DDS243-1

INTERIM

DESIGN DATA SHEET

DEPARTMENT OF THE NAVY, BUREAU OF SHIPS

1 January 1960

DDS4301
PROPULSION SHAFTING
Supersedes DDS4301, dated 1 May 1957

DDS4301-a. References

1. General Specifications for Ships of the U. S. Navy, sections S9-3 and S43-1.
2. F. M. Lewis, Torsional Vibration in the Diesel Engine, Trans. Society Naval Architects and Marine Engineers, 1925.
3. J. P. Den Hartog, Mechanical Vibrations, McGraw-Hill Publ. Co.
4. W. Ker Wilson, Practical Solution of Torsional Vibration Problems, J. Wiley and Sons.
5. S. Timoshenko, Strength of Materials, D. Van Nostrand Co.
6. R. J. Roark, Formulas for Stress and Strain, McGraw-Hill Publ., Co.
7. C. R. Soderberg, Factor of Safety and Working Stress, A. S. M. E. Trans., APM-52-2.
8. Jacobsen, Lydik S., "Torsional Stress Distributions in Prismatical Bars" (ASME-AMD Trans.).
9. Kane, J. R. and McGoldrick, R. T., "Longitudinal Vibrations of Marine Propulsion-Shafting Systems," Trans. SNAME, vol. 57.
10. Bureau of Ships plan, ~~No 5000-S4300-P-1985661~~ 810-2145807 (five sheets).
11. EES Report 040034C of 15 July 1953.
12. Mil. Spec. MIL-S-15058 - Synthetic Rubber Coating.
13. Maleev, V. L. - "Machine Design," International Textbook Co.
14. MIL-STD-167 (SHIPS) - Mechanical Vibrations of Shipboard Equipment.

15. E. Panagopulos - Calculations of Shaft Vibrations - Trans. SNAME, 1950.

16. N.H. Jasper - A Design Approach to the Problem of Critical Whirling Speed - DTMB Report 890.

17. Torsional Vibration Analysis - BUSHIPS Report 371-V-19.

18. F.P. Porter - Harmonic Coefficients of Engine Torque Curves, Journal of Applied Mechanics, March 1943.

DDS4301-b. General Design

Note: Data in regard to off-center thrust moments on the propeller shafts and torsional vibratory torques at full power and critical speeds is to be considered tentative, pending further tests and evaluation of existing test data. Also the effect of cold rolling of shafts at the propeller is under evaluation and cannot be included at this time.

1. The basis for the design of propulsion shafting for naval ships shall be as follows:

(a) All propulsion shafting shall be designed for full power plus an additional torque imposed by slowing up of the propeller when the ship is making a turn at maximum power. This additional torque shall be equal to 20 percent of the full power torque for both single and multi-shaft ships, except for ships with geared diesel engine propulsion and diesel-electric (A.C.) propulsion, where 10% (instead of 20%) additional torque shall be used.

In addition, the design of shafting for ships driven by reciprocating engines shall be checked for the torsional critical speed at which the vibratory stresses are largest.

Freedom from longitudinal and lateral vibration and vibratory stresses due to torsional vibration shall be in accordance with reference (14).

(b) Propulsion shafting shall be designed to the following factors of safety, based on the yield strength and fatigue limit in air for complete stress reversals in bending. In no case shall the bending stress at the shaft surface resulting from propeller overhang and off-center thrust exceed 6,000 p.s.i. ($k_b \times S_b$).

Shaft	Ship Type			
	Surface Ships other than Ice Breakers	Ice Breakers	Submarines	
			Single Shaft	Multi-Shaft
Propeller	2.00	3.50	2.25	2.00
Intermediate	2.00	2.25	2.25	2.00
Stern Tube	2.00	2.25	2.25	2.00
Line	1.75	2.25	2.00	1.75

(c) All surfaces of steel shafts, including couplings and flanges, that are exposed to the sea, shall be covered with rubber in accordance with reference 12. Details of construction are given in reference 10.

(d) In calculating stresses, the steady stress and the alternating stress shall be calculated separately. The steady stress is the resultant stress due to the steady torque and thrust. The alternating stress is the resultant stress due to the alternating torque and to bending, including bending due to off-center thrust.

(e) Inasmuch as the alternating stress alone must be multiplied by appropriate stress concentration factors, it is necessary, especially where the alternating stresses are large, to avoid high localized stresses. This shall be accomplished by the use of generous fillets, and by avoiding the drilling of holes into the shafting to secure keys, sleeves, oil baffles, etc. Likewise, welding shall be prohibited except where specifically authorized.

(f) A multiplier of 2 shall be used for converting shearing stress to tensile stress. All calculations are then to be based upon tension and compression.

(g) The final stress reduced to tensile or compressive stress should fall within the triangle shown in figure 4.

(h) Shafting 6 inches and less in diameter shall be made solid; above 6 inches, bored. The inside diameter chosen will depend upon the application; normally it will be approximately two-thirds of the outside diameter.

(i) Caution should be used in the computation of alternating stresses, particularly when they are engine excited, as in the case of Diesel engines and reciprocating steam engines, where a more elaborate torsional analysis may be r quired.

DDS4301-c. Symbols

A = Shaft cross sectional area, square inches.
B = Number of blades of propeller.
C = Damping coefficient, inch-pounds-sec. per radian.
c = a constant.
D = Outside diameter of shaft, inches.
d = Inside diameter of shaft, inches.
D_o = Outer diameter of bronze sleeve shrunk on shaft, inches (see reference 10).
D_p = Propeller dia., inches.
e = Efficiency, expressed as a decimal fraction.
E = Modulus of elasticity (see Table 5), lbs. per in.^{2}
EHP = Effective horsepower.
f = Cycles per minute.
FL = Fatigue or endurance limit for complete reversal of stress (see Table 5).
G = Modulus of shear (see Table 5), lbs. per in.^{2}
I = Mass moment of inertia = WR^2/g, lb.-in.-sec.^{2}
I_b = Bending moment of inertia of section, in.^{4}
I_e = Mass moment of inertia of engine, lb.-in.-sec.^{2}
i = Moment of inertia of shaft about diameter (inches) 4 = $\pi(D^4 - d^4)/64$.
I_p = Mass moment of inertia of propeller about its axis, lb.-in.-sec.^{2}
I_x = Mass moment of inertia of propeller about diameter, lb.-in.-sec.^{2} = $1/2\ I_p$ (add 60% for entrained water).
J = Polar moment of inertia of section, in.^{4}
K = Torsional stiffness of shaft, in.-lb. per radian.
k_b = Stress concentration factor in bending.
k_t = Stress concentration factor in torsion.
L = Length, inches.
L_p = Moment arm of propeller assembly assumed from center of gravity of propeller assembly to a point one shaft diameter forward of after end of strut bearing, for a wood or single rubber strip bearing.
M = Bending moment, inch-pounds.
M_ℓ = Bending moment in line shaft, inch-pounds.
M_g = Gravity bending moment at strut bearing, inch-lbs.
$M_{o.c.}$ = Bending moment due to off-center thrust, inch-lbs.
M_p = Sum of M_g and $M_{o.c.}$
MIP = Mean indicated pressure, p.s.i.
m_1 = Propeller mass, see p. 18.
m_2 = Gear mass, see p. 18.
n = Number of firings per revolution of Diesel engine.
n_b = Number of coupling bolts.
RPM = Revolutions per minute.
Q = Mean or steady torque, inch-lbs.
Q_a = Alternating indicated engine torque expressed in percent of mean indicated engine torque.

Q_p = Alternating propeller torque, expressed in percent of mean propeller torque.
r = Radius of fillet, inches.
S_b = Compressive stress due to bending, p. s. i.
S_c = Compressive stress due to thrust, p. s. i.
S_s = Shear stress due to torsion, p. s. i.
SHP = Shaft horsepower.
PC = Propulsive coefficient
T = Propeller thrust, pounds.
t = Thrust deduction factor, expressed as a decimal fraction.
V = Ship's speed, knots.
w = Weight per unit length of shaft, lbs. /inch.
W_p = Weight of propeller, including nut, hub cap, packing, and shaft stub, pounds (in air).
W_{prop} = Weight of propeller, nut, cap, and packing.
YP = Yield point, p. s. i. (see Table 5).
ϕ = Vibration amplitude, radians.
Ω = Angular velocity, r. p. m.
μ = Shaft mass per inch = w/g lb. sec. ^{2}/inch^2
ω = Circular frequency, radians per second.
$K_1 K_2$ = Longitudinal stiffnesses of shaft, lbs. /inch (see Fig. 6).
K_o = Stiffness of thrust bearing and foundation in series, lbs. per inch.
K_e = Stiffness of thrust bearing and engine foundation (see Fig. 7), lbs. per inch.

DDS4301-d. Detail design

1. Steady stress -

(a) The torque:

$$Q = \frac{33,000 \text{ SHP} \times 12}{2\pi \text{RPM}} = 63,025 \frac{\text{SHP}}{\text{RPM}} \tag{1}$$

(b) The shear stress due to torque:

$$S_s = \frac{QD}{2J} = \frac{16}{(\pi)} \frac{QD}{(D^4 - d^4)} = 5.1 \frac{QD}{D^4 - d^4} \text{ (hollow shaft)} \tag{2a}$$

$$S_s = \frac{5.1Q}{D^3} \text{ (solid shaft)} \tag{2b}$$

See paragraph DDS4301-b-1(a) for value of Q to be used for design.

(c) The thrust:

$$T = \frac{33,000 \text{ EHP}}{6080V/60} \times \frac{1}{1-t} = 326 \frac{\text{EHP}}{V} \times \frac{1}{1-t} \tag{3}$$

For submarine shafting, the thrust is increased by an amount equal to the maximum submergence pressure multiplied by the area of the propeller shaft.

Table 1

Values of Thrust Deduction, t

	Number of Propellers		
	1	2	4
AO & Cargo	.20		
DE	.03		
DD		.05	
LSD, DE, PCE, AM, AD, LPH, CVE		.12	
LSM, TAG, AO		.24	
AGB		.30	
CL, CA, CB, CV (no skegs)			.10
CV (skegs)			.18

The above values of "t" are average and should be used only in case model basin data for a specific application is not available.

(d) The compressive stress due to thrust:

$$S_c = \frac{T}{A} = \frac{T}{(D^2 - d^2)\,\pi/4} = \frac{1.273T}{(D^2 - d^2)} \quad (4)$$

(e) The resultant steady stress, reduced to compressive stress:

$$S_{steady\ resultant} = \sqrt{S_c^{\,2} + (2S_s)^2} \quad (5)$$

2. Alternating stress -

(a) The bending moment at the propeller strut bearing, due to propeller overhang is:

$$M_g = W_p L_p \quad (6)$$

(b) The bending moment at the propeller strut bearing due to off-center thrust and additive to the gravity moment, equation (6), is approximated by the following:

SURFACE SHIPS

For single shaft ships $M_{o.c.} = 2M_g$ (7a)

For multi-shaft ships $M_{o.c.} = M_g$ (7b)

SUBMARINES

For single shaft submarines $M_{o.c.} = 0$

For multi-shaft submarines $M_{o.c.} = M_g$

Data is being accumulated on off-center thrusts for different types of ships and will be incorporated in equations (7a) and (7b) when available.

(c) The resulting bending moment is:

SURFACE SHIPS

$$M_p = M_g + M_{o.c.} = 3M_g \text{ for single shaft ships} \quad (8a)$$

$$= 2M_g \text{ for multi-shaft ships} \quad (8b)$$

SUBMARINES

$$M_p = M_g \text{ for single shaft subs} \quad (8c)$$

$$M_p = 2M_g \text{ for multi-shaft subs} \quad (8d)$$

The stress due to M_p shall not exceed 6,000 p.s.i. ($k_b \times S_b$).

(d) The maximum bending moment in a line shaft with many more-or less equally spaced bearings at distance, L, apart is, approximately:

$$M_{\ell} = \frac{wL^2}{12} \quad (9)$$

Note: The graphical method of determining the bending moment should be used where greater accuracy is desired.

(e) The alternating bending stress in a shaft:

$$S_b = \frac{MD}{2I_b} = \frac{32}{\pi} \times \frac{MD}{D^4 - d^4} = \frac{10.2M \cdot D}{D^4 - d^4} \text{ (Hollow shaft)} \quad (10a)$$

$$S_b = \frac{10.2M}{D^3} \text{ (Solid shaft)} \quad (10b)$$

Note: (a) A separate calculation must be made for the line shafting and for the propeller and stern tube shafting.

Note: (b) In computing the bending stress due to M_p, the inside diameter "d" shall be assumed equal to the larger bore of the propeller shaft.

(f) The alternating torsional stress at the first critical speed can be calculated in a simple manner only when the installation belongs to one of the four types listed below: In these cases, the system is considered to be a two-mass system, with a connecting shaft (spring) of uniform diameter:

(1) Geared turbine drives, usually with two turbine branches, provided the torsional stiffness of the propulsion shafting is small in comparison with the torsional stiffness of the turbine shafting. This is usually the case.

(2) Electric drives with or without gears.

(3) Hydraulic coupling drives, where there is a certain torsional flexibility in the propeller shaft between the propeller and the driven coupling half.

(4) Diesel drives, with or without intermediate gears, provided the torsional stiffness of the line shaft is less than one-fourth as large as the torsional stiffness of the full length of engine shafting.

Note: In cases 1, 2, and 3, the excitation is entirely in the propeller; in case 4 the excitation is either in the propeller or in the engine. The derivation of all formulas is given in paragraph DDS4301-e.

(g) Vibratory torsional stress at first critical speed, cases (1), (2), and (3) above.

(1) For cases (1), (2), and (3) above, the vibration amplitude at the propeller (in radians) at the first torsional critical, or resonant speed is:

$$\Phi_{prop.} = 0.0025 \frac{Q_p}{B} \qquad (11)$$

$$Q_p = \frac{\text{alternating propeller torque}}{\text{mean propeller torque}} \times 100 \qquad (12)$$

Approximate values of Q_p are given in Table 2.

Table 2*

Alternating Propeller Torque, Percentage

	Single Shaft			Multi-Shaft					
				Without Skegs			With Skegs		
No. blades	4	5	6	4	5	6	4	5	6
Q_p				7.5					

*Data not given is under accumulation. Where data is not given, use Q_p = 7.5.

The shear stress in the propeller shaft (at the critical speed) due to alternating torque is:

$$S_s = \frac{GD}{2L} \Phi_{prop.} \left[\frac{I_p + I_e}{I_e}\right] \qquad (13)$$

I_p includes 25% due to entrained water.

(2) In steam turbine installations, I_e usually is very large compared to I_p. Then the above formula is very nearly equal to:

$$S_s = \frac{GD}{2L} \Phi_{prop.} = 0.00125\ Q_p \frac{GD}{BL} \qquad (13a)$$

(3) In case there are gears in the installation so that the engine and propeller run at different speeds, the mass moments of inertia of the fast-running members have to be multiplied by the square of their speed ratio before insertion into equation 13. (For example, in a turbine drive with a 200 r.p.m. propeller shaft, 1,000 r.p.m. intermediate gears, and 5,000 r.p.m. turbines, the moments of inertia of the intermediate gears are to be multiplied by $(1000/200)^2 = 25$, and the moments of inertia of the turbines are to be multiplied by $(5000/200)^2 = 625$. The values so multiplied must be inserted in equation 13.)

h) Vibratory torsional stress at first critical speed, diesel drive.

(1) Equations 11, 12 and 13 apply not only to drive types 1, 2, 3 as stated, but to diesel drives, case 4, as well, when the critical speed is excited by the propeller. However, in installations of case 4 type, the main excitation is normally caused by the engine. For this case, equation 13 is still applicable, but equation 11 is modified to:

$$\Phi_{prop.} = \frac{Q_a}{400\,ne} \times \frac{135}{MIP} \left(\frac{RPM_{rated}}{RPM_{critical}}\right) \frac{2I_p}{I_e} \qquad (14)$$

where n is the number of firings per revolution of the engine, e is the mechanical efficiency of the engine, and Q_a is the ratio of engine exciting torque to mean full-load engine torque expressed in percent:

$$Q_a = \frac{\text{alternating engine torque}}{\text{mean indicated engine torque}} \times 100 \qquad (15)$$

(2) Values for Q_a differ for various types of diesel engines and preferably should be obtained from the manufacturer. For

preliminary design, the values of Q_a are to be taken from Table 3. More comprehensive values of alternating engine torques are given in references 17 and 18.

Table 3
Alternating Engine Torque, Percentage

n =	2	2-1/2	3	3-1/2	4	4-1/2	5
Q_a =	160	130	110	90	65	45	35
n =	5-1/2	6	6-1/2	7	7-1/2	8	8-1/2
Q_a =	25	18	14	11	9	7	6
n =	9	9-1/2	10	10-1/2	11	11-1/2	12
Q_a =	5	4	3-1/2	3	2-1/2	2	1-1/2

Note: It is clear that the half-integer values of n occur only in four-cycle engines, where n is equal to half the number of cylinders; whereas, for two-cycle diesels, n is equal to the number of cylinders.

(i) Vibratory torsional stress at full power, geared turbine drive.

In the case of long shaft ships with geared turbine drive, the first torsional critical speed is usually below one-half the full power r.p.m. The second torsional critical usually lies 15 percent or more above the full power r.p.m. In such cases, the torsional vibratory stress at full power may be estimated with sufficient accuracy from Table 4.

Table 4*
Full Power Vibratory Stress, Percentage

	Single Shaft		Multi-shaft	
	Skeg	No skeg	Skeg	No skeg
F.P. vibratory stress as % of F.P. steady shear stress				5

*Data not given is under accumulation.

(j) The stress calculated by equation (13) occurs only at the first torsional critical speed, while at other speeds, well removed from the torsional critical, the stress is negligibly small. The torsional critical speed of the propeller shaft for propeller excitation is:

$$RPM_{critical} = \frac{30}{\pi B}\sqrt{\frac{K(I_e + I_p)}{I_e \times I_p}} \quad (16)$$

where K, the torsional stiffness of the propulsion shafting, is:

$$K = \frac{GJ}{L} = \frac{G \times \pi(D^4 - d^4)}{32L} \quad (17)$$

(k) Usually the engine inertia is many times greater than the propeller inertia. For geared turbine drives or similar installations in which I_e may be considered infinite as compared to I_p, equation (16) may be replaced by

$$RPM_{critical} = \frac{30}{\pi B}\sqrt{\frac{K}{I_p + 0.33\,(I_{shaft})}} \quad (17a)$$

(l) The critical speed of the engine shaft in the case of engine excitation is:

$$RPM_{critical} = \frac{30}{\pi n}\sqrt{\frac{K(I_e + I_p)}{I_e \times I_p}} \quad (18)$$

with K defined by equation 17. Note that equation (16) refers to the propulsion shafting and equation (18) to the engine shaft, which have different speeds in case reduction gears are used.

(m) Resultant alternating stress: The resultant alternating stress is found, first by multiplying the bending and torsion component each by its appropriate stress concentration factor, and then by combining, thus:

$$S_{alternating\ resultant} = \sqrt{(k_b S_b)^2 + (2k_t S_s)^2} \quad (19)$$

The principal points of stress concentration in propeller shafting occur at the corners of keyways; in propeller shafting of controllable pitch propeller installations, at the fillet of the propeller hub flange; in stern tube shafting, at flange fillets and keyways; in line shafting, at the fillets between the shaft and its flange, or at holes drilled in the shaft. The General Specifications for Ships of the U.S. Navy prohibit the drilling of holes in propulsion shafting where avoidable.

The stress concentration factor, k_t for torsional stress at keyway fillets, is a function of the ratio of the fillet radius r_1 in the corner of the keyways to the depth of keyway H. Values of k_t are to be taken from figure 1. The fillet radius should be made as large as practical and under no circumstances should it ever be smaller than $r_1/H = 0.1$.

Table 5

Mechanical Properties of Shafting Materials

Material	Government specification	$E \times 10^6$	$G \times 10^6$	Tensile strength p.s.i.	Yield strength (.01% offset) p.s.i.	Tensile F.L. (in air) complete reversals, p.s.i.
Steel						
Class 1	MIL-S-23284	29.5	♦11.75	95,000	75,000	47,500
Class 2	MIL-S-23284	29.5	11.75	80,000	55,000	40,000
Class 3	MIL-S-23284	29.0	12.00	75,000	45,000	34,000
Class 4	MIL-S-23284	29.0	11.75♦	60,000	35,000	27,000
Alloy #2	MIL-S-890	29.0	11.8	120,000	105,000	60,000
Alloy #4	MIL-S-890	29.0	11.8 or 11.9	120,000	100,000	60,000
Class An (Mo-Va)	MIL-S-890	29.0	12.0	80,000	45,000	40,000
An (2.75 Ni)	MIL-S-890	29.0	11.7	80,000	45,000	40,000
Class B	MIL-S-890	29.0	11.9	60,000	30,000	27,000
Class Bs (special)	MIL-S-890	29.0	11.9	75,000	40,000	34,000
Class Hg	MIL-S-890	29.0	11.6	95,000	65,000	47,500
Monel (Ni-Cu alloy)						
1/2" dia. to 3-1/2" incl.	QQ-N-281	26.0	9.5	87,000	60,000	37,500
Over 3-1/2" dia. to 4" incl.	QQ-N-281	26.0	9.5	84,000	55,000	36,000
K Monel	QQ-N-286	26.0	9.5	140,000	100,000	50,000
Aluminum bronze						
0.5" to 1.0" dia. incl.	MIL-B-15939	16.0	7.2	105,000	55,000	35,000
Over 1"-2.0" incl.	MIL-B-15939	16.0	7.2	105,000	50,000	35,000
Over 2"-3.0" incl.	MIL-B-15939	16.0	7.2	95,000	45,000	30,000
Over 3" dia.	MIL-B-15939	16.0	7.2	85,000	42,500	26,000

Note: Symbol ♦ shows change to replacement page.

The stress concentration factor, k_t, for torsional stress at the fillet of a coupling flange depends on the fillet radius r, the outside shaft diameter D, and the outer diameter of the flange, D_{flange}, and is shown on Fig. 2.

Note: Bending and tensile stresses are affected by notches, corrosion pits, etc., in approximately the same manner. The stress concentration factor k_t, in bending or tension due to a keyway is unity, and provided the ends of the keyway are properly faired into the shaft (see reference 10, sheet 5) stress concentration can be neglected at key end.

(n) The influence of flange fillets on the bending or tensile stresses is shown on figure 3.

Note: Oil holes drilled normal to the surface of a shaft usually have a diameter which is small with respect to the shaft diameter, and a stress concentration factor of three (3) for the bending stress should be used. As stated before, such holes are prohibited, except when unavoidable. Equation (19) has to be applied to a number of points of the shafting which indicate high stress, such as the bottom of a keyway or the fillet of a flange. It is clear that the stress concentration factors in a flange and in a keyway do not occur at the

STRESS CONCENTRATION FACTOR AT KEYWAY FILLET, IN TORSION.

REF. (a) JACOBSEN, L.S.- "TORSIONAL STRESS DISTRIBUTION IN PRISMATIC BARS." TRANSACTIONS, A.S.M.E.-AMD.
(b) MALEEV-"MACHINE DESIGN."

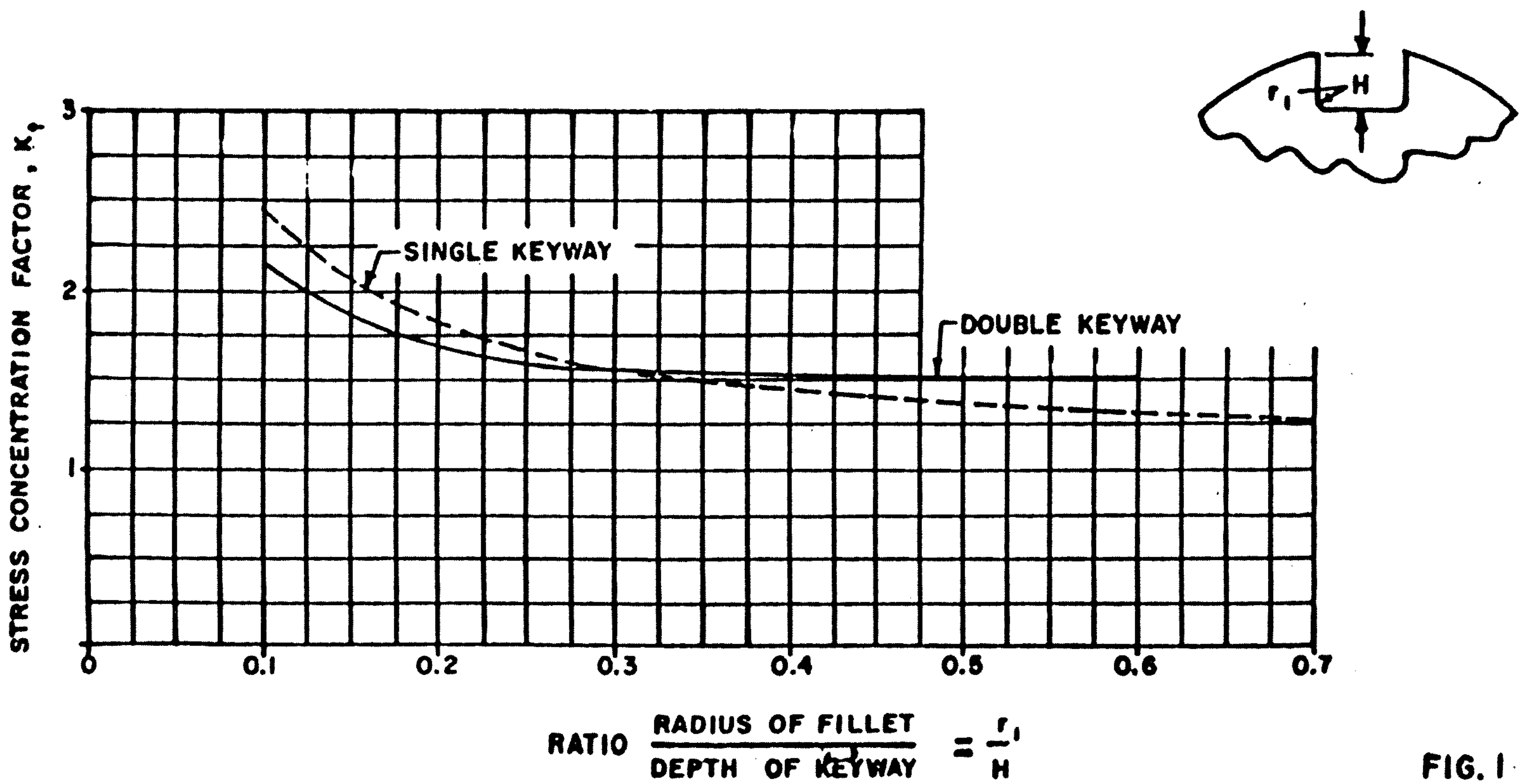


FIG. 1

DDS4301

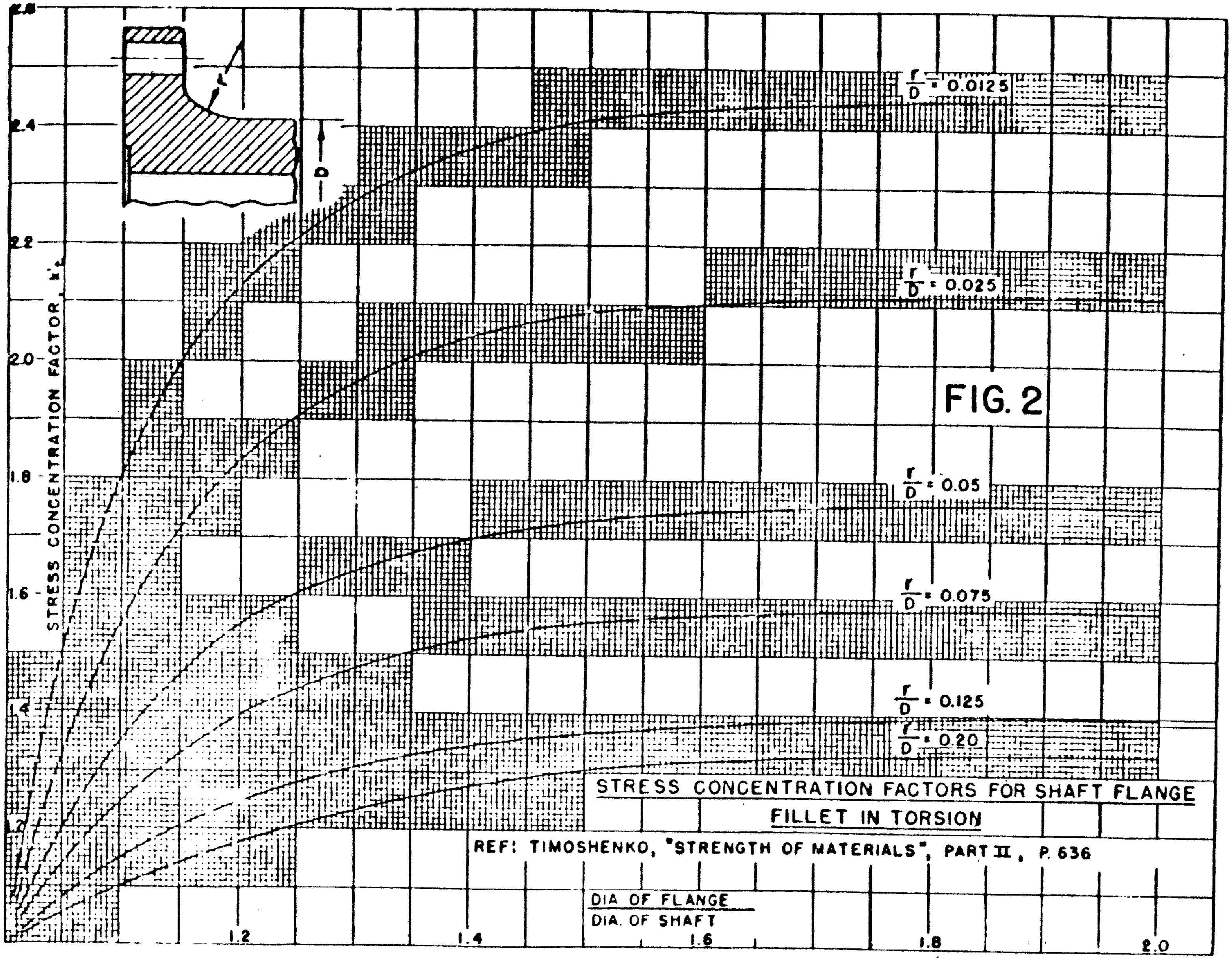
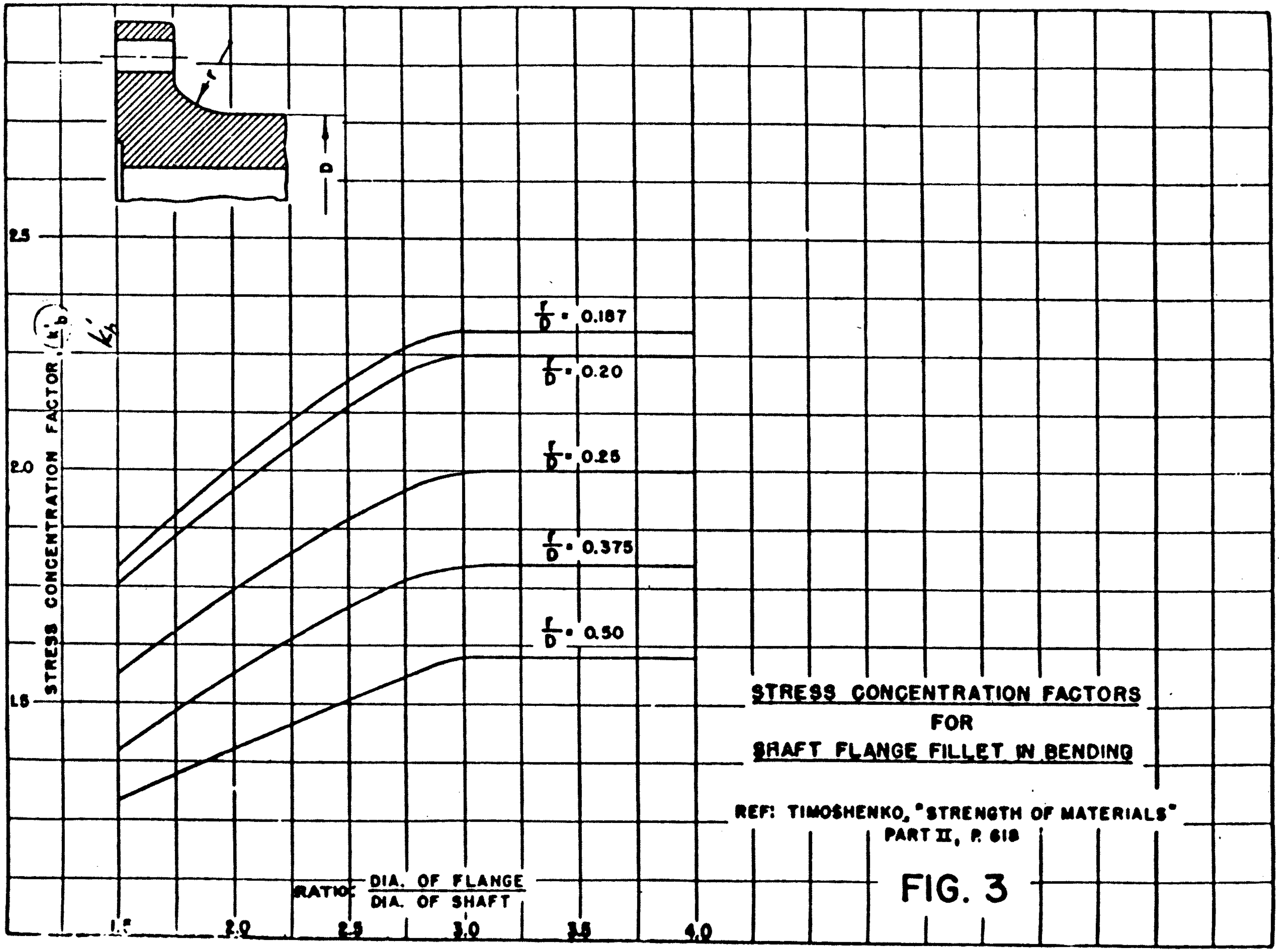


FIG. 2

STRESS CONCENTRATION FACTORS FOR SHAFT FLANGE FILLET IN TORSION

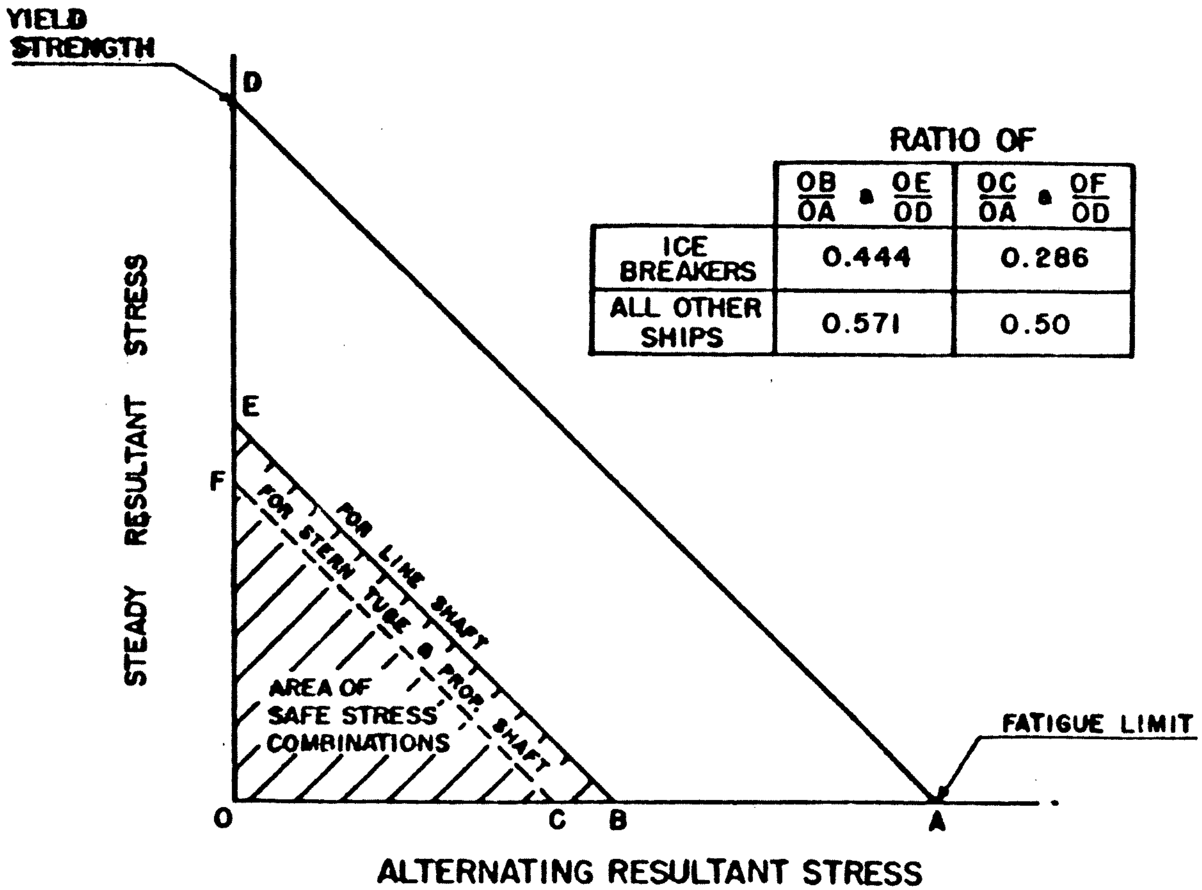
REF: TIMOSHENKO, "STRENGTH OF MATERIALS", PART II, P. 636



STRESS CONCENTRATION FACTORS
FOR
SHAFT FLANGE FILLET IN BENDING

REF: TIMOSHENKO, "STRENGTH OF MATERIALS" PART II, P. 618

FIG. 3



RATIO OF

	$\frac{OB}{OA} = \frac{OE}{OD}$	$\frac{OC}{OA} = \frac{OF}{OD}$
ICE BREAKERS	0.444	0.286
ALL OTHER SHIPS	0.571	0.50

FIG. 4

same point and, therefore, only one of these factors, the highest, should be entered into equation (19) at a time.

3. Combined stress -

(a) The resultant steady stress, equation (5), should be plotted as the ordinate and the resultant alternating stress, equation (19), should be plotted as the abscissa of figure 4 and the combined point should lie within one of the triangles below the heavy line, giving the required safety factor.

(b) Expressed in a formula, this condition is:

$$\frac{S_{\text{steady resultant}}}{YP} + \frac{S_{\text{alternating resultant}}}{FL} = \frac{1}{F.S.} \qquad (20)$$

This should be done for the highest rated speed (where the steady stress is largest), and for the critical speed or largest alternating stress, if it happens to be within the operating range. In both cases equation (20) should be satisfied. The torque used at highest rated speed is defined in paragraph DDS4301-b-1(a).

4. Whirling critical speed -

The bearings of the propulsion shafting should be placed sufficiently close together so that the fundamental whirling critical speed of the shaft is at least fifteen percent above the running speed. The critical speed is excited by unbalance, either in the shaft or propeller and therefore coincides with the fundamental shaft frequency. The most vulnerable part is the tail shaft and its fundamental frequency is approximately (ref. 15) given by equation (21). When shafting of non-uniform diameter is used, an equivalent length, based upon a single diameter, must be used. See sample calculation, DDS4301-f. Equation (21) does not apply where the propeller, or heavy coupling, is placed between bearings. W_{prop} should be increased by 25 percent for entrained water.

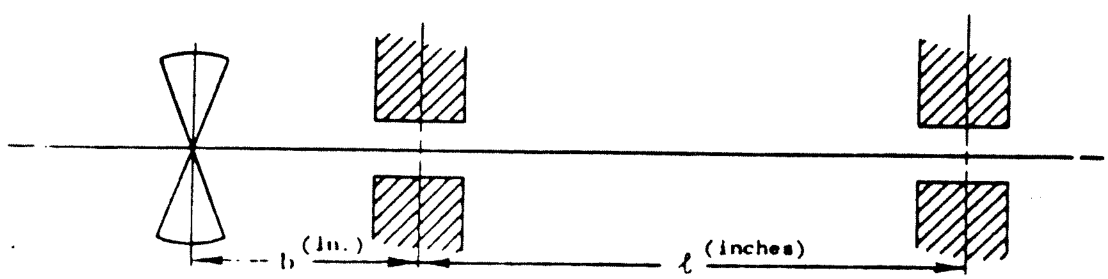


Figure 5

$$f = \frac{30}{\pi}\sqrt{\frac{IE}{I_x\left(b + \frac{\ell}{3}\right) + \frac{W_{prop}b^2}{g}\left(\frac{b}{2} + \frac{\ell}{3}\right) + \mu\left(\frac{b^4}{8} + \frac{\ell b^3}{9} + \frac{7\ell^4}{360}\right)}} \qquad (21)$$

NOTE: Symbol ♦ shows change to replaced page.

5. Longitudinal critical frequencies -

(a) The longitudinal critical speeds, computed by equation 22, shall be restricted to the lower half of, or 15% above, the maximum propeller r.p.m.

$$RPM = \frac{\omega \times 60}{2\pi \times \text{No. of propeller blades}} \qquad (22)$$

The circular frequency, ω, is computed by the following equations and depends upon the configuration of the main propulsion mass elastic system.

(1) The main thrust bearing aft of the reduction gear, motor or engine, figure 6.

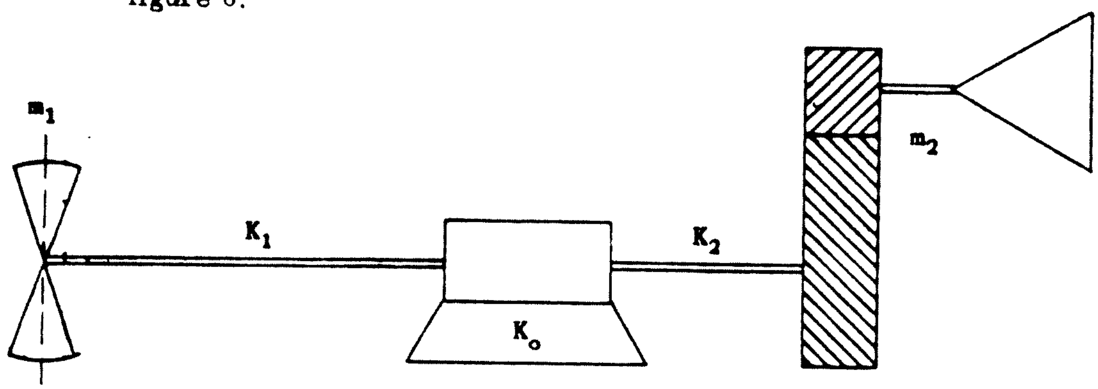


Figure 6

◆ $$\omega^4 - \omega^2 \left[\frac{m_1K_2(K_1 + K_0) + m_2K_1(K_2 + K_0)}{m_1m_2(K_1 + K_2 + K_0)}\right] + \frac{K_1K_2K_0}{m_1m_2(K_1 + K_2 + K_0)} = 0 \qquad (23)$$ ◆

K_1 = stiffness of shafting between propeller and thrust bearing, pounds/inch.

K_2 = stiffness of shafting between thrust bearing and gear, motor or engine, pounds/inch.

m_1 = [weight of propeller plus 60% (for entrained water) plus one-half the weight of shafting between the propeller and thrust bearing] divided by 386, pounds-second^2/inch.

*m_2 = [weight of bull gear and pinion plus one-half the weight of shafting between thrust bearing and bull gear] divided by 386, pounds-second^2/inch.

*If no splined shaft is used between engine and reduction gear, the mass of the rotating parts of motor or engine must be included.

(2) The main thrust bearing located in the reduction gear, motor, or engine, figure 7.

NOTE: Symbol ◆ shows change to replaced page.

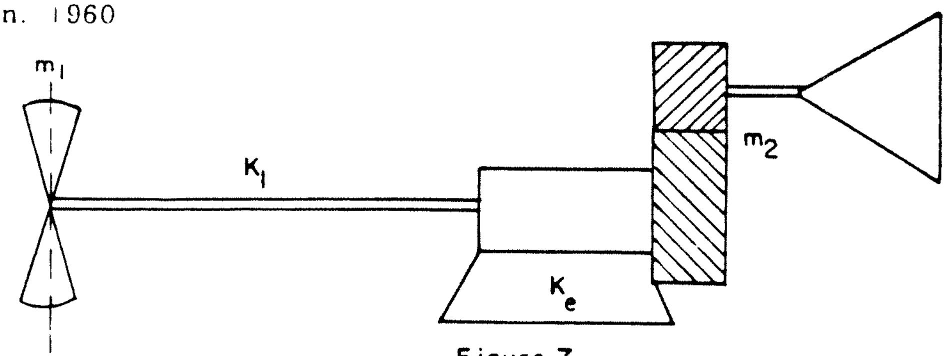


Figure 7

$$\omega^4 - \omega^2 \left[\frac{K_1}{m_1} + \frac{K_1 + K_e}{m_2}\right] + \frac{K_1 K_e}{m_1 m_2} = 0 \qquad (24)$$

K_1 = stiffness of shafting between propeller and gear, pounds/inch.

K_e = stiffness of thrust bearing and the thrust bearing-gear-engine foundation in series, pounds/inch.

m_1 = same as above.

m_2 = (weight of rotating elements of gear and motor or engine plus the weight of wet condenser plus one-fourth the weight of machinery foundations plus one-half the weight of shafting between the propeller and the bull gear) divided by 386, pounus-second ^{2}/inch.

For further information, see reference 9.

6. Key design -

(a) Nomenclature

D = Propeller shaft diameter, inches.

D_m = Mean diameter of shaft taper in propeller, inches.

B_c = Effective length of key, inches.

W = Width of key, inches.

N_1 = Number of keys.

S_s = Shearing stress in key, p.s.i.

S_c = Compressive stress, p.s.i.

H = Depth of keyway (straight side plus corner radius) at mid-length of taper, inches.

b_1 = Contact depth of keyway (straight side only) at mid-length of taper, inches D_m - H Keyway corner radius, inches (see ref. 10, sheet 5).

NOTE: Symbol ♦ shows change to replaced page.

(b) Design formula

$$W = \frac{2Q}{N_1 B_e D_m S_s} \qquad (25)$$

$$b_1 = \frac{2Q}{N_1 D_k B_e S_c} \qquad (26)$$

(c) Shaft taper at outboard coupling and propeller -

Shaft taper at outboard coupling and propeller hub is specified on reference (10) to be 1 inch on diameter per foot of length for shafts over 3-1/4 inches in diameter.

The details of small shafting, 3-1/4 inch diameter and under, shall be in accordance with S. A. E. standards, with tapers 3/4-inch in diameter per foot of length.

(d) In general, key material shall be similar to shaft material. Allowable shearing stress S_s in keys, for use in formula (25) are given in table 6. Table 7 gives values of allowable compressive stress S_c, for use in formula (26). These allowable stresses are based, respectively, on the yield strength in shear and ultimate compressive strength of the materials and a factor of safety of five.

Table 6

Material	Government specification	Allowable shearing stress, S_s	
		1 key	2 or more keys
Steel:			
Class 1	MIL-S-23284	11, 250	7, 500
Class 2	MIL-S-23284	8, 250	5, 500
Class 3	MIL-S-23284	6, 750	4, 500
Class 4	MIL-S-23284	5, 250	3, 500
Class Alloy #4	MIL-S-890	15, 000	10, 000
Class Alloy #2	MIL-S-890	15, 750	10, 500
Class HG	MIL-S-890	9, 750	6, 500
Class An	MIL-S-890	6, 750	4, 500
Class Bs	MIL-S-890	6, 000	4, 000
Class B	MIL-S-890	4, 500	3, 000
Monel	QQ-N-281	7, 800	5, 200
K Monel	QQ-N-286	15, 000	10, 000
Aluminum Bronze	QQ-B-679	7, 350	4, 900
Manganese Bronze, half hard rolled	QQ-B-728	5, 250	3, 500

Table 7

Material	Government specification	Allowable compressive stress (p.s.i.) S_c	
		1 key	2 or more keys
Steel:			
Class 1	MIL-S-23284	28,500	19,000
Class 2	MIL-S-23284	24,000	16,000
Class 3	MIL-S-23284	22,500	15,000
Class 4	MIL-S-23284	18,000	12,000
Class Alloy #4	MIL-S-890	36,000	24,000
Class Alloy #2	MIL-S-890	36,000	24,000
Class HG	MIL-S-890	28,500	19,000
Class An	MIL-S-890	24,000	16,000
Class Bs	MIL-S-890	22,500	15,000
Class B	MIL-S-890	18,000	12,000
Monel (Ni-Cu. Alloy)	QQ-N-281	27,000	18,000
K Monel (Ni-Cu-Al Alloy)	QQ-N-286	42,000	28,000
Aluminum Bronze	QQ-B-679	25,500	17,000
Manganese Bronze, half hard, rolled	QQ-B-728	19,500	13,000

7. Details of couplings -

(a) Couplings in line shaft. (See ref. 10, sheet 3.)

All dimensions are expressed in terms of the shaft diameter D.
Bore diameter d = 0.65D.
Diameter of junction of fillet with flat portion of flange = 1.20D.
Flange thickness = 0.20D.

Clearance in the bolt holes is to be 0.002 inch in diameter. The fillet between shaft and flange has two radii of curvature, 0.2D and 0.05D, as shown on ref. 10, sheet 3.

All coupling bolt material to be BuShips Specs. MIL-S-890 alloy No. 2.

For surface combat vessels, use a shear stress of 13,000 p.s.i.; for all other vessels, use a shear stress of 9,000 p.s.i.

(b) Inboard stern tube shaft coupling. (See ref. 10, sheet 4.)

The bolts should be bodybound across the thrust collar. They should be tightened hard so as to transmit the torque by friction. There should a shrink fit between the stern tube shaft and coupling so that the torque shall be transmitted to the stern tube shaft by means of friction. The keys provide additional safety. There should be a snug fit on the face where the thrust collar mates with the stern tube shaft sleeve, in order to transmit reverse thrust without end play. Since a shrink fit of the sleeve makes such a line-up difficult, the sleeve should be shrunk on, leaving some clearance adjacent to the thrust collar, this clearance to be taken up by shims in the assembly.

Note: Use same bolt material and allowable shear stresses as are used for line shaft and outboard flange coupling.

(c) Outboard coupling. (See ref. 10, sheets 3 and 4.)

The outboard coupling shall be of the flange type, ref. 10, sheet 3, unless distances between bearing are too short to allow assembly and disassembly. In the latter case it will be necessary to use a muff type outboard coupling, ref. 10, sheet 4. The latter coupling should be shrunk on. The taper should be one part in twelve, or 1 inch per foot of taper length. There are to be 2 keys in each shaft end and one cross key to hold the shaft on the sleeve longitudinally. All dimensions, in terms of the outboard shaft diameter, are shown on ref. 10, sheet 4.

The coupling designs shown in reference 10, are recommended and approved by the Bureau of Ships. They are not mandatory. Other designs may be submitted and will be considered.

DDS4301-e. Derivation of some equations.

1. More elaborate discussion of the calculation of alternating stress appears in references 2 to 7, inclusive. For installation which do not fall in any of the four categories (1) to (4) of subparagraph DDS4301-d-2(f), a complete analysis of the torsional vibration characteristics must be made in order to find the alternating stress of equation (19). The method of performing such analysis is completely described in any one of references 2, 3, and 4.

2. Derivation of equation 11. This equation applies to the critical or resonant speed and is derived by equating the alternating energy supplied by the propeller to the energy dissipated in damping by the propeller. The energy input per cycle is:

$$\text{Work input per cycle} = \pi \times \text{torque} \times \text{angle} = \pi \left(\frac{\varphi_p}{100} \cdot Q_{mean} \right) \Phi_{prop} \qquad \text{(a)}$$

The damping dissipation in the propeller is:

$$\text{Work dissipated per cycle} = C \pi \omega \Phi^2_{prop.} \qquad \text{(b)}$$

By page 250 of reference 3, $C = 2(dQ/d\Omega)$ corresponding to twice the slope of the propeller mean torque curve. Assuming a parabolic-speed curve, Fig. 8, we have

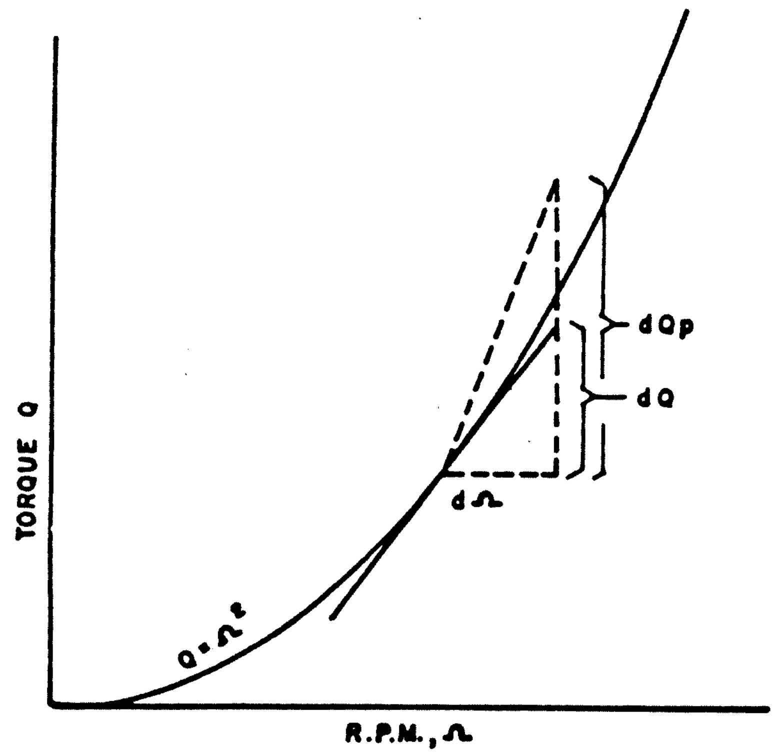


Figure 8

$$Q = c\Omega^2 \text{ whence } \frac{dQ}{d\Omega} = 2c\Omega.$$

Substituting in
$$C = 2\,\frac{dQ}{d\Omega},$$

we get $C = 4c\Omega$. We can write the equation for mean torque,

$$\frac{Q}{c\Omega} = \Omega,$$

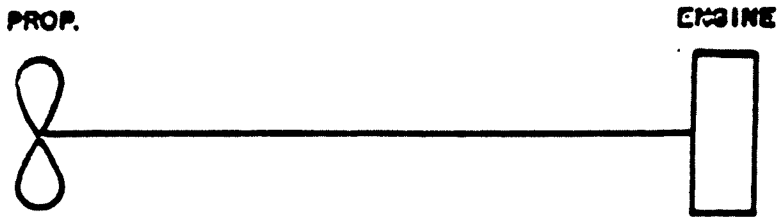
whence
$$C = 4\,\frac{Q}{\Omega}.$$

Substituting this value for C in equation (b) and remembering that $\omega = B\Omega$, we get,

$$\text{Work dissipated per cycle} = 4\,\frac{Q}{\Omega}\,\pi B\Omega\Phi^2_{prop.} = 4\pi QB\Phi^2_{prop.}$$

Equating this expression for work dissipated with input work, equation (a) gives

$$4\pi QB\Phi^2_{prop.} = \pi\left(\frac{Q_p}{100}\cdot Q\right)\Phi_{prop.} \text{ and } \Phi_{prop.} = 0.0025\,\frac{Q_p}{B}$$



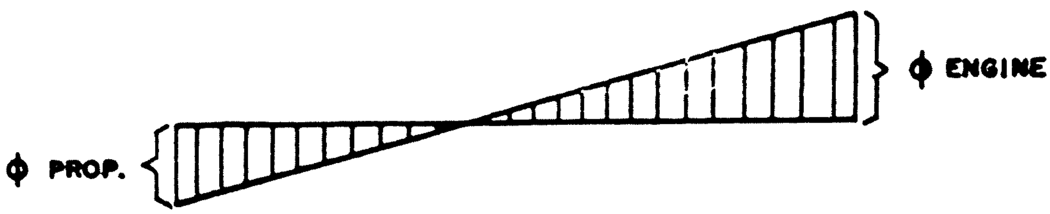


Figure 9

3. Derivation of equation 13. For installations described in cases (1), (2), or (3) of subparagraph DDS4301-d-2(c), consisting of two concentrated inertias joined by a shaft having torsional flexibility, the "elastic curve" of natural vibration is shown in Fig. 9, where the amplitude of vibration at the engine is:

$$\Phi_{engine} = \Phi_{prop.} \frac{I_{prop.}}{I_{engine}} \qquad (d)$$

Consequently, the angle of twist along the entire shaft length is:

$$\Phi_{shaft} = \Phi_{engine} + \Phi_{prop.} = \Phi_{prop.} \left[\frac{I_{prop.} + I_{engine}}{I_{engine}}\right] \qquad (e)$$

The shear stress in any shaft is related to the twist angle by the formula;

$$S_s = \frac{GD\Phi_{shaft}}{2L} \qquad (f)$$

Combination of equations (e) and (f) leads to equation (13).

4. Derivation of equation 14 - This equation, like equation (11), is based on a balance of energy put in (this time by the engine) and the energy dissipated by the propeller. The energy input of the engine is:

$$\text{Work/cycle} = \pi \times \text{Torque} \times \text{Angle} = \pi \frac{Q_a Q_{indicated}}{100} \times \Phi_{engine} \qquad (g)$$

The work dissipated by the propeller is as in equation (c):

$$\text{Work/cycle} = 4\pi Q_{prop.} \times n \times \Phi^2_{prop.} \qquad \text{(h)}$$

We write n here instead of B, since with engine excitation the number of vibrations per revolution equals the number of engine firings n per revolution.

Equate (g) to (h), and substitute the relation (d) between the engine and propeller amplitudes. Further note that $Q_{prop.} = eQ_{indicated\ engine}$. Values for n in table 3 are based on a rated MIP of 135 p.s.i. and must be corrected for each particular engine. This leads to equation (14).

DDS4301-f. Sample calculations

1. General characteristics

It is required to check the diameter of the propeller shaft of a proposed destroyer. The following data is relevant:

S.H.P./shaft	= 35,000
Propeller R.P.M.	= 320
Speed, knots	= 33.1
Propeller diameter	= 13'-3"
Number of propeller blades	= 4
Propeller weight, lbs.	= 18,800 (includes, cap, nut, packing)
Propeller WR^2 in air	= 18.4×10^6 lb. (inches)^2.
Tail shaft O.D./I.D.	= 17.75/10.5
Tail shaft weight/ft.	= 550 lbs.
Tail shaft material	= An steel
Moment arm, L_p, of propeller, inches	= 54.6
Propulsive coefficient (percent)	= 62.5
Two keyways in propeller shaft	
Keyway depth	= 2-3/4"
Fillet radius in keyway	= 3/8"

The tail shaft is assumed to be bored to a diameter of 10-1/2" for its entire length, although the bore in way of strut bearing is 3-1/2". This assumption is on the side of safety regarding the values of stresses in the region of the strut bearing.

2. Steady stress, propeller shaft

(a) Shear stress

$$D^2 - d^2 = (17.75)^2 - (10.5)^2 = 204.8 \text{ inches.}^2$$

$$D^4 - d^4 = 87109 \text{ inches}^4$$

Full power torque, $Q = 63025 \frac{SHP}{RPM} = \frac{63025 \times 35000}{320} = 6{,}900{,}000$ **in. lbs.**

Design shear stress, $S_s = 5.1 \frac{QD \times 1.2}{D^4 - d^4} = \frac{5.1 \times 6.9 \times 10^6 \times 1.2 \times 17.75}{87109}$

$$= 8610 \text{ p.s.i.}$$

(b) Compressive stress, propeller shaft.

$$\text{Thrust, } T = 326 \frac{EHP}{V} \times \frac{1}{(1 - t)}$$

From Table 1, thrust deduction t = 0.035.

$$T = \frac{326 \times 35000 \times .625}{33.1} \times \frac{1}{(1 - 0.035)} = 223{,}000 \text{ lbs.}$$

Compressive stress, due to thrust $= S_c = \frac{1.273\ T}{D^2 - d^2} = \frac{1.273 \times 223{,}000}{204.8} = 1385$ p.s.i.

(c) Resultant steady stress

$$S_{s.r.} = \sqrt{(S_c)^2 + (2S_s)^2} = \sqrt{(1385)^2 + (2 \times 8610)^2} = 17{,}280 \text{ p.s.i.}$$

3. **Alternating stress, propeller shaft.**

$M_g = W_p L_p = 18800 \times 54.6 = 1{,}027{,}000$ in. lbs.
$M_{o.c.} = M_g = 1{,}027{,}000$ in. lbs.
$M_p = M_g + M_{o.c.} = 2{,}054{,}000$ in. lbs.

Bending stress at propeller, $S_b = 10.2\ M_p \frac{D}{D^4 - d^4}$

$$= 10.2 \times 2{,}054{,}000 \times \frac{17.75}{87109} = 4260 \text{ p.s.i.}$$

This stress is within the allowable maximum of 6000 p.s.i.

The torsional vibratory stress, excited by the propeller at full power, has a low value for a geared turbine driven destroyer. From Table 4, this stress is estimated to be 5% of the steady shear stress at full power.

$$S_s = 0.05 \times 8610 = 430 \text{ p.s.i.}$$

The alternating stresses S_b and S_s occur in way of propeller strut bearing. Although the propeller keyway does not extend into the bearing, the alternating resultant stress is computed to include a torsional stress concentration factor K_t of 1.9 (from Fig. 1) for vibratory shear stress. Since the bending stress is parallel to the keyway axis, no bending stress concentration takes place.

Alternating resultant stress,

$$S_{a.r.} = \sqrt{(S_b)^2 + (2k_t\, S_s)^2} = \sqrt{(4260)^2 + (2 X 1.9 \times 430)^2}$$

$$= 4570 \text{ p.s.i.}$$

4. Factor of safety

From equation (20)

$$\frac{S_{s.r.}}{Y.P.} + \frac{S_{a.r.}}{F.L.} = \frac{1}{F.S.}$$

$$\frac{17280}{45,000} + \frac{4570}{40,000} = 0.498 = \frac{1}{F.S.}$$

$$F.S. = 2.01$$

5. Alternating shear stress at first torsional critical speed.

By equation (13a) the alternating, or vibratory, shear stress at the first torsional critical speed is:

$$S_s = 0.00125\ Q_p \frac{GD}{BL}$$

From Table 2, Q_p has a value of 7.5. Length L = 111 ft.

$$S_s = \frac{0.00125 \times 7.5 \times 11.6 \times 10^6 \times 17.75}{4 \times 12 \times 111} = 360 \text{ p.s.i.}$$

6. First torsional critical speed.

This is given by equation (17a)

$$R.P.M._{crit.} = \frac{30}{\pi B}\sqrt{\frac{K}{I_p + .33\,(I_{shaft})}}$$

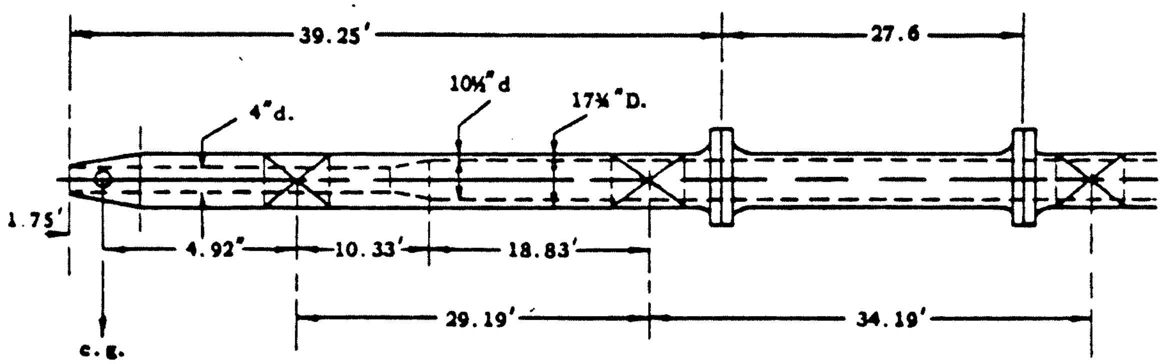
The equivalent torsional stiffness constant of the shaft is, $K = 64 \times 10^6$ in. lbs. per radian. The polar mass moment of inertia of the propeller, including

-5% for entrained water, is 23 x 10^6 lb. in.^{2}, or 59,500 lb.in.sec.^{2} One-third of the WR^2 of the shafting is 5466 lb.in.sec.^{2}

$$RPM_{crit.} = \frac{30}{4\pi}\sqrt{\frac{64 \times 10^6}{59{,}500 + 5466}} = 75.0$$

At 75.0 R.P.M. the S.h.p. transmitted is so low that a computation of factor of safety is not warranted.

7. Whirling Critical Speed



Formula (21) is applicable.

Due to the change in the bore of the tail shaft, the equivalent lengths "b" nd "ℓ" in this formula must be computed. The equivalent lengths will be referred to the 10-1/2" bored shaft.

$$\text{Equivalent length "b"} = 4.92 \times \frac{\overline{17.75}^4 - \overline{10.5}^4}{(17.75)^4 - 4^4} = 4.34 \text{ ft.}$$

$$= 52.08 \text{ in.}$$

$$\text{Equivalent length of 10.33 ft. section} = 10.33 \times \frac{\overline{17.75}^4 - \overline{10.5}^4}{\overline{17.75}^4 - \overline{4}^4}$$

$$= 9.1 \text{ ft.} = 109.2 \text{ in.}$$

quivalent length "ℓ" = 9.1 + 18.83 = 27.93 ft. = 335.2 inches

ropeller WR^2 (in air) about its axis = 18.4 x 10^6 lb.in.^{2}

Propeller WR^2 (in air) about its diameter = 9.2 x 10^6 lb.in.^{2}

Adding 60% for entrained water, the mass moment of inertia is:

$$I_x = \frac{9.2 \times 10^6 \times 1.6}{386} = 38135 \text{ lb.in.sec.}^2$$

Moment of inertia, i of shaft section about diameter:

$$i = \frac{\pi(\overline{17.75}^4 - \overline{10.5}^4)}{64} = 4276 \text{ in.}^4$$

Propeller mass, allowing 25% for entrained water:

$$\frac{W_p}{g} = \frac{18800 \times 1.25}{386} = 60.88 \frac{\text{lb. sec.}^2}{\text{inch}}$$

$$\text{Shaft mass per inch} = \frac{550}{386 \times 12} = 0.119 \frac{\text{lb. sec.}^2}{\text{inch}^2}.$$

$$f = \frac{30}{\pi}\sqrt{\frac{29 \times 10^6 \times 4276}{38135\left(52 + \frac{335.2}{3}\right) + 60.88(52)^2\left(\frac{52}{2} + \frac{335.2}{3}\right) + 0.119\left[\frac{\overline{52}^4}{8} + \frac{335.2(52)^3}{9} + \frac{7(335.2)^4}{360}\right]}}$$

$f = 439$ c.p.m.

In a destroyer, the excitation causing a whirling critical is usually due to unbalance of the rotating parts and is hence of rotational rather than propeller olade frequency. The whirling critical at 439 c.p.m. is therefore sufficiently above the full power r.p.m. to avoid resonance.

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