p} \theta_O \right] \right\}$$

$$= (1) \left[1 - \frac{0.198(0.3)}{2}\right] \frac{(147.412)^2}{(30 \times 10^6)(0.375)} \{1 - 0.00229\ [(5.169)\ (-0.693)$$

$$+ 36.645\ (5.072)\ (0.721)]\} = 0.00131 \text{ in.*}$$

$$D_{H\Delta p} = \Delta p \left[1 + \frac{K_1 \nu}{2}\right] \frac{R_H^2}{E_H t_H} \left\{ 1 - C_{H\Delta p} \left[(\cosh \eta_{1,H\Delta p} \theta_H) \cos \eta_{2,H\Delta p} \theta_H \right.\right.$$

$$\left.\left. + G_{H\Delta p} (\sinh \eta_{1,H\Delta p} \theta_H) \sin \eta_{2,H\Delta p} \theta_H \right] \right\}$$

$$= (1) \left[1 + \frac{0.690(0.3)}{2}\right] \frac{(101.55)^2}{(30 \times 10^6)(0.9)} \{1 - (0.1015)\ [(3.072)\ (-0.216)$$

$$+ (1.5282)\ (2.905)\ (0.976)]\} = 0.000264 \text{ in.*}$$

$$D_{Op} = D_{Hp} = 0 \text{ since } p = 0$$

Therefore, the strut load is:

$$W = \frac{(D_{O\Delta p} - D_{H\Delta p}) + (D_{Op} - D_{Hp})}{\frac{L_s}{A_s E_s} + D_{Os} - D_{Hs}}$$

$$= \frac{(0.00131 - 0.000264) + (0 - 0)}{(0.578 + 2.524 - 0.361)\ 10^{-6}} = 382 \text{ lb}$$

The hoop load T(x) in the outer hull frame is, by equation (3-9):

$$T(x) = \left[P_c R_O - W(\gamma_x)\right]\left\{\frac{A_{fO}}{A_{TO}}\left[\frac{a-x}{a}\right] + \frac{x}{a}\right\}$$

since: $$A_{fO}/A_{TO} = \frac{1.878}{6.391} = 0.294$$

$$\text{let } f_1 = \frac{A_{fO}}{A_{TO}}\left[\frac{a-x}{a}\right] + \frac{x}{a} = (0.294)\left[\frac{0.192-x}{0.192}\right] + \frac{x}{0.192}$$

then, for x = 0 , that is, the midbay location:

$$f_1 = (0.294)\left[\frac{0.192 - 0}{0.192}\right] + \frac{0}{0.192} = 0.294$$

for $x = a$, the "at-strut" location:

$$f_1 = (0.294)\left[\frac{0.192 - 0.192}{0.192}\right] + \frac{0.192}{0.192} = 1.0$$

Now,

$$P_c R_O = L_{e,O}\,(\Delta p)\,R_O$$

$$= (12.035)\ (1)\ (147.412) = 1774 \text{ lb}$$

thus,

$$T(x = 0) = [1774 - 382\ (2.620)]\ (0.294)$$

$$= 227 \text{ lb at midbay}$$

$$T(x = a) = [1774 - 382\ (2.572)]\ (1.0)$$

$$= 791 \text{ lb at strut}$$

The moment in the outer hull frame is given by equation (3-10):

$$M_f(x) = WR_O\ (\xi_x)$$

thus,

$$M_f(x = 0) = (382)\ (147.412)\ (0.0158) = 890 \text{ in-lb at midbay}$$

$$M_f(x = a) = (382)\ (147.412)\ (-0.0322) = -1813 \text{ in-lb at strut}$$

C-3.1.2.3 Calculation of Strut-Supported MBT Stresses

(A) Strut Stress

- The axial stress in the strut is:

$$\sigma_s = \frac{W}{A_s} = \frac{382}{1.915} = 199 \text{ psi}$$

(B) Outer Hull Plating - Longitudinal Stress at the Frame

- Calculate the various terms in equation (4-1):

$$\frac{1.734\ K'}{(2V'N'-K'^2)(1-\nu^2)^{1/2}} = \frac{1.734\ (1.038)}{[2(1.018)(1.020)-(1.038)^2]\sqrt{1-(0.3)^2}} = 1.888$$

$$\frac{A_{TO}}{t_O} + H_M' \frac{A_{fO}}{t_O} - L_{e,O} = \frac{6.391}{0.375} + (-.009569)\frac{1.878}{0.375} - 12.035 = 4.960 \text{ in.}$$

$$\frac{(\Delta p)R_O}{A_{TO}}\left[1 - \frac{\nu}{6}\rho\right] = \frac{(1)(147.412)}{6.391}\left[1 - \frac{0.3}{6}(0.837)\right] = 22.100$$

$$\frac{A_{TH}}{t_H} + H_M \frac{A_{fH}}{t_H} - L_{e,H} = \frac{24.989}{0.90} + (-0.3424)\frac{10.02}{0.90} - 16.632 = 7.321$$

$$\frac{p'\ R_H^2 E_O}{A_{TH}R_O E_H}\left[1 - \frac{\nu}{2}\right] = \frac{(1)(101.55)^2\ (30 \times 10^6)}{(24.989)(147.412)(30 \times 10^6)}\left[1 - \frac{0.3}{2}\right] = 2.380$$

where:

$$p' = p + \Delta p = 1.0$$

$$\frac{(\Delta p)\ R_O \rho}{6t_O} = \frac{(1)(147.412)(0.837)}{6(0.375)} = 54.837$$

Therefore, the longitudinal stress in the outer hull plating at the frame is:

(i) At the midbay location, x = 0:

$$\sigma_L\ (x = 0) = \pm\ (1.888) \left\{ \frac{382(147.412)^2}{16.023} (-0.000075) + \frac{382}{6.391} (2.620) + (22.100)\ (4.960) + (2.380)\ (7.321) \right\} + 54.837$$

$$= \begin{matrix} 517 \text{ psi for inside surface} \\ -407 \text{ psi for outside surface} \end{matrix}$$

(ii) At the strut location, x = a:

$$\sigma_L\ (x = a) = \pm\ (1.888) \left\{ \frac{382(147.412)^2}{16.023} (0.00007918) + \frac{382}{6.391} (2.572) + (22.100)\ (4.960) + (2.380)\ (7.321) \right\} + 54.837$$

$$= \begin{matrix} 662 \text{ psi for inside surface} \\ -553 \text{ psi for outside surface} \end{matrix}$$

(C) Circumferential Stress in the Frame Flange

$$Z_f = \frac{I_O}{d_O + \frac{t_O}{2} - Y_O} = \frac{16.023}{4 + \frac{0.375}{2} - 0.923} = 4.908 \text{ in}^3$$

Therefore, the circumferential stress in the frame flange is:

$$\sigma_{\phi f}\ (x = 0) = \frac{227}{6.391} - \frac{890}{4.908} = -146 \text{ psi at midbay}$$

$$\sigma_{\phi f}\ (x = a) = \frac{791}{6.391} - \frac{-1813}{4.908} = 493 \text{ psi at strut}$$

(D) Circumferential Stress in the Plating at the Frame

$$Z_p = \frac{I_O}{Y_O + \frac{t_O}{2}} = \frac{16.023}{0.923 + \frac{0.375}{2}} = 14.429 \text{ in}^3$$

Thus the stress is:

$$\sigma_{\phi p}\ (x = 0) = \frac{227}{6.391} + \frac{890}{14.429} + (0.3)(517 \text{ or } -407)$$

$$= \begin{array}{l} 252 \text{ psi for inside surface at midbay location} \\ -25 \text{ psi for outside surface at midbay location} \end{array}$$

$$\sigma_{\phi p}\ (x = a) = \frac{791}{6.391} + \frac{-1813}{14.429} + 0.3\ (662 \text{ or } -553)$$

$$= \begin{array}{l} 197 \text{ psi for inside surface at the strut location} \\ -168 \text{ psi for outside surface at the strut location} \end{array}$$

(E) Total Longitudinal Outer Hull Stress at the Frame

The stress is:

$$\sigma_{Lp}\ (x = 0) = \sigma_L(x = 0) + \nu \left[\frac{T(x = 0)}{A_{TO}} + \frac{M_f(x = 0)}{Z_p} \right]$$

$$= 517 + (0.3)\ [97.2]$$

$$= 546 \text{ psi for inside surface at midbay location}$$

or, $\sigma_{Lp}\ (x = 0) = -407 + (0.3)\ [97.2]$

$$= -378 \text{ psi for outside surface at midbay location}$$

$$\sigma_{Lp}\ (x = a) = \sigma_L\ (x = a) + \nu \left[\frac{T(x = a)}{A_{TO}} + \frac{M_f(x = a)}{Z_p} \right]$$

$$= 662 + (0.3)\ [-1.9]$$

$$= 661 \text{ psi for inside surface at the strut location}$$

or, $\sigma_{Lp}\ (x = a) = -553 + 0.3\ [-1.9]$

$$= -554 \text{ psi for outside surface at the strut location}$$

(F) Longitudinal Stress at the Wing Bulkhead

Calculate the various terms in equation (4-6):

$$\frac{1.734\ K'(1-\nu^2)^{-1/2}}{2V'N' - (K')^2} = 1.888$$

$$\frac{(\Delta p)R_O\rho}{6t_O} = 54.837$$

$$\frac{p'R_H^2E_O}{A_{TH}R_OE_H}\left[1 - \frac{\nu}{2}\right] = 2.380$$

$$\frac{A_{TH}}{t_H} + H_M\left[\frac{A_{fH}}{t_H}\right] - L_{e,H} = 7.321$$

$$1 - \frac{\nu}{6}\rho = 1 - \frac{0.3}{6}(0.837) = 0.958$$

$$\frac{b'_B t_O^3}{b'_B t_O^3 + L_{e,O}t_B^3} = \frac{16\ (0.375)^3}{16\ (0.375)^3 + (12.035)\ (0.50)^3} = 0.359$$

$$\frac{L_{e,O}\ t_O}{A_{TO}} = \frac{12.035\ (0.375)}{6.391} = 0.706$$

$$\frac{(\Delta p)\ R_O}{t_O} = \frac{(1)\ (147.412)}{0.375} = 393.099$$

Thus the longitudinal stress at the wing bulkheads:

$$\sigma_{LB} = \pm\ (1.888)\ \{[393.099]\ [(1-.009569)+(0.706)(0.359)]\ [0.958] + [2.380]\ [7.321]\} + 54.837$$

= 972 psi or -863 psi, for inside and outside surfaces, respectively

(G) Circumferential Stress at the Wing Bulkhead

The circumferential stress is, per Appendix A of Reference 5:

$$\sigma_{CB} = \nu \, \sigma_{LB}$$

Thus, $\sigma_{CB} = 0.3\ (972) = 292$ psi for inside surface

or, $\sigma_{CB} = 0.3\ (-863) = -259$ psi for outside surface

(H) Outer Hull Plating Stresses at Midway Between Frames

Stresses in the outer hull plating midway between frames when the ship broaches are, per equations (4-7) and (4-8):

(i) Circumferential stress, σ_ϕ:

$$\sigma_\phi = \frac{(\Delta p)\ R_O}{t_O} + \nu \frac{(\Delta p)\ R_O \rho}{6t_O}$$

$$= 393.099 + (0.3)\ (54.837)$$

$$= 410 \text{ psi}$$

(ii) Longitudinal stress, $\sigma_{\frac{L}{2}}$:

$$\sigma_{\frac{L}{2}} = \frac{(\Delta P)\ R_O \rho}{6t_O} + \nu \frac{(\Delta P)\ R_O}{t_O}$$

$$= 54.837 + (0.3)(393.099)$$

$$= 173 \text{ psi}$$

The calculated stresses are relatively low, with the largest stress of 972 psi occurring at the wing bulkhead. This is considered reasonable since the blow pressure exerted is only 1 psi, a relatively small level of pressure.

Had the stresses been calculated as due to a specific design load as given in Table I or Table II in Appendix A, the resulting stresses would have been of much higher values and should have been evaluated to determine if they may exceed the allowable, using the design limits given in this DDS.

C-4 CALCULATION OF LOADS AND STRESSES FOR END STIFFENED BALLAST TANK STRUCTURE

C-4.1 Calculation of Loads and Stresses in Primary Longitudinal Stiffeners

The following load and stress calculations are illustrated with a blow pressure of 1.0 psi.

C-4.1.1 Structural Dimensions

The end stiffened tank has the following structural characteristics:

Longitudinal stiffeners - 20.4 # PL (HTS) x 18" deep (web)

30.6 # PL (HTS) x 10" width (flg)

Outer hull plating - 20.4 # PL (HTS)

$L_L = l_L =$ average spacing between longitudinal frames = 30 in.

w_L = effective width of plating attached to longitudinal stiffeners

= 50 t for HTS

= 25 in.

Therefore, the following data is applicable for the longitudinal stiffeners:

w_L = 25 in., t_O = 0.5 in. (shell plating)

b_O = 0.5 in, d_O = 18 in. (web)

w_f = 10 in., h_f = 0.75 in. (flange)

C-4.1.2 Cross-Sectional Property Calculations

- Area calculations

Shell area: $A_{sh} = w_L (t_O) = 25 (0.5) = 12.5 \text{ in}^2.$

Web area: $A_W = b_O (d_O - h_f) = 0.5 (17.25) = 8.625 \text{ in}^2.$

Flange area: $A_{flg} = w_f (h_f) = 10 (0.75) = 7.5 \text{ in}^2.$

Frame area: $A_f = A_W + A_{flg} = 8.625 + 7.5 = 16.125 \text{ in}^2.$

Total area: $A_{TL} = A_{sh} + A_f = 12.5 + 16.125 = 28.625 \text{ in}^2.$

- Inertia calculations

Shell inertia: $$I_{sh} = \frac{w_L t_O^3}{12} = \frac{25(0.5)^3}{12} = 0.260 \text{ in}^4.$$

Web inertia: $$I_W = \frac{b_O (d_O - h_f)^3}{12} = \frac{0.5(17.25)^3}{12} = 213.873 \text{ in}^4.$$

Flange inertia: $$I_{flg} = \frac{w_f h_f^3}{12} = \frac{10(0.75)^3}{12} = 0.352 \text{ in}^4.$$

- Neutral-axis location:

$$\overline{NA}_O = \frac{A_{sh}\left(\frac{t_O}{2}\right) + A_W\left(t_O + \frac{d_O - h_f}{2}\right) + A_{flg}\left(t_O + d_O - \frac{h_f}{2}\right)}{A_{TL}}$$

$$= \frac{12.5(0.25) + 8.625(0.50 + 8.625) + 7.5(0.50 + 18 - 0.375)}{28.625}$$

$= 7.608$ in. from outer hull surface

Therefore, $$Y_O = \overline{NA}_O - \frac{t_O}{2} = 7.608 - 0.25$$

$= 7.358$ in. from mid-thickness of hull plating

- The total inertia about the neutral axis is:

$$I_O = I_{sh} + A_{sh}(Y_O)^2 + I_W + A_W\left(t_O + \frac{d_O - h_f}{2} - \overline{NA}_O\right)^2 + I_{flg}$$

$$+ A_{flg}\left(t_O + d_O - \frac{h_f}{2} - \overline{NA}_O\right)^2$$

$$= 0.260 + 12.5\,(7.358)^2 + 213.873 + 8.625\,(9.125 - 7.608)^2 + 0.352 +$$

$$7.5\,(18.125 - 7.608)^2$$

$$= 1740.64 \text{ in}^4.$$

- The section modulus is:

$$Z_{pL} = \frac{I_O}{Y_O + \frac{t_O}{2}} = \frac{1740.64}{7.358 + 0.25} = 228.79 \text{ in}^3$$

$$Z_{fL} = \frac{I_O}{d_O + \frac{t_O}{2} - Y_O} = \frac{1740.64}{18.25 - 7.358} = 159.81 \text{ in}^3$$

C-4.1.3 Load and Stress Calculations

- The bending moment at the end of a longitudinal beam element is:

$$M_X = \frac{(\Delta P) L_L l_L^{\,2}}{12}$$

$$= \frac{(1)\,(30)\,(30)^2}{12} = 2250 \text{ in-lb}$$

- Therefore, the stress in the flange of the longitudinal stiffener is, per equation (6-1):

$$\sigma_f = \frac{M_X}{Z_{fl}} = \frac{2250}{159.81} = 14 \text{ psi}$$

o Similarly, the stress in the outer hull plating of the longitudinal stiffener is, from equation (6-2):

$$\sigma_p = \frac{M_x}{Z_{pL}} = \frac{2250}{228.79} = 10 \text{ psi}$$

The stress values as calculated above are relatively low since a low value of 1 psi blow pressure is used. Had the design load as given in Tables I and II in Appendix A been used as blow pressure, the resulting stresses would have been higher.

Furthermore, the stiffened end tanks' structures must also be designed for the external loads as described in Section 5.1 and given in Reference (7). These external loads might induce much higher stresses than the internal blow pressure does.

C-4.2 Calculation of Loads and Stresses for Primary Transverse Stiffeners

The following load and stress calculations are illustrated with a blow pressure of 1.0 psi.

C-4.2.1 Structural Dimensions

For transverse stiffeners, the following data apply:

Plating:	L_o	= 30 in.,	t_o	= 0.50 in.
Web:	d_o	= 8.5625 in.,	b_o	= 0.375 in.
Flange:	w_f	= 7 in.,	h_f	= 0.5625 in.
	θ_t	= 30°,	l_T	= 27 in.
	σ_y	= 80000 psi,	E	= 30×10^6 psi

Thus, effective plating width is:

$$w_L = 2t_O \sqrt{E/\sigma_y} = (2)\ (0.50) \sqrt{(30 \times 10^6)/80000} = 19.365 \text{ in.}$$

C-4.2.2 Cross-Sectional Properties and Parameters Calculations

To calculate stress in the transverse stiffeners, we proceed to calculate the various cross-sectional properties as follows:

- o Area calculations

Plating area: $A_{pl} = w_L(t_O) = 19.365\ (0.5) = 9.683 \text{ in}^2.$

Web area: $A_w = b_O\ (d_O - h_f) = (0.375)(8.5625 - 0.5625) = 3 \text{ in}^2.$

Flange area: $A_{flg} = w_f(h_f) = 7\ (0.5625) = 3.938 \text{ in}^2.$

Frame area: $A_f = A_w + A_{flg} = 3 + 3.938 = 6.938 \text{ in}^2.$

Total area: $A_{TO} = A_f + A_{pl} = 6.938 + 9.683 = 16.621 \text{ in}^2.$

- o Inertia calculations

Plating inertia: $I_{pl} = \dfrac{w_L t_O^3}{12} = \dfrac{19.365\ (0.50)^3}{12} = 0.202 \text{ in}^4.$

Web inertia: $I_w = \dfrac{b_O(d_O - h_f)^3}{12} = \dfrac{0.375\ (8.5625 - 0.5625)^3}{12} = 16.0 \text{ in}^4$

Flange inertia: $I_{flg} = \dfrac{w_f h_f^3}{12} = \dfrac{7\ (0.5625)^3}{12} = 0.104 \text{ in}^4.$

- o Neutral-axis location:

$$\overline{NA}_O = \frac{A_{pl}\left(\frac{t_O}{2}\right) + A_w\left(t_O + \frac{d_O - h_f}{2}\right) + A_{flg}\left(t_O + d_O - \frac{h_f}{2}\right)}{A_{TO}}$$

$$= \frac{9.683\ (0.25) + 3\ (4.219) + 3.938\ (8.218)}{16.621}$$

$$= 2.854 \text{ in. from outer hull surface}$$

Therefore:

$$Y_O = \overline{NA}_O - \frac{t_O}{2} = 2.854 - 0.25$$

$$= 2.604 \text{ in. from mid-thickness of hull plating}$$

o The total inertia about the neutral axis is:

$$I_O = I_{pl} + A_{pl}\,(Y_O)^2 + I_w + A_w\left(t_O + \frac{d_O - h_f}{2} - \overline{NA}_O\right)^2 + I_{flg}$$

$$+ A_{flg}\left(t_O + \frac{d_O - h_f}{2} - \overline{NA}_O\right)^2$$

$$= 0.202 + 9.683\,(2.604)^2 + 16 + 3\,(4.219 - 2.854)^2 + 0.104$$

$$+ 3.938\,(8.218 - 2.854)^2$$

$$= 200.860 \text{ in}^4.$$

o Also, $R_T = R_O - \overline{NA}_O$

$$= 130 - 2.854 = 127.146 \text{ in.} \quad \text{use } R_O = 130 \text{ in.}$$

o Then, $J = \dfrac{R_T^{\,2}}{A_{TO} I_O} + \dfrac{1}{A_{TO}^{\,2}}$

$$= \frac{(127.146)^2}{16.621(200.860)} + \frac{1}{(16.621)^2}$$

$$= 4.846 \text{ in}^{-4}$$

$$Q = 0.174\left(\frac{R_T^{\,4}}{I_O^{\,2}}\right) + 18.614\left(\frac{R_T^{\,2}}{A_{TO} I_O}\right) + \frac{18.44}{A_{TO}^{\,2}}$$

$$= 0.174\left(\frac{(127.146)^4}{(200.860)^2}\right) + \frac{18.614\,(127.146)^2}{16.621\,(200.860)} + \frac{18.44}{(16.621)^2}$$

$$= 1217.330 \text{ in}^{-4}$$

$$Z_{PL} = \frac{I_O}{Y_O + \frac{t_O}{2}} = \frac{200.860}{2.064 + 0.25} = 86.802 \text{ in}^3$$

$$Z_{fL} = \frac{I_O}{d_O + \frac{t_O}{2} - Y_O} = \frac{200.860}{8.25 - 2.604} = 35.576 \text{ in}^3$$

Thus, $Z_{min} = Z_{fL} = 35.576 \text{ in}^3$.

C-4.2.3 Calculation of Loads and Stresses

$$P_T = -(\Delta p)\, L_O R_T + 14.346 \left[\frac{(\Delta p)\, L_O R_T J}{Q}\right]$$

$$= -(1)\,(30)\,(127.146) + 14.346 \left[\frac{(1)\,(30)\,(127.146)\,(4.846)}{1217.330}\right]$$

$$= -3596.54 \text{ lb}$$

$$M_T = -3.919 \left[\frac{(\Delta p) L_O R_T{}^2 J}{Q}\right]$$

$$= -3.919 \left[\frac{(1)\,(30)(127.146)^2\,(4.846)}{1217.330}\right]$$

$$= -7566.19 \text{ in-lb}$$

- The maximum stress in the primary transverse stiffener is:

$$\sigma_t = \frac{M_t}{Z_{min}} + \frac{P_T}{A_{TO}}$$

$$= \frac{-7566.19}{35.576} + \frac{-3596.54}{16.621}$$

$$= -429 \text{ psi}$$

C-4.3 Load and Stress Calculations for Secondary Transverse Stiffeners

The following load and stress calculations are illustrated with a blow pressure of 1.0 psi.

C-4.3.1 Structural Dimensions

For secondary transverse stiffeners, the following data apply:

Plating:	$L_O = 30$ in.,	$t_O = 0.50$ in.
Web:	$d_O = 5.125$ in.,	$b_O = 0.3125$ in.
Flange:	$w_f = 5.75$ in.,	$h_f = 0.5$ in.
	$\theta_t = 30^\circ$	$l_T = 27$ in.
	$\sigma_y = 80000$ psi,	$E = 30 \times 10^6$ psi

C-4.3.2 Load and Stress Calculations

$$\Delta p_t = \frac{\Delta p}{2} \left(1 + \cos \frac{\theta_t}{2}\right)$$

$$= \frac{1.0}{2} (1 + \cos 15^\circ) = 0.983 \text{ psi}$$

$$M_{ST} = \frac{(\Delta p_t)\, L_O\, l_T^2}{12}$$

$$= \frac{(0.983)(30)(27)^2}{12} = 1791.5 \text{ in-lb}$$

Therefore, the stress in the secondary transverse stiffener is:

$$\sigma_{end} = \frac{M_{ST}}{Z_{min}} + \frac{\Delta p\, L_O\, R_T}{A_{TO}}$$

$$= \frac{1791.5}{22} + \frac{(1.0)\ (30)\ (127.146)}{16.621}$$

$$= 311 \text{ psi}$$

In the above, A_{TO}, R_T, and Z_{min} are assumed values. They can be calculated using the section properties as given above. The procedures used in the calculations are similar to those presented in Sections C-4.2.2 and C-4.2.3 and are not repeated here.

C-4.4 Calculation of Load and Stress for Outer Hull Plating

The following load and stress calculations are illustrated with a blow pressure of 1.0 psi.

C-4.4.1 Longitudinally Stiffened End Tank

The following sectional properties apply for the present example of outer hull plating in a longitudinally stiffened end tank:

$$t_O = 0.375 \text{ in.}$$

$$a' = 75 \text{ in.}, \quad b' = 30 \text{ in.}$$

since a'/b' = 2.5 , Tables A and B in Section 6.4.1 gives:

$$\epsilon = 0.0284, \quad \phi = 0.498$$

$$n'_1 = 0.125, \quad n'_2 = 0.304$$

then,

$$A' = \frac{(n'_1)^3 \; t_O^{\,2}}{\epsilon} \; \frac{a'^4}{b'}$$

$$= \frac{(0.125)^3 \; (0.375)^2}{0.0284} \; (2.5)^4 = 0.378$$

$$B' = -\frac{(\Delta p) \; (n'_1)^3 \; a'^4}{Et_O}$$

$$= -\frac{(1.0)(0.125)^3 \; (75)^4}{(30 \times 10^6)(0.375)} = -0.005493$$

$$F = \sqrt{\frac{B'^2}{4} + \frac{A'^3}{27}}$$

$$= \sqrt{\frac{(-0.005493)^2}{4} + \frac{(0.378)^3}{27}} = 0.04481$$

$$Z = \sqrt[3]{-\frac{B'}{2} + F} + \sqrt[3]{-\frac{B'}{2} - F}$$

$$= \sqrt[3]{-\frac{-.005493}{2} + .04481} + \sqrt[3]{-\frac{-.005493}{2} - .04481} = 0.0145$$

$$\Delta p_1 = \frac{Z\ E\ t_O^3}{\epsilon\ b'^4}$$

$$= \frac{(0.0145)\ (30 \times 10^6)\ (0.375)^3}{(0.0284)\ (30)^4} = 0.99719 \text{ psi}$$

$$\Delta p_2 = \frac{Z^3\ E\ t_O}{n'^3_1\ a'^4}$$

$$= \frac{(0.0145)^3\ (30 \times 10^6)\ (0.375)}{(0.125)^3\ (75)^4} = 0.00055 \text{ psi}$$

Since it is required that $\Delta p_1 + \Delta p_2 = \Delta p \doteq 1.0$, it is considered reasonable to set:

$$\Delta p_1 = 0.99 \quad \text{and} \quad \Delta p_2 = 0.01$$

based on the above calculations.

Then the combined maximum stress in the outer hull plating is:

$$\sigma_{LOHP} = \phi \frac{\Delta p_1 \, b'^2}{t_O^2} + n'_2 \left[E \left(\frac{\Delta p_2 \, a'}{t_O} \right)^2 \right]^{1/3}$$

$$= (0.498) \frac{(0.99)(30)^2}{(0.375)^2} + (0.304) \left[(30 \times 10^6) \left(\frac{(0.01)(75)^2}{0.375} \right) \right]^{1/3}$$

$$= 3155.3 + 149.9$$

$$= 3305 \text{ psi}$$

C-4.4.2 Transversely Stiffened End Tank

For this example, the various structural characteristics and sectional properties derived in Example C-3 are used. This adoption of previous data not only serves the illustration purpose but also simplifies the procedures. The main characteristics of the tank are:

Plating thickness, t_O	0.375 in.
Frame spacing, L_O	27.000 in.
T-frame web thickness, b_O	0.230 in.
Radius to outer hull surface, $R_O + \frac{t_O}{2}$	147.600 in.
Longitudinals at 90° intervals	

Thus we have:

$A'_O = 1.919$ in^2.

$\alpha_O = 0.188$

$\beta_O = 0.00846$

$F_1 = 0.437$

$F_2 = 0.00766$

$F_3 = -1.877$ (from Figure 33, for θ = 4.663 rad. $\gamma = 0.0$)

$F_4 = 0.510$

and, equations (6-10a) and (6-10b) give:

$$\sigma_h = \frac{\Delta p \ R_O}{t_O} = \frac{(1.0)\ (147.412)}{0.375} = 393.1 \text{ psi}$$

$$k = \frac{(1 - \nu/2)\alpha_O}{\alpha_O + \beta_O + F_1\ (1 - \beta_O)}$$

$$= \frac{(1 - 0.3/2)(0.188)}{0.188 + 0.00846 + 0.437(1-0.00846)} = 0.2537$$

The circumferential stress in the outer hull plating at midbay is, per equation (6-6):

$$\sigma_\phi = \sigma_h\ [1 + k\ (-F_2 \pm 0.3\ F_4)]$$

$$= 393.1\ [1 + 0.2537\ (-0.00766 \pm 0.3\ (0.510))]$$

$$= 408 \text{ psi or } 377 \text{ psi}$$

The longitudinal stress in the outer hull plating at midbay is, per equation (6-7):

$$\sigma_{\frac{L}{2}} = \sigma_h\ (0.5 \pm k\ F_4)$$

$$= 393.1\ (0.5 \pm 0.2537\ (0.510))$$

$$= 247 \text{ psi or } 146 \text{ psi}$$

The circumferential stress at the frame is, per equation (6-8):

$$\sigma_{\phi p}(x) = \sigma_h\ [1 + k\ (-1 \pm 0.3\ F_3)]$$

$$= 393.1\ [1 + 0.2537\ (-1 \pm 0.3\ (-1.877))]$$

$$= 237 \text{ psi or } 350 \text{ psi}$$

The longitudinal stress at the frame is, per equation (6-9):

$$\sigma_L(x) = \sigma_h\ (0.5 \pm k\ F_3)$$

$$= 393.1\ [0.5 \pm 0.2537\ (-1.877)]$$

$$= 384 \text{ psi or } 9 \text{ psi}$$

C-4.5 Bulkhead Stress Calculations

The stress calculations for MBT bulkhead and its primary and secondary stiffeners are illustrated with a design load of 30 psi for emergency blow condition (see Table I for design limit).

C-4.5.1 Flat Plating Thickness

Assuming the following data applies:

$\Delta p = 30$ psi

$a' = 48$ in., $b' = 48$ in.

$\sigma_y = 80{,}000$ psi (yield strength)

Then, $\sigma_a = 1.25\,(\sigma_y) = 100{,}000$ psi

and,

$$C = \frac{0.125}{3 + 4\left(\frac{b'}{a'}\right)^4}$$

$$= \frac{0.125}{3 + 4\,(1)^4} = 0.017857$$

The required thickness of the flat plating in MBT bulkheads is then:

$$t_B = \left[\frac{6C\,(\Delta p)(b')^2}{\sigma_a}\right]^{1/2}$$

$$= \left[\frac{6(0.017857)(30)(48)^2}{100{,}000}\right]^{1/2} = 0.27 \text{ in.}$$

C-4.5.2 Primary Stiffeners

Assuming the following data applies:

$\Delta p = 30$ psi

$L_{BSP} = 46$ in. $\quad t_B = 0.50$ in

$S_b = 48$ in. $\quad \sigma_y = 80{,}000$ psi

$Z_B = 64$ in^3. $\quad E_B = 30 \times 10^6$ psi

then the maximum stress on the primary bulkhead stiffeners at mid-span is:

$$\sigma_{BSP} = \pm \frac{(\Delta p)\, S_b\, (L_{BSP})^2}{24\, Z_B} \left[3 - \frac{S_b}{L_{BSP}}\right]^2$$

$$= \pm \frac{(30)(48)(46)^2}{24\,(64)} \left[3 - \frac{48}{46}\right]^2$$

$$= \pm\ 3791 \text{ psi}$$

C-4.5.3 Secondary Stiffeners

Assuming the following data applies:

$\Delta p = 30$ psi

$L_{BSS} = 30$ in.

$Z_B = 38$ in^3.

$S_{bs} = 27$ in.

then the maximum stress in the secondary bulkhead stiffeners at the end of the span is:

$$\sigma_{BSS} = \pm \frac{(\Delta p)\, S_{bs}}{12\, L_{BSS}\, Z_B} \left[L_{BSS}^3 + \frac{S_{bs}^3}{8} - \frac{L_{BSS}\, S_{bs}^2}{2}\right]$$

$$= \pm \frac{(30)(27)}{12(30)(38)} \left[(30)^3 + \frac{(27)^3}{8} - \frac{(30)(27)^2}{2}\right]$$

$$= \pm\ 1097 \text{ psi}$$

APPENDIX D - VIRTUAL WORK

D-1 GENERAL

The stress analysis of the primary transverse stiffener (Section 6.3.1) is based on representing the stiffener as a 90° circular arch, fixed supported at each end, with a uniformly distributed pressure load, Δp, acting on it. Since the structure as represented is indeterminate, the method of virtual work is used to solve for the reactions at the fixed ends of the arch and is presented in the following pages.

Step 1 presents the governing equations for the virtual work principle which requires that the deformation energy (left-hand side of equation) due to the action of the pressure loadings, V_O, P_O, and M_O, balances out the potential energy (right-hand side of equation) resulting from the action of the reactions V, P, and M. Steps 3 through 5 give the resulting forces and moment due to the pressure loading given in Step 2. Step 6 gives the resulting forces and moment due to the action of the unit loads, V_A, P_V, and M_A. Step 7 calculates the deformation energy due to the action of the pressure loading-induced forces and moments, and the potential energy due to the action of the unit load-induced forces and moments. Step 8 is the standard solution for a set of linear equations. Steps 9 through 12 are self-explanatory.

For configurations other than the case presented herein (i.e., longitudinal stiffeners at intervals other than 90°), the appropriate angle should be substituted into the integrals of Step 7 for evaluation. Steps 8 and 9 remain unchanged. Substituting values obtained from Step 7 into the determinants developed in Step 9 results in new values for the determinants.

These may then be simplified in a manner similar to Steps 10 and 11.

To apply this method and obtain the reaction for alternate configurations, the variables used in the virtual work solution may be correlated to those used in Equation 6-3 as follows:

$P = P_T$;

$W = \Delta p\ L_O$;

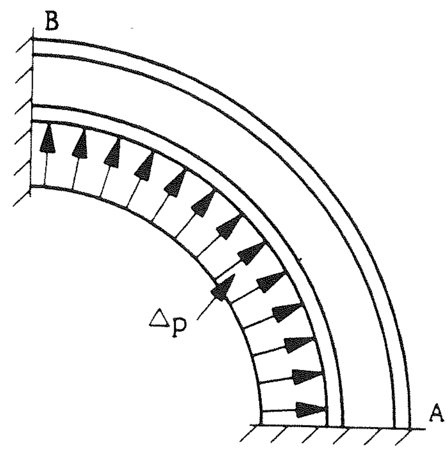
$R = R_T$;

$M = M_T$;

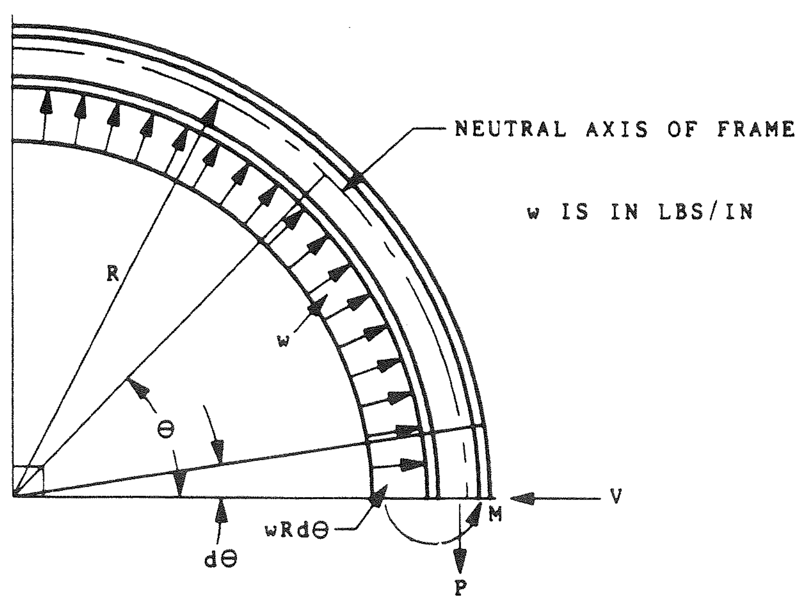
$A = A_{TO}$; and

$I = I_O$.

DERIVATION OF THE REACTIONS FOR A FIXED ENDED CIRCULAR ARCH UNDER A UNIFORM INTERNAL PRESSURE



A circumferential stiffener is designed in 90° segments as an arch under a uniform internal pressure fixed at both ends. The ends of the arch are assumed fixed because of the constraints caused by the vertical centerline bulkhead at one end and a longitudinal stiffener at the other end.



VIRTUAL WORK SOLUTION

1. Equations

$$\Delta V = V\delta_{VV} + P\delta_{VP} + M\alpha_{VM}$$

$$\Delta P = V\delta_{PV} + P\delta_{PP} + M\alpha_{PM}$$

$$\alpha_{M} = V\alpha_{MV} + P\alpha_{MP} + M\alpha_{MM}$$

2. Load on an Element

At the 0° end of the arch $\quad LOAD \doteq wRd\theta$

3. Moment Due to Loading

At the 0° end of the arch $\quad M_O = \int_0^\theta (wRd\theta)(R\sin\theta) = wR^2(1-\cos\theta)$

4. Axial Force Due to Loading

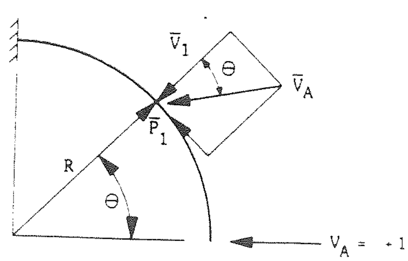
At the 0° end of the arch $\quad P_O = \int_0^\theta wR\sin\theta d\theta = wR(1-\cos\theta)$

5. Shear Force Due to Loading

V_O is assumed negligible and therefore not included in this solution.

6. Unit Loads

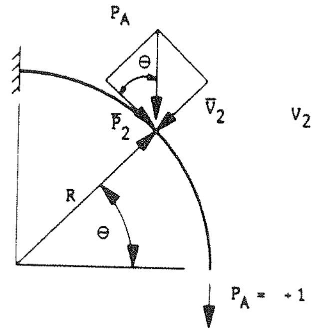
(Sign Convention)



$\bar{P}_1 = \sin\theta$

$\bar{M}_1 = -R\sin\theta$

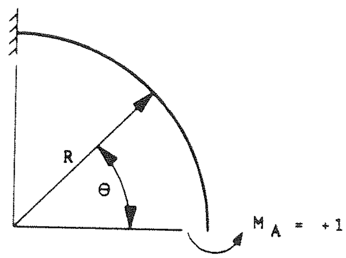
$\bar{V}_1$ is Neglected



$\bar{P}_2 = \cos\theta$

$\bar{M}_2 = -R(1-\cos\theta)$

$\bar{V}_2$ is Neglected



$\bar{P}_3 = 0$

$\bar{M}_3 = 1.0$

$\bar{V}_3 = 0$

7. Terms

NEGLIGIBLE

$$\Delta_V = \int_0^{\pi/2} \left[B \frac{\bar{V}_1 V_O}{GA} + \frac{\bar{P}_1 P_O}{EA} + \frac{\bar{M}_1 M_O}{EI} \right] Rd\theta = -\frac{wR^2}{2EA} - \frac{wR^4}{2EI}$$

NEGLIGIBLE

$$\Delta_P = \int_0^{\pi/2} \left[B \frac{\bar{V}_2 V_O}{GA} + \frac{\bar{P}_2 P_O}{EA} + \frac{\bar{M}_2 M_O}{EI} \right] Rd\theta = \frac{wR^2}{EA}\left[1 - \pi/4\right] - \frac{wR^4}{EI}\left[\frac{3\pi}{4} - 2\right]$$

NEGLIGIBLE

$$a_M = \int_0^{\pi/2} \left[B \frac{\bar{V}_3 V_O}{GA} + \frac{\bar{P}_3 P_O}{EA} + \frac{\bar{M}_3 M_O}{EI} \right] Rd\theta = \frac{wR^3}{EA}\left[\pi/2 - 1\right]$$

$$\delta_{VV} = \int_0^{\pi/2} \left[B \frac{\bar{P}_1 \bar{P}_1}{EA} + \frac{\bar{M}_1 \bar{M}_1}{EI} \right] Rd\theta = \frac{\pi R}{4EA} + \frac{\pi R^3}{4EI}$$

$$\delta_{PP} = \int_0^{\pi/2} \left[\frac{\bar{P}_2 \bar{P}_2}{EA} + \frac{\bar{M}_2 \bar{M}_2}{EI} \right] Rd\theta = \frac{\pi R}{4EA} + \frac{R^3}{EI} \left[\frac{3\pi}{4} - 2 \right]$$

$$a_{MM} = \int_0^{\pi/2} \left[\frac{\bar{P}_3 \bar{P}_3}{EA} + \frac{\bar{M}_3 \bar{M}_3}{EI} \right] Rd\theta = \frac{\pi R}{2EI}$$

$$\delta_{PV} = \delta_{VP} = \int_0^{\pi/2} \left[\frac{\bar{P}_1 \bar{P}_2}{EA} + \frac{\bar{M}_1 \bar{M}_2}{EI} \right] Rd\theta = -\frac{R}{2EA} + \frac{R^3}{2EI}$$

$$a_{VM} = a_{MV} = \int_0^{\pi/2} \left[\frac{\bar{P}_1 \bar{P}_3}{EA} + \frac{\bar{M}_1 \bar{M}_3}{EI} \right] Rd\theta = -\frac{R^2}{EI}$$

$$a_{PM} = a_{MP} = \int_0^{\pi/2} \left[\frac{\bar{P}_2 \bar{P}_3}{EA} + \frac{\bar{M}_2 \bar{M}_3}{EI} \right] Rd\theta = -\frac{R^2}{EI} \left[\pi/2 - 1 \right]$$

8. <u>Cramer's Rule</u>

$$P = \frac{|A_P|}{|A|} \qquad V = \frac{|A_V|}{|A|} \qquad M = \frac{|A_M|}{|A|}$$

Where:

$$|A| = \begin{vmatrix} \delta_{VV} & \delta_{VP} & a_{VM} \\ \delta_{VP} & \delta_{PP} & a_{PM} \\ a_{VM} & a_{PM} & a_{MM} \end{vmatrix}$$

$$|A_P| = \begin{vmatrix} \delta_{VV} & \Delta_V & a_{VM} \\ \delta_{VP} & \Delta_P & a_{PM} \\ a_{VM} & a_M & a_{MM} \end{vmatrix}$$

$$|A_V| = \begin{vmatrix} \Delta_V & \delta_{VP} & a_{VM} \\ \Delta_P & \delta_{PP} & a_{PM} \\ a_M & a_{PM} & a_{MM} \end{vmatrix}$$

$$|A_M| = \begin{vmatrix} \delta_{VV} & \delta_{VP} & \Delta_V \\ \delta_{VP} & \delta_{PP} & \Delta_P \\ a_{VM} & a_{PM} & a_M \end{vmatrix}$$

9. <u>Expanding the Determinant's</u>

$$|A| = \delta_{VV}\delta_{PP}a_{MM} - \delta_{VV}a_{PM}^{2} - a_{MM}\delta_{VP}^{2} - \delta_{PP}a_{VM}^{2}$$

$$|A_P| = \delta_{VV}\Delta_P a_{MM} - a_M a_{PM}\delta_{VV} - \delta_{VP}a_{MM}\Delta_V$$

$$|A_V| = \delta_{PP}a_{MM}\Delta_V - \Delta_V a_{PM}^{2} - \delta_{VP}\Delta_P a_{MM}$$

$$|A_M| = \delta_{VV}\delta_{PP}a_M - \Delta_P a_{PM}\delta_{VV} - \delta_{VP}a_M$$

10. Substituting and Combining Terms From No. 7 Into No. 9

For Convenience, the constant term "E" will be suppressed from here on until all computations are completed.

$$|A| = \frac{R^7}{32I^3}\left[\pi^3 - 20\pi + 32\right] + \frac{R^5}{32I^2A}\left[2\pi^3 - 24\pi + 32\right] + \frac{R^3}{32IA^2}\left[\pi^3 - 4\pi\right]$$

$$|A_P| = -\frac{wR^8}{32I^3}\left[\pi^3 - 20\pi + 32\right] - \frac{wR^6}{32I^2A}\left[2\pi^3 - 4\pi^2 - 16\pi + 32\right] - \frac{wR^4}{32IA^2}\left[\pi^3 - 4\pi^2 + 4\pi\right]$$

$$|A_V| = -\frac{wR^6}{32I^2A}\left[4\pi^2 - 8\pi\right] - \frac{wR^4}{32IA^2}\left[4\pi^2 - 8\pi\right]$$

$$|A_M| = \frac{wR^7}{32I^2A}\left[4\pi^2 - 24\pi + 32\right] + \frac{wR^5}{32IA^2}\left[4\pi^2 - 24\pi + 32\right]$$

11. Solving for the Reactions - Using Cramer's Rule

$$P = \frac{|A_P|}{|A|} = -wR + \frac{\left[4\pi^2 - 8\pi\right]\left[\frac{wR^3}{AI} + \frac{wR}{A^2}\right]}{\frac{R^4}{I^2}\left[\pi^3 - 20\pi + 32\right] + \frac{R^2}{AI}\left[2\pi^3 - 24\pi + 32\right] + \frac{1}{A^2}\left[\pi^3 - 4\pi\right]}$$

Evaluating the π Terms

$$P = -wR + \frac{14.346\ wR\left[\frac{R^2}{AI} + \frac{1}{A^2}\right]}{.174\frac{R^4}{I^2} + 18.614\frac{R^2}{AI} + \frac{18.44}{A^2}}$$

$$V = \frac{|A_V|}{|A|} = \frac{-wR\left[4\pi^2 - 8\pi\right]\left[\frac{R^2}{AI} + \frac{1}{A^2}\right]}{\frac{R^4}{I^2}\left[\pi^3 - 20\pi + 32\right] + \frac{R^2}{AI}\left[2\pi^3 - 24\pi + 32\right] + \frac{1}{A^2}\left[\pi^3 - 4\pi\right]}$$

Evaluating the π Terms

$$V = \frac{-14.346\ wR\left[\frac{R^2}{AI} + \frac{1}{A^2}\right]}{.174\frac{R^4}{I^2} + 18.614\frac{R^2}{AI} + \frac{18.44}{A^2}}$$

$$M = \frac{|A_M|}{|A|} = \frac{wR^2\left[\frac{R^2}{AI} + \frac{1}{A^2}\right]\left[4\pi^2 - 24\pi + 32\right]}{\frac{R^4}{I^2}\left[\pi^3 - 20\pi + 32\right] + \frac{R^2}{AI}\left[2\pi^3 - 24\pi + 32\right] + \frac{1}{A^2}\left[\pi^3 - 4\pi\right]}$$

Evaluating the π Term

$$M = \frac{-3.919\ wR^2\left[\frac{R^2}{AI} + \frac{1}{A^2}\right]}{.174\frac{R^4}{I^2} + 18.614\frac{R^2}{AI} + \frac{18.44}{A^2}}$$

12. Simplifying the Reactions

Since the stiffener has a constant modulus of elasticity "E", the term divides out of the denominator and numerator.

Let $Q = .174\frac{R^4}{I^2} + 18.614\frac{R^2}{AI} + \frac{18.44}{A^2}$, and

$$J = \frac{R^2}{AI} + \frac{1}{A^2}$$

Therefore,

$$P = -wR + 14.346\frac{wRJ}{Q}$$

$$V = -14.346\frac{wRJ}{Q}$$

$$M = -3.919\frac{wR^2J}{Q}$$

For a 90° Arch.

Where: w is in lb/in;

A is in in^2;

I is in in^4; and

R is in in.

As noted earlier, the variables used here can be correlated in those in Equation 6-3 as follows:

$P = P_T$;

$W = \Delta_P L_O$;

$R = R_T$;

$M = M_T$;

$A = A_{TO}$; and

$I = I_O$.