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DEPARTMENT OF THE NAVY
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ELECTRICAL SYSTEM INTERFACE - VOLTAGE & CURRENT HARMONIC CALCULATIONS

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320-2-a. References

(a) MIL-STD-1399, Section 300, Interface Standard for Shipboard Systems, Electric Power, Alternating Current.

(b) R. Yacamini and J.C. de Olivera, "Harmonic in Multiple Convertor Systems: a Generalized Approach", IEE proceeding, Volume 127, part B, No. 2, pages 96-106, March,1980.

(c) MPR-250, Handbook for the Calulation of Current and Voltage Harmonics on 3-Phase Shipboard Power Distriution Systems Due to Controlled Static Power Supplies. MPR Associates, November, 1970.

(d) W. A. Lewis, "The Transmission of Electric Power - Unbalances and System Disturbances, "Volume II, Illinois Instiute of Technology Press, 1953.

(e) C. F. Wagner and R. D. Evans, "Symmetrical Components", McGraw Hill Book Co., 1933.

(f) E. Kreszig, "Advanced Engineering Mathematics", J. Wiley & Sons, Inc., 1972.

(g) E. Uhlmann, "Power Transmission by Direct Current", Springer Verlag, 1975.

(h) Kimbark, E. W.,"Direct Current Transmission", J. Wiley & Sons, 1971.

(i) Harmonic II Computer Program for Voltage Harmonics, NAVSEA 56Z14.

320-2-b. Scope

This design data sheet outlines a procedure for the calculation of shipboard electrical system voltage harmonics in a balanced system caused by current harmonics generated by user equipments.

This design data sheet also provides methods for the calculation of noncharacteristic current harmonics, current harmonic reduction by multiphase transformers, and voltage unbalances due to unbalanced loads.

320-2-c. Symbols and Abbreviations

Symbols	Terms/ Parameters	Units
a	- Unit vector for symmetrical components. $a = 1\underline{/120^\circ}$, $a^2 = 1\underline{/240^\circ}$	--
a_n, b_n, c_n	- Fourier coefficients at the nth harmonic.	--
C_1	- Correlation constant. $C_1 = 0.9$ for synchronous generator with non salient poles and synchronous generator with salient poles and damper windings. $C_1 = 0.7$ for synchronous generator with salient poles and no damper windings.	--
E_d	- Dc voltage of user equipment with load.	Volt
E_{do}	- Dc voltage of user equipment with no-load.	Volt
$\overline{E}_{1g}, \overline{E}_{2g}$	- Generator positive and negative sequence voltages.	Volt
f	- Fundamental frequency.	Hertz
I_m	- Maximum single phase line-to-line short circuit current.	Ampere
I_n	- Nth current harmonic.	Ampere
$\overline{I}_u$	- Unbalanced Phase load current.	Ampere
$\overline{I}_{nAk}, \overline{I}_{nBk}, \overline{I}_{nCk}$	- Currents at the nth harmonic in the primary phase A, phase B, and phase C windings of a phase-shifting transformer number k.	Ampere
$\overline{I}_{nak}, \overline{I}_{nbk}, \overline{I}_{nck}$	- Currents at the nth harmonic in the secondary phase A, phase B, and phase C windings of a phase-shifting transformer number k.	Ampere
I_{nrk}, I_{nik}	- Real and imaginary components of primary winding currents $\overline{I}_{nAk}$, $\overline{I}_{nBk}$, or $\overline{I}_{nCk}$.	Ampere

$\overline{I}_{SYAn}, \overline{I}_{SYBn}, \overline{I}_{SYCn}$	- Phase A, phase B, and phase C secondary wye winding currents at the nth harmonic of a secondary delta wye transformer.	Ampere
$\overline{I}_{SDAn}, \overline{I}_{SDBn}, \overline{I}_{SDCn}$	- Phase A, phase B, and phase C secondary delta winding currents at nth harmonic of a secondary delta wye transformer.	Ampere
k	- Postive integers indicating number of phase shifting transformers.	--
kVA	- Complex power rating of equipment.	kVA
L	- Inductance of a circuit element.	Henry
l_c	- Cable length.	Feet
n	- Harmonic orders.	--
N_{2k}, N_{3k}	- Number of turns for secondary windings of a phase shifting transformer number k.	--
P_c	- Number of identical cables in parallel per phase.	--
pf	- Power factor.	--
pu	- Per unit.	--
R	- Total resistance of an unbalanced phase load.	Ohm
R_c	- Cable resistance.	$\Omega/10^3$ft
R_g	- Generator resistance.	pu
R_2	- Line-to-neutral negative sequence resistance of a network.	Ohm
R_t	- Transformer resistance.	Ohm

u	- Commutation angle for a rectifier load.	Degree
$\overline{V}_{ab}, \overline{V}_{bc}, \overline{V}_{ca}$	- Line-to-line voltages.	Volt
V_A	- Rated voltage of equipment or system.	Volt
V_B	- Common voltage base.	Volt
V_{av}	- Average magnitudes of line-to-line voltages.	Volt
$\overline{V}_0, \overline{V}_1, \overline{V}_2$	- Zero, positive, and negative sequence voltages.	Volt
V_2 -- V_n	- 2nd through nth harmonic voltages.	Volt
V_{max}	- Highest magnitude of line-to-line voltage.	Volt
V_{min}	- Lowest magnitude of line-to-line voltages.	Volt
V_{nom}	- Nominal value of fundamental voltage.	Volt
V_{THD}	- Total voltage harmonic distortion.	Volt
V_{unb}	- Voltage unbalance.	Volt
X	- Total reactance of a single-phase load.	Ohm
X_c	- Cable reactance at the fundamental frequency.	$\Omega/10^3$ft
X_d''	- Generator direct axis subtransient reactance.	Ohm
X_{2g}	- Generator negative sequence reactance.	Ohm
X_2	- Line-to-neutral negative reactance of a network. It is the sum of a generator negative sequence reactance, a transformer, and cable reactance.	Ohm
X_t	- Transformer inductive reactance.	Ohm
$\overline{Z}$	- Total impedance of a single-phase unbalanced load.	Ohm
Z_A	- Impedance of circuit elements or system at rated voltage V_A.	Ohm

Z_B	- Impedance of circuit elements or system at common voltage base V_B.	Ohm
$\overline{Z}_2$	- Line-to-neutral negative sequence impedance of a network.	Ohm
$\overline{Z}_{0g}$, $\overline{Z}_{1g}$, $\overline{Z}_{2g}$	- Generator zero, positive, and negative sequence impedances.	Ohm
$\overline{Z}_{0u}$, $\overline{Z}_{1u}$, $\overline{Z}_{2u}$	- Zero, positive, and negative sequence components of a 3-phase unbalanced impedance.	Ohm
$\overline{Z}_{gn}$	- Generator equivalent phase-to-neutral impedance at the nth harmonic.	Ohm
$\overline{Z}_{cn}$	- Cable equivalent phase-to-neutral impedance at the nth harmonic.	Ohm
$\overline{Z}_{tn}$	- Transformer equivalent phase-to-neutral impedance at the nth harmonic.	Ohm
$\overline{Z}_{ln}$	- Line voltage regulator equivalent phase-to-neutral impedance at the nth harmonic.	Ohm
$\overline{Z}_{Tn}$	- Total impedance of a system circuit at the nth harmonic.	Ohm
α_p , α_n	- Positive and negative firing angle for rectifier loads.	Ohm
β	- Difference from the ideal value of 180° between centers of the positive and negative current waveforms.	Degree
ε_p , ε_n	- Variation from a normal 120° of positive and negative current pulse widths.	Degree
Ψ_k	- Angle between the primary and secondary windings of a phase-shifting transformer number k.	Degree
*	- Symbol for multiplication.	--

320-2-d. General.

Nonlinear loads are the major source of voltage and current waveform distortions in a ship electrical distribution system. The distorted current and voltage consist of the fundamental frequency and harmonics of the fundamental.

Current harmonics in the electrical distribution system may be generated by power sources such as generators, static frequency changers or by user equipment such as power converters. In this design data sheet, only nonlinear user equipment is analyzed as a source of current harmonics. The current harmonics flow through the system components and generate voltage harmonics throughout the system. These current harmonics could be in any order. For example, balanced 6-pulse and 12-pulse converter loads would cause characteristic current harmonics in the orders of $6n \pm 1$ and $12n \pm 1$ respectively. If the system is unbalanced due to unbalanced load, noncharacteristic current harmonics are produced and may be in even or triplen(multiple of three) orders.

Multiphase transformers help to cancel current harmonics by phase shifting the currents in the secondary windings. Certain current harmonics are greatly reduced in the primary windings. Balanced 3-phase, 3-wire transformer primary windings would have no triplen or even current harmonics. The balanced 3-phase, 4-wire transformer secondary windings may have all triplen but not even current harmonics.

When a 400Hz MG or static frequency changer is used to convert the 60Hz to 400Hz, current harmonics must be calculated separately for each system(60Hz and 400Hz).

320-2-e. Voltage Harmonic Calculations.

The procedures outlined below are a simplified version to calculate voltage harmonics at any point in a balanced system due to current harmonics generated by an user equipment. This simplified version does not take into consideration the distribution of some current harmonics flowing into other user equipment(parallel with the generator which also acts as a load as far as the current harmonics are concerned). For more accurate calculation including all user equipments, the Harmonic II computer program should be used. It is necessary to know the orders and magnitudes of these currents prior to performing the analysis. Current harmonics may be calculated or experimentally determined. Section 320-2-f will outline procedures and methods to calculate these currents. The assumptions and approximations have been made as follows:

- Impedances of buses and circuit breakers are negligible.
- Skin effect and distributed capacitance of cables are negligible.
- The effect of induction motors on the voltage harmonic is not included.
- Generator resistance is neglected if it is less than 10% of its reactance value.

- Generator voltage waveform is basically sinusoidal.
- Generator negative sequence reactance shall be used for the calculation of its impedance at the nth harmonic. The negative sequence reactance is related to the subtransient reactance as follows:
  - For synchronous generator with non-salient poles or a synchronous generator with salient poles and damper windings, the negative sequence reactance is equal to the subtransient reactance, e.g. $X_{2g} = X_d''$.
  - For synchronous generator with salient poles and no damper windings, the negative sequence reactance is approximately 1.7 times that of the subtransient reactance, e.g. $X_{2g} = 1.7X_d''$.
- For solid state frequency changers, the negative sequence impedance would most likely not be a linear function of the frequencies because output filters in the frequency changers would have poles and zeros. The solid state frequency changer impedance curves at various harmonics (Z_n vs frequency) should be obtained from manufacturers.

1. **Procedure for Calculation of Voltage Harmonic.**

Consider a typical power distribution system as shown in Figure 1 on page 36. The 3n current harmonics are assumed to be negligible and are to be ignored. To calculate the voltage harmonics at any point in the system, the following steps should be applied.

Step 1.

Determine the equivalent phase-to-neutral impedance of each component in the system at the nth harmonic from the following formulas:

Generator: $$\bar{Z}_{gn} = \left[\{R_g + j(n)(C_1)(X_{2g})\}\left(\frac{(V_A)^2}{(1000)(kVA)}\right)\right]$$

Transformer: $$\bar{Z}_{tn} = [R_t + j(n)(X_t)]\left(\frac{(V_A)^2}{(1000)(kVA)}\right)$$

Cable: $$\bar{Z}_{cn} = \frac{[R_c + j(n)(X_c)]l_c}{(P_c)(1000)}$$

Line Voltage Regulator :

$$\bar{Z}_{ln} = j(n)(2)(\pi)(f)(L)$$

Note:

1. The term $\left(\frac{(V_A)^2}{(1000)(kVA)}\right)$ for ohmic conversion is used when the component impedances are expressed in pu.

2. Each component impedance is calculated at its own rated voltage.

Step 2.

Convert each component impedance at its own rated voltage base V_A to a chosen common voltage base V_B by using the following equation:

$$Z_B = Z_A \left(\frac{V_B}{V_A} \right)^2$$

Step 3.

Calculate the total system impedance on the common voltage base:

$$\overline{Z}_{Tn} = \overline{Z}_{gn} + \overline{Z}_{cn} + \overline{Z}_{tn} + \overline{Z}_{ln}$$

Note:

It is necessary to calculate only the total impedances up to the point (from load toward generator) where the voltage harmonic will be calculated.

Step 4.

Calculate current harmonics generated by user equipment using procedures in section 320-2-f, unless provided by equipment manufacturer or by measurement.

Step 5.

Calculate voltage harmonics at any point in the system from the following equations:

(1) Determine the individual voltage harmonic.

$$V_n = | I_n * Z_{Tn} |$$

(2) Determine the percent of individual voltage harmonic.

$$V_n(\%) = \frac{V_n}{V_{nom}} * 100\%$$

(3) Determine V_{THD}.

$$V_{THD} = \sqrt{V_2^2 + V_3^2 + V_4^2 + \ldots + V_n^2}$$

(4) Determine the percent of V_{THD}.

$$V_{THD}(\%) = \frac{V_{THD}}{V_{nom}} * 100$$

2. Sample Voltage Harmonic Calculation.

Calculate the voltage harmonics at the generator terminal for a 400Hz power distribution system in Figure 1., on page 36.

Step 1.

Calculate all impedances of the circuit elements in the system.

Generator:

300kW, 0.8 pf, 450V, 400 Hz, $X_{2g} = X_d'' = 0.12$ pu, $C_1 = 0.9$, $R_g = 0$

$$\bar{Z}_{gn} = \frac{jn\ (0.9)\ (0.12)\ (450)^2}{(1000)\ (300/0.8)}$$

$$= j\ 0.05832n\ \Omega \quad \text{on the 450V base}$$

Transformer:

The transformer bank consists of three 25-kVA transformers with a 450 volt delta-connected primary and 120/208-volt wye-connected secondary. The transformer impedance at 400 Hz is 0.0097 + j0.0187 pu.

$$\bar{Z}_{tn} = \frac{[0.0097 + j\ n\ (0.0187)]\ (120)^2}{(1000)\ (75)}$$

$$= 0.00186 + j0.00359n\ \Omega \text{ on the 120V base.}$$

Cable:

There is a total of 88 feet of three T-125 cables in parallel. Each T-125 cable has an impedance of 0.11 + j0.17Ω per 1000 feet(cables #1 and #2 in Figure 1 on page 36).

$$\bar{Z}_{cn1} = \frac{[0.11 + j\ 0.17\ (n)]\ (88)}{(3)\ (1000)}$$

$$= 0.003227 + j0.004987n\ \Omega \text{ on the 450V base.}$$

There is a total of 66 feet of three T-125 cables in parallel(cables #3 and #4 in Figure 1 on page 36).

$$\overline{Z}_{cn2} = \frac{[0.11 + j\ 0.17\ (n)](66)}{(3)\ (1000)}$$

$$= 0.00242 + j0.00374n\ \Omega \text{ on the 120V base.}$$

Line Voltage Regulator:

$$\overline{Z}_{1n} = j(n)(2)(\pi)(400)(25 * 10^{-6})$$

$$= j0.0628n\ \Omega \text{ on the 120V base.}$$

Step 2.

Convert the impedances to the common voltage base. With the chosen common voltage base V_B of 120V, the impedance Z_B is:

$$Z_B = Z_A \left(\frac{120}{450}\right)^2$$

$$= 0.071 Z_A$$

Then,

$$\overline{Z}_{gn} = j\ 0.05832\ (n)\ (0.071)$$

$$= j\ 0.00415\ (n)\ \Omega$$

$$\overline{Z}_{cn1} = [0.00323 + j\ 0.00499(n)](0.071)$$

$$= 0.00023 + j\ 0.00035\ (n)\ \Omega$$

Step 3.

Calculate the total system impedance in the common voltage base 120V:

$$\overline{Z}_{gn} = 0 + j\ 0.00415n\ \Omega$$

$$\overline{Z}_{tn} = 0.00186 + j\ 0.00359n\ \Omega$$

$$\overline{Z}_{cn1} = 0.00023 + j\,0.00035n\ \Omega$$

$$\overline{Z}_{cn2} = 0.00242 + j\,0.00374n\ \Omega$$

$$\overline{Z}_{1n} = 0 \quad + j\,0.0628n\ \ \Omega$$

The total system impedance on the 120V base is:

$$\overline{Z}_{Tn} = \overline{Z}_{gn} + \overline{Z}_{cn1} + \overline{Z}_{cn2} + \overline{Z}_{tn} + \overline{Z}_{1n}$$

$$\overline{Z}_{Tn} = 0.00451 + j\,0.0746n\ \Omega$$

For $n = 5,\ 7,\ 11,$ and 13

$$\overline{Z}_{T5} = 0.00451 + j\,0.0746(5)$$

$$= 0.373\ \underline{/89.3^\circ}\ \Omega$$

$$\overline{Z}_{T7} = 0.00451 + j\,0.0746(7)$$

$$= 0.522\ \underline{/89.5^\circ}\ \Omega$$

$$\overline{Z}_{T11} = 0.00451 + j\,0.0746(11)$$

$$= 0.821\ \underline{/89.7^\circ}\ \Omega$$

$$\overline{Z}_{T13} = 0.00451 + j\,0.0746(13)$$

$$= 0.970\ \underline{/89.7^\circ}\ \Omega$$

Step 4.

Calculate current harmonics generated by user equipment. Current harmonics in this case have been provided by the user equipment manufacturer. The currents, in rms, for the AN/USM-470(V) are:

$I_5 = 4.8$ amp, $I_7 = 3.5$ amp, $I_{11} = 1.0$ amp

$I_{13} = 0$ (13th and higher harmonics are negligible)

AN/USM-470 has only 3 current harmonics, the others are negigible. For another piece of equipment other current harmonics maybe present, including triplen and even current harmonics.

Step 5.

Calculate the voltage harmonic.

(1). Calculate individual voltage harmonic.

$$V_5 = |I_5 * \overline{Z}_{T5}|$$
$$= 4.8(0.373)$$
$$= 1.79 \text{ volts}$$

$$V_7 = |I_7 * \overline{Z}_{T7}|$$
$$= 3.5(0.522)$$
$$= 1.83 \text{ volts}$$

$$V_{11} = |I_{11} * \overline{Z}_{T11}|$$
$$= 1.0(.821)$$
$$= 0.82 \text{ volts}$$

$$V_{13} = 0$$

(2). Calculate total voltage harmonic distortion:

$$V_{THD} = \sqrt{V_5^2 + V_7^2 + V_{11}^2}$$
$$= \sqrt{(1.79)^2 + (1.83)^2 + (0.82)^2}$$
$$= 2.69 \text{ volts}$$

(3). Calculate the percent of V_{THD}:

$$V_{THD}(\%) = \frac{V_{THD}}{V_{nom}} * 100$$
$$= \frac{2.69}{120} * 100$$
$$= 2.24$$

320-2-f. Noncharacteristic Current Harmonic

Perfectly balanced conditions are rarely achieved in practice, and noncharacteristic current harmonics are produced when phase currents are unbalanced. A nonlinear load is likely to produce current harmonics of all orders in a system, but low orders of noncharacteristic current harmonics are normally much smaller than those of the adjacent characteristic current harmonics. For example the 4th current harmonic is normally much smaller than the 5th one.

Unbalanced conditions are resulted from three-phase voltage unbalances, from differences of generator or transformer reactances between phases, or from different firing angles of converters (nonlinear loads) in each phase.

Appendix 1 shows the derivation of noncharacteristic current harmonics for nonlinear loads. The current waveform is shown in Figure 1-1, on page 37.

1. Noncharacteristic Current Harmonic Calculations.

Step 1.

Obtain the firing angle(α) of rectifier loads for each phase from equipment manufacturer. The normal firing angle is 30°, which produces a 120° current pulse width in each phase. The angle (α) is designated as (α_p) and (α_n) in the negative and positive cycles respectively of the current waveform.

Step 2.

Determine the commutation angle u for each phase from equation:

$$\frac{E_d}{E_{do}} = \frac{\cos(\alpha) + \cos(\alpha + u)}{2}$$

This equation must be solved iteratively for u. The no-load and with load dc voltages must be known or estimated for new equipment. If the equipment being analyzed is similar to the existing one, the angle u can be determined by measuring an oscilloscope trace of a current from the existing equipment. The above equation is derived in reference (g), page 87.

Step 3.

Estimate β from the current waveform by taking the absolute value of the quantity 180° minus the angle between centers of the positive and negative waveforms.

Step 4.

Calculate a_n and b_n from the following equations derived in Appendix 1:

$$a_n = \frac{-I_m}{un^2\pi}[\cos(n\alpha_p) - \cos n(\alpha_p + u) + \cos n(\beta - \alpha_n)$$

$$- \cos n(\beta - \alpha_n + u) + \cos(n\pi)[\cos n(u - \alpha_p)$$

$$- \cos(n\alpha_p) - \cos n(\alpha_n + \beta) + \cos n(\alpha_n + \beta + u)]], \quad (100)$$

$$b_n = \frac{-I_m}{un^2\pi}[\sin(n\alpha_p) - \sin n(\alpha_p + u) + \sin n(\beta - \alpha_n)$$
$$- \sin n(\beta - \alpha_n + u) + \cos(n\pi)[\sin(n\alpha_p) + \sin n(u - \alpha_p)$$

$$- \sin n(\alpha_n + \beta) + \sin n(\alpha_n + \beta + u)]], \quad (101)$$

Step 5.

Calculate current harmonic I_n.
The components a_n and b_n can be combined to give current harmonic values. These currents should be normalized. The normalizing value is the fundamental current for a square wave, which is:

$$I_f = \frac{2\sqrt{3}}{\pi} * I_m$$

$$= 1.10266 * I_m$$

Then,

$$I_n(\%) = \frac{\sqrt{a_n^2 + b_n^2}}{(1.10266)I_m} * 100$$

2. **Sample Noncharacteristic Current Harmonic Calculations.**

Calculate current harmonics I_n (for this example arbitrarily choose n = 1, 3, 13, and 20) using the simplified waveform shown in Figure 1-3, on page 45.

Step 1.

From Figure 1-3:

$\alpha_p = 27°$, $\alpha_n = 29°$,

Step 2.

From Figure 1-3:

u = 0.001 radian/or 0.0573°

Step 3.

From Figure 1-3:

$\beta = 0°$

Step 4.

a. Calculate a_n from equation (100):

For n = 1

$$a_1 = -318.31 I_m \{\cos(27°) - \cos(27.0573°) + \cos(-29°) - \cos(-28.9427°) - 1[\cos(-26.9427°) - \cos(27°) - \cos(29°) + \cos(29.0573°)]\}$$

$$= -0.00053 I_m$$

As above, other calculated values for a_n are:

$a_3 = -0.00006 I_m$, $a_{13} = -0.00062 I_m$, $a_{20} = -0.02046 I_m$

b. Calculate b_n from equation (101):

For n = 1

$$b_1 = -\,318.31I_m\{\sin(27°) - \sin(27.0573°) + \sin(-29°)$$
$$- \sin(-28.9427°) - 1[\sin(27°) + \sin(-26.9427°)$$
$$- \sin(29°) + \sin(29.0573°)]\}$$
$$= -\,318.31I_m(-0.00353)$$
$$= +\,1.124I_m$$

As above, other calculated values for b_n are:

$b_3 = +\,0.0443I_m$, $b_{13} = +\,0.0952I_m$, $b_{20} = -\,0.00020I_m$

Step 5.

Calculate current harmonics I_n in percent:

$$I_n(\%) = \frac{\sqrt{a_n^2 + b_n^2}}{1.10266I_m} * 100$$

For:

n = 1, then $I_1(\%) = 101.94$

n = 3, then $I_3(\%) = 4.02$

n = 13, then $I_{13}(\%) = 8.63$

n = 20, then $I_{20}(\%) = 1.86$

Notice that when u = 0°, a_n is negligible for odd harmonics, and b_n is negligible for even harmonics.

Table I, on page 34, lists additional calculated current harmonics up to the 37th.

320-2-g. **Current Harmonic Reduction by Multiphase Transformers.**

There are some transformers with multiple secondary windings which could reduce current harmonics in the primary windings. When transformer secondary windings are phase-shifted certain current harmonics in the primary windings will be reduced.

Figure 2-1 shows several transformer configurations that provide phase shift. Multiphase transformers generally reduce only characteristic current harmonics, but their effectiveness is somewhat reduced when unbalanced conditions are encountered.

A 6-phase(12-pulse)transformer will reduce the 5th and 7th current harmonics and their multiples even in an unbalanced system. Equations for current harmonic reduction by multiphase transformers are derived in Appendix 2.

1. **Calculation of Current Harmonic Reduction by a 6-phase Secondary Delta Wye Transformers.**

The transformer connection is shown in Figure 2-2, on page 51.

Step 1.

Calculate the currents in the secondary windings.

a. Calculate the secondary currents in the Wye windings.

Phase A: Secondary current $\overline{I}_{SYAn}$ is calculated from:

$$\overline{I}_{SYAn} = \sum_{n=1}^{37} [a_{nA}\cos(n\Theta) + b_{nA}\sin(n\Theta)]$$

Where a_{nA} and b_{nA} are calculated from equations (100) and (101) derived in Appendix 1.

Phase B: Secondary current $\overline{I}_{SYBn}$ is calculated from:

$$\overline{I}_{SYBn} = \sum_{n=1}^{37} [a_{nB}\cos(n\Theta) + b_{nB}\sin(n\Theta)]$$

With:

$$a_{nB} = \frac{-Im}{un^2\pi}\Big(\cos n(\alpha_p + 30°) - \cos n(\alpha_p + 30° + u) \tag{200}$$

$$+ \cos n(\beta + 30° - \alpha_n) - \cos n(\beta + 30° - \alpha_n + u)$$

$$+ \cos(n\pi)[\cos n(u + 30° - \alpha_p) - \cos n(\alpha_p + 30°)$$

$$- \cos n(\alpha_n + 30° + \beta) + \cos n(\alpha_n + 30° + \beta + u)]\Big),$$

$$b_{nB} = \frac{-I_m}{un^2\pi}\Big(\sin n(\alpha_p + 30°) - \sin n(\alpha_p + 30° + u) \tag{201}$$

$$+ \sin n(\beta + 30° - \alpha_n) - \sin n(\beta + 30° - \alpha_n + u)$$

$$+ \cos(n\pi)[\sin n(\alpha_p + 30°) + \sin n(u + 30° - \alpha_p)$$

$$- \sin n(\alpha_n + 30° + \beta) + \sin n(\alpha_n + 30° + \beta + u)]\Big)$$

Phase C: Secondary current $\overline{I}_{SYCn}$ is calculated from:

$$\overline{I}_{SYCn} = \sum_{n=1}^{37} [a_{nC}\cos(n\Theta) + b_{nC}\sin(n\Theta)]$$

Where a_{nC} and b_{nC} can be calculated also from equations (200) and (201) with a phase shifted angle of (-30°).

b. Calculate the secondary current in the Delta windings.

Phase A: Secondary current $\overline{I}_{SDAn}$ is calculated from:

$$\overline{I}_{SDAn} = \frac{(\overline{I}_{SYBn} + \overline{I}_{SYCn})}{\sqrt{3}}$$

Step 2.

Calculate the corresponding primary current harmonic I_p from equation:

$$I_p = \left| \frac{(\overline{I}_{SYAn} + \overline{I}_{SDAn})}{2} \right|$$

The primary current must equal to half of corresponding secondary currents so that the primary transformer rating will remain the same for the 6-phase(12-pulse) transformer as for the 3-phase(6-pulse) transformer.

Step 3.

Normalize the current I_p:

$$I_n = \frac{I_p}{1.10266 I_m}$$

Step 4.

Express the primary current harmonic I_n in percent :

$$I_n(\%) = \left| \frac{0.5 * (\overline{I}_{SYAn} + \overline{I}_{SDAn})}{1.10266 I_m} \right| * 100$$

2. **Sample Current Harmonic Reduction by a 6-Phase Secondary Delta Wye Transformer with Identical Phase Currents in the secondary windings.**

The waveform and firing angles will be the same as for the sample noncharacteristic harmonic calculation in 320-2-f. Even though the waveform used in Figure 1-3 on page 45, is not symmetrical, this waveform is used for all three phases and, for a balanced system, the 5th, 7th, 17th, 19th..current harmonics will be cancelled in the primary windings. Noncharacteristic current harmonics will not be cancelled, but may be reduced.

For this example, assume the system is balanced so that all phase currents are identical. For each phase let:

$$\alpha_p = 27°, \ \alpha_n = 29°, u = 0.0573°, \ \beta = 0°$$

A. Calculate the 5th current harmonic in the primary winding.

Step 1.

a. Calculation of currents in the secondary Wye windings.

Phase A current $\bar{I}_{SYA5}$

The analysis in section 320-2-f showed that $a_n = 0$ for the odd harmonics when u is approximately equal to zero. Therefore, all a_n terms will be zero.

Then, by substituting the values of α_n, α_p, u, and β in equations (100) and (101):

$$a_{5A} = 0 \quad \text{(n is odd and u is small)}$$

$$b_{5A} = -\ 12.732 I_m\{\sin(135°) - \sin(135.286°) + \sin(-145°)$$

$$- \sin(-144.714°) - 1[\sin(135°) + \sin(-134.714°)$$

$$- \sin(145°) + \sin(145.286°)]\}$$

$$= -\ 0.1943 I_m$$

Then the 5th current harmonic of secondary wye phase-A is:

$$\bar{I}_{SYA5} = -\ 0.1943 I_m \sin(5\Theta)$$

Note: This value corresponds to:

$$I_{5A}(\%) = \frac{\sqrt{a_{5A}^2 + b_{5A}^2}}{1.10266 I_m} * 100$$

$$I_{5A}(\%) = \frac{0.1943}{1.10266} * 100$$

$$= 17.62$$

This value is listed in Table I, on page 34, for the 5th current harmonic.

Phase B current $\overline{I}_{SYB5}$

Similarly, replace values of α_n, α_p, u, and β in equations (200) and (201):

Then,
$$a_{5B} = 0 , \quad b_{5B} = -8.8344 I_m$$
$$\overline{I}_{SYB5} = -8.8344 I_m \sin(5\Theta)$$

Phase C current $\overline{I}_{SYC5}$

Again, as above by using equations (202) and (203):

Then,
$$a_{5C} = 0, \quad b_{5C} = +9.171 I_m$$
$$\overline{I}_{SYC5} = 9.171 I_m \sin(5\Theta)$$

b. Calculation of currents in the secondary Delta windings.

From equation in step (1), the current in the Secondary Delta phase A is:

$$\overline{I}_{SDA5} = (1/\sqrt{3})(-8.8344 + 9.171) I_m \sin(5\Theta)$$

Step 2.
The corresponding primary current harmonic I_p is:

$$I_p = |0.5 [-0.1943 + (-8.8344 + 9.171)/1.732]| I_m$$

Step 3.
Normalize the current I_p:

$$I_n = \frac{|0.5[-0.1943 + (-8.8344 + 9.171)/1.732] I_m|}{1.10266 I_m}$$

Step 4.
Finally, the 5th current harmonic in the primary winding is:

$$I_5(\%) = \frac{|0.5[-0.1943 + (-8.8344 + 9.171)/1.732]|}{1.10266} * 100$$
$$= 0$$

Thus the 5th current harmonic is completely eliminated in the primary winding for a balanced 6-phase(12-pulse) secondary Delta Wye transformer.

B. Calculate the 13th current harmonic in the primary windings.

By following steps (1) through (4) in the above example, the 13th primary current harmonic in percent is:

$$I_{13}(\%) = 8.61$$

Thus the 13th current harmonic is not cancelled in a 6-phase secondary Delta-Wye transformer. This value also appears in Table I on page 34.

3. Sample Current Harmonic Reduction by a 6-phase Secondary Delta-Wye Transformer with Different Phase Currents in the Secondary Windings.

Use the same conditions as in the above example with the exception that in :

Phase A: $\alpha_p = 27°$, $\alpha_n = 29°$, $u = 0.0573°$, $\beta = 0°$

Phase B: $\alpha_p = \alpha_n = 30°$, $u = 0.0573°$, $\beta = 0°$

Phase C: $\alpha_p = \alpha_n = 30°$, $u = 0.0573°$, $\beta = 0°$

By following step (1) through step (4) in the above example, the 5th current harmonic in percent is found to be:

$$I_5(\%) = 1.19$$

So there is a very small maginitude of the 5th current harmonic due to the differences in the phase currents.

4. Current Harmonic Reduction by Multiphase Zig-Zag Transformers.

Another method of current harmonic reduction by multiphase transformers is to have their secondary windings connected in a zig-zag configuration. Equations for calculating these current harmonics are derived in Appendix 2.

A. Current Harmonics for a 6-phase Transformer with Different Phase Currents in the Secondary Windings.

See Figures 2-4 and 2-5, for transformer configuration and current harmonic component elimination.

Step 1.

Calculate primary current harmonics resulting from secondary winding currents in the two phase-shifting transformers:

$$\overline{I}_{nA1} = I_{nr1} + jI_{ni1}$$

$$\overline{I}_{nA2} = I_{nr2} + jI_{ni2}$$

Where:

$$I_{nA1} = I_{nr1} = I_{na1}, \text{ since } N_{31} = 0,\ I_{ni1} = 0$$

$$I_{nr2} = 0.577 I_{na2}\cos(n30°) - 0.577 I_{nc2}\cos(-n210°), \quad (214)$$

$$I_{ni2} = 0.577 I_{na2}\sin(n30°) - 0.577 I_{nc2}\sin(-n210°), \quad (215)$$

Step 2.

Calculate the nth primary current harmonic in percent from equation:

$$I_n(\%) = \sqrt{(I_{nr1} + I_{nr2})^2 + (I_{ni1} + I_{ni2})^2} * \frac{100}{2n}, \quad (216)$$

B. **Sample Current Harmonic Reduction by a 6-phase Zig-Zag Transformer With Unequal Phase Currents in the Secondary Windings.**

Calculate the 7th current harmonic, if currents $I_{7a1} = 1.5$ pu, and $I_{7a2} = I_{7c2} = 1.0$ pu

Step 1.

Using equations (214), and (215), the real and imaginary components of I_{7a1} and I_{7a2} currents at the 7th harmonic are:

$$I_{7A1} = I_{7r1} = I_{7a1} = 1.5\text{pu},\ N_{31} = 0,\ I_{7i1} = 0$$

$$I_{7r2} = -1.0 \text{ pu}, \quad I_{7i2} = 0$$

Step 2.

From equation (216), the 7th current harmonic in percent is:

$$I_7(\%) = (\sqrt{.5^2} / 2*7) * 100$$

$$= 3.57$$

The 7th current harmonic does not cancel. However, it is much smaller than the 7th current harmonic for a standard 3-phase (6-pulse: $I_7(\%) = 14.28$ with $\alpha_p = \alpha_n = 30°, u = 0°, \beta = 0°$) transformer.

Table II, on page 35, lists additional calculated current harmonics up to the 49th.

C. Current Harmonics Reduction by a 9-phase Zig-Zag Transformer with Equal Phase Current in the Secondary Windings.

Figure 2-6, on page 62, shows the transformer configuration.

Step 1.

Calculate primary current harmonics resulting from secondary winding currents for the three phase-shifting transformers from equations:

$$\bar{I}_{na1} = I_{nr1} + jI_{ni1}$$

$$\bar{I}_{na2} = I_{nr2} + jI_{ni2}$$

$$\bar{I}_{na3} = I_{nr3} + jI_{ni3}$$

Where:

$I_{nr1} = I_{na1}$, since $n_{31} = 0$, $I_{ni1} = 0$

$$I_{nr2} = 0.742 I_{na2} \cos(n20°) - 0.395 I_{nc2} \cos(-n220°), \quad (221)$$

$$I_{ni2} = 0.742 I_{na2} \sin(n20°) - 0.395 I_{nc2} \sin(-n220°), \quad (222)$$

$$I_{nr3} = 0.395 I_{na3} \cos(n40°) - 0.742 I_{nc3} \cos(-n200°), \quad (223)$$

$$I_{ni3} = 0.395 I_{na3} \sin(n40°) - 0.742 I_{nc3} \sin(-n200°), \quad (224)$$

Step 2.

Calculate the nth primary current harmonics in percent from equation:

$$I_n(\%) = \sqrt{(I_{nr1} + I_{nr2} + I_{nr3})^2 + (I_{ni1} + I_{ni2} + I_{ni3})^2} * \frac{100}{3n}, \quad (225)$$

D. Sample Current Harmonic reduction by a 9-phase Zig-Zag Transformer with Equal Phase Current.

For this example, calculate arbitrarily the 13th current harmonics in the primary windings.

Let the magnitudes of currents $\overline{I}_{13a1}$, $\overline{I}_{13c1}$, $\overline{I}_{13a2}$, and $\overline{I}_{13c2}$ equal to 1.0 pu.

Step 1.

Calculate the real and imaginary components of currents $\overline{I}_{13A1}$, $\overline{I}_{13A2}$, and $\overline{I}_{13A3}$ at the 13th harmonics as follows:

$$\overline{I}_{13A1} = I_{13r1} = I_{13a1} = 1.0, \text{ since } N_{31} = 0,\ I_{13i1} = 0$$

$$I_{13r2} = 0.742\cos(260°) - 0.395\cos(-2860°) = -0.500$$

$$I_{13i2} = 0.742\sin(260°) - 0.395\sin(-2860°) = -0.866$$

$$I_{13r3} = 0.395\cos(520°) - 0.742\cos(-2600°) = -0.500$$

$$I_{13i3} = 0.395\sin(520°) - 0.742\sin(-2600°) = 0.866$$

Step 2.

Calculate the 13th primary current harmonic in percent as follow:

$$I_{13}(\%) = \sqrt{(1.0 - 0.5 - 0.5)^2 + (0 + 0.866 - 0.866)^2} * \frac{100}{3*13} = 0$$

Thus for a 9-phase zig-zag transformer, the 13th primary current harmonic is eliminated if the system is balanced.

Table III, on page 35, lists additional calculated current harmonics up to the 49th.

Note: See Appendix 2 on page 59 for the calculation of current harmonic reduction by a 12-phase shifting transformer.

320-2-h. Voltage Unbalance Calculations.

Voltage unbalances are caused by the following load conditions:

- o Single-phase unbalance in which one of the three phase loads is different from the other two equal loads ($Z_a \neq Z_b = Z_c$).
- o Three-phase unbalance in which all three phase loads are different ($Z_a \neq Z_b \neq Z_c$).

All equations for voltage calculations are derived in appendix 3.

1. Voltage Unbalances for Single-phase Unbalance.

Two methods will be used to calculate the voltage unbalances.

Method 1. -- Single-phase Unbalanced Current $\overline{I}_u$ is known.

Step 1.

Calculate line-to-line voltages, $\overline{V}_{ab}$, $\overline{V}_{bc}$, and $\overline{V}_{ca}$ from the following equations:

$$\overline{V}_{ab} = (3/2)(1 - \overline{I}_u\overline{Z}_2) + j(\sqrt{3}/2)(1 + \overline{I}_u\overline{Z}_2) \qquad (302)$$

$$\overline{V}_{bc} = - j\sqrt{3}(1 + \overline{I}_u\overline{Z}_2) \qquad (303)$$

$$\overline{V}_{ca} = (3/2)(\overline{I}_u\overline{Z}_2 - 1) + j(\sqrt{3}/2)(1 + \overline{I}_u\overline{Z}_2) \qquad (304)$$

Where:

$$\overline{Z}_2 = R_2 + jX_2$$

Generally cable resistances are assumed to be balanced, and generator negative sequence resistence is negligible.

Step 2.

Calculate the percent voltage unbalance:

$$V_{unb}(\%) = \frac{(V_{max} - V_{min})}{V_{av}} * 100$$

Where:

$$V_{av} = \frac{|\overline{V}_{ab}| + |\overline{V}_{bc}| + |\overline{V}_{ca}|}{3}$$

Sample Voltage Calculations for Method 1.

Let consider a 450V, 60Hz, 1000kW generator with:

$X_{2g} = 0.2$ pu, $R_g = 0$. So that $\overline{Z}_2 = 0.2\underline{/90°}$ pu.

The three-phase currents in pu are:

$\overline{I}_a = 0.3\ \underline{/-36.86°}$

$\overline{I}_b = 0.3\ \underline{/-36.86°} + 0.1\ \underline{/-53.13°}$

$\overline{I}_c = 0.3\ \underline{/-36.86°}$

$\overline{I}_u = 0.1\ \underline{/-53.13°}$, is the unbalanced current in phase B. load current.

Step 1.

Calculate line-to-line voltages as follow:

$$\overline{V}_{ab} = (3/2)[1 - (0.1\ \underline{/-53.1°} * 0.2\ \underline{/90°})]$$
$$+j(\sqrt{3}/2)[1 + (0.1\underline{/-53.1°}).(0.2\underline{/90°})]$$
$$= 1.5 - 0.03\ \underline{/36.9°} + j(0.866 + 0.0173\underline{/36.9°})$$
$$= 1.7006\ \underline{/30.4°}\ \text{pu}$$

$$\overline{V}_{bc} = -j\sqrt{3}[1 + (0.1\ \underline{/-53.1°}) * (0.2\ \underline{/90°})]$$
$$= 1.760\ \underline{/270.7°}\ \text{pu}$$

$$\overline{V}_{ca} = 1.5[(0.1\ \underline{/-53.1°}) * (0.2\ \underline{/90°}) - 1]$$
$$+ j\frac{\sqrt{3}}{2}[1 + (0.1\ \underline{/-53.1°}) * (0.2\ \underline{/90°})]$$
$$= 1.736\ \underline{/148.9°}\ \text{pu}$$

So $V_{max} = 1.736$ pu, $V_{min} = 1.7006$ pu

$V_{av} = 1.732$ pu

Step 2.

Calculate the percent voltage unbalance.

$$V_{unb}(\%) = \frac{(1.760 - 1.7006) * 100}{1.732}$$

$$= 3.43$$

Method 2. -- Single-phase Unbalanced Impedance is known.

Step 1.

Calculate phase voltages from the following equations:

$$\overline{V}_a = K[R + jX], \tag{308}$$

$$\overline{V}_b = K\left((-\frac{R}{2} + \frac{\sqrt{3}}{2}X + \sqrt{3}X_2) - j(\frac{\sqrt{3}}{2}R + \frac{X}{2}) \right), \tag{309}$$

$$\overline{V}_c = K\left((-\frac{R}{2} - \frac{\sqrt{3}}{2}X - \sqrt{3}X_2) + j(\frac{\sqrt{3}}{2}R - \frac{X}{2}) \right), \tag{310}$$

Step 2.

Calculate line-to-line voltages from equations:

$$\overline{V}_{ab} = \overline{V}_a - \overline{V}_b, \tag{311}$$

$$\overline{V}_{bc} = \overline{V}_b - \overline{V}_c, \tag{312}$$

$$\overline{V}_{ca} = \overline{V}_c - \overline{V}_a, \tag{313}$$

Step 3.

Calculate the percent voltage unbalance:

$$V_{unb}(\%) = \frac{V_{max} - V_{min}}{V_{av}} * 100$$

Sample Voltage Calculations for Metod 2.

Let the unbalanced phase impedance be:

$$\overline{Z} = R + jX$$
$$= 6 + j7.8 \text{ pu}$$

And

$$\overline{Z}_2 = \overline{Z}_{2g} = jX_2 \quad , (R_2=0)$$
$$= j0.20 \text{ pu}$$

Step 1.

The phase voltages are obtained by substituting the value of R, X, and X_2 into equations (308), (309), and (310) in step (1) :

$$\overline{V}_a = K[6 + j7.8]$$

$$\overline{V}_b = K[4.1 -j9.1]$$

$$\overline{V}_c = K[-10.1 + j1.3]$$

Step 2.

The line-to-line voltages are:

$$\overline{V}_{ab} = K[1.9 + j16.9]$$

$$= 17.006K\underline{/83.6^\circ} \text{ pu}$$

$$\overline{V}_{bc} = K[14.2 - j10.4]$$

$$= 17.60K\underline{/-36.2^\circ} \text{ pu}$$

$$\overline{V}_{ca} = K[-16.1 - j6.5]$$

$$= 17.36K\underline{/202^\circ} \text{ pu}$$

Step 3.
The percent of voltage unbalance is:

$$V_{unb}(\%) = \frac{K[17.60 - 17.006]}{K17.32} * 100$$

$$= 3.43$$

Voltage unbalances due to the unbalanced phase load for various load power factors may be found from Figure 3-5 on page 72.

2. Voltage Calculations for Three-Phase Unbalanced Load.

In this case all three phase loads are different.

Step 1.
Calculate line-to-line voltages from the following equations:

$$\dot{\overline{V}}_{ab} = K_2\{(a^2 - 1)(2\overline{Z}_b\overline{Z}_a + \overline{Z}_b^2) + 3\overline{Z}_{2g}(a^2\overline{Z}_b - \overline{Z}_a)\}, \qquad (320)$$

$$\overline{V}_{bc} = K_2\{(a - a^2)(\overline{Z}_b^2 + 3\overline{Z}_{2g}\overline{Z}_b + 2\overline{Z}_a\overline{Z}_b)\}, \qquad (321)$$

$$\overline{V}_{ca} = K_2\{(1 - a)(\overline{Z}_b^2 + 2\overline{Z}_a\overline{Z}_b) + 3\overline{Z}_{2g}(\overline{Z}_a - a\overline{Z}_b)\}, \qquad (322)$$

Step 2.
Calculate the percent voltage unbalance.

$$V_{unb}(\%) = \frac{V_{max} - V_{min}}{V_{av}} * 100$$

Note: If $\overline{Z}_a$ is very large with respect to $\overline{Z}_b$ (i.e, $\overline{Z}_a = 1000\overline{Z}_b$) the single phase case is duplicated.

Sample Voltage Calculations for Three-Phase Unbalance

Let

$\overline{Z}_b = j1.0$ pu , (power factor = 0)

$\overline{Z}_a = j1.5$ pu

$\overline{Z}_{2g} = j0.2$ pu

Step 1.

Calculate line-to-line voltages:

$$\overline{V}_{ab} = K_2\{(a^2 - 1)[2(j1)(j1.5) + (j1)^2] + 3(j0.2)(j1)(a^2 - 1.5)\}$$

$$= K_2\{-4(-1.5 - j.866) - 0.6(-2 - j0.866)\}$$

$$= K_2(7.2 + j3.984)$$

$$= 8.22K_2\underline{/28.9°} \text{ pu}$$

$$\overline{V}_{bc} = K_2\{(a - a^2)[(j1)^2 + 3(j0.2)(j1) + 2(j1.5)(j1)]\}$$

$$= K_2\{(-1 - 3 - 0.6)j\sqrt{3}\}$$

$$= 7.97K_2\underline{/-90°} \text{ pu}$$

$$\overline{V}_{ca} = K_2\{[(j1)^2 + 2(j1.5)(j1)](1-a) + 3(j0.2)[j1.5 - a(j1)]\}$$

$$= K_2\{-4(1.5 - j0.866) - 0.6(2 - j0.866)\}$$

$$= K_2(-7.2 + j3.98)$$

$$= 8.22K_2\underline{/151.1°} \text{ pu}$$

Step 2.

Calculate percent voltage unbalance.

$$V_{unb}(\%) = \frac{K_2(8.22 - 7.97)}{8.137K_2} * 100$$

$$= 3.07$$

TABLE I

CURRENT HARMONICS IN AN ASYMMETRICAL CURRENT WAVEFORM

Waveform of Figure 1-3/page 45, u = 0.0573°, α_p = 27°, α_n = 29°, β = 0°.

n	Calculated I_n (%)	Harmonics by 1/n Rule (%)
1	101.94	100.00
2	1.13	50.00
3	4.02	33.33
4	0.75	25.00
5	17.62	20.00
6	1.97	16.67
7	15.74	14.30
8	1.44	12.50
9	3.92	11.11
10	0.35	10.00
11	6.34	9.10
12	1.83	9.09
13	8.63	7.70
14	1.69	7.14
15	3.72	6.67
16	0.07	6.25
17	2.85	5.90
18	1.60	5.56
19	5.69	5.30
20	1.86	5.00
21	3.43	4.76
22	0.48	4.55
23	1.12	4.30
24	1.31	4.17
25	3.93	4.00
26	1.93	3.85
27	3.08	3.70
28	0.85	3.57
29	0.12	3.40
30	0.96	3.33
31	2.71	3.20
32	1.91	3.13
33	2.68	3.03
34	1.17	2.94
35	0.47	2.90
36	0.58	2.78
37	1.79	2.70
.	.	.
.	.	.
n	I_n	1/n

TABLE II

6-Phase(12-Pulse)Transformer Current Harmonics with Unequal Phase Currents in the Secondary Windings.

Harmonic No.(n)	I_n (%)
5	5.00
7	3.57
11	11.36
13	9.62
17	1.47
19	1.32
23	5.43
25	5.00
29	0.86
31	0.81
35	3.57
37	3.38
41	0.61
43	0.58
47	2.66
49	2.55
.	.
.	.
n	I_n

TABLE III

9-Phase(18-Pulse)Transformer Current Harmonics with Equal Phase Currents in the Secondary Windings.

Harmonic No.(n)	I_n (%)
5	0.00
7	0.00
11	0.00
13	0.00
17	5.88
19	5.26
23	0.00
25	0.00
29	0.00
31	0.00
35	2.86
37	2.70
41	0.00
43	0.00
47	0.00
49	0.00
.	.
.	.
n	I_n

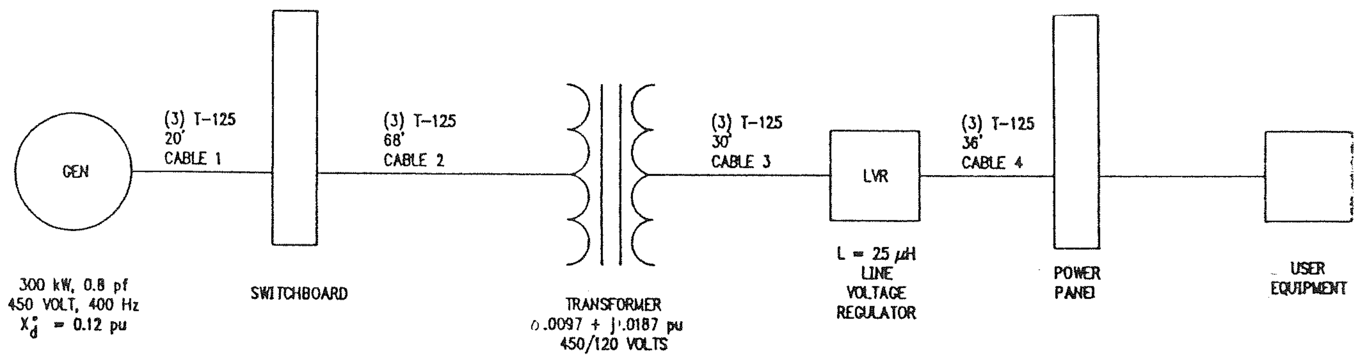


Figure I: Sample Power Distribution System.

APPENDIX 1

CALCULATION OF NONCHARACTERISTIC CURRENT HARMONICS FOR NONLINEAR LOADS

Figure 1-1 shows a typical single-phase current waveform that will be analyzed. This waveform assumes a trapezoidal current, which approximates many rectifier controlled loads with large inductance. Figure 1-2 shows a circuit that will produce a trapezoidal current.

The following assumptions have been made in developing the equations for noncharacteristic current harmonics.

- All commutation angles are approximated as straight lines.
- The current harmonics will vary with commutation angle u.
- All turn-on and turn-off commutation angles are equal.
- The firing angle is assumed to be close to 30°, so that the triplen harmonics will nearly be cancelled.
- All shunt capacitances (cable charging, power factor correction) are negligible.
- Current harmonics and voltages are not calculated for short circuit conditions. Only load currents (balanced or unbalanced) are considered.
- The load of a power converter is inductive, and large enough so that the current will be constant outside of the commutation interval.

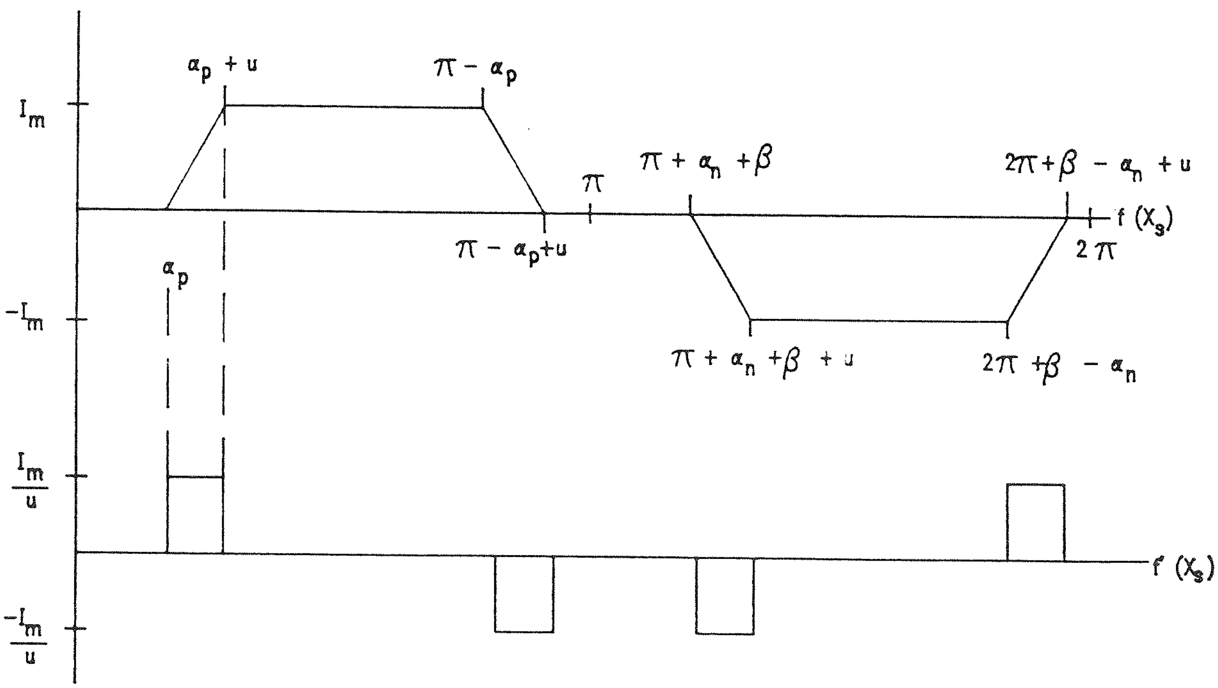


Figure 1-1: Current Waveform with Imbalances

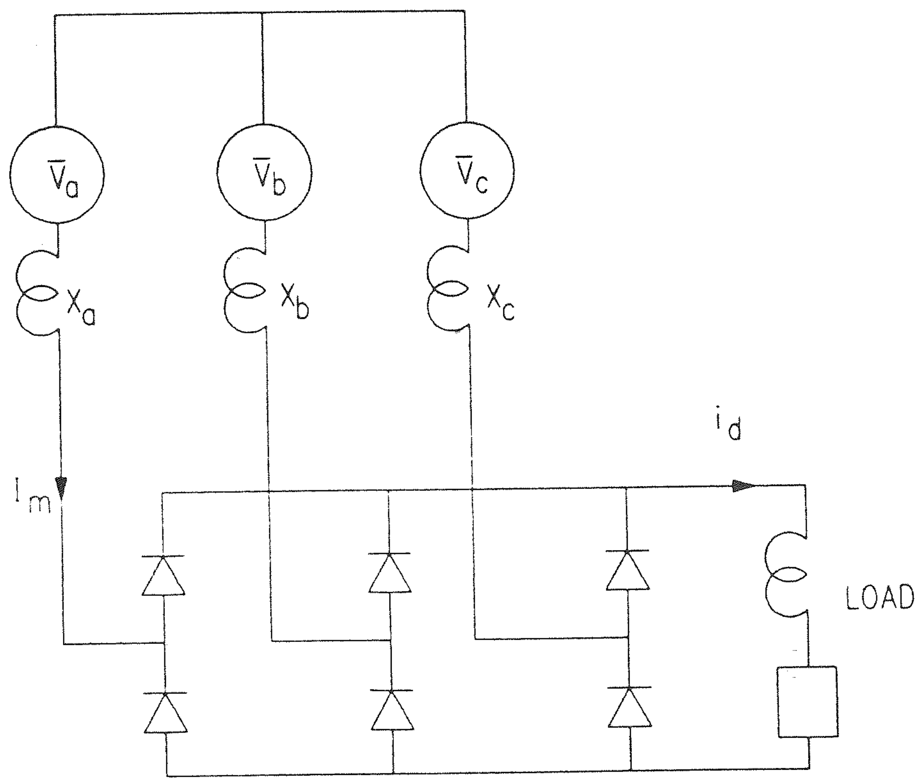


Figure 1-2: Typical 6-Pulse Converter

The Fourier analysis of the waveform in Figure 1-1 may be obtained relatively simple without integration. The Fourier coefficients may be calculated from the jump function and its derivatives. A jump is the difference between the right-hand and left-hand limits of the function or its derivatives. An upward jump is positive, and a downward jump is negative. The following notations are used:

j_s = jump of function at point X_s.

j'_s = jump of first derivative at X_s.

j''_s = jump of second derivative at X_s.

If the function or its derivative is continuous at X_s, then the value of j_s (or j'_s) is zero.

X_s = jump point of waveform.

$f(X_s)$ = value of function at X_s.

$f'(X_s)$ = value of first derivative at X_s.

The waveform in Figure 1-1 is represented by equation:

$$F_n(\Theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \{a_n \cos(n\Theta) + b_n \sin(n\Theta)\}$$

For this analysis, the dc component a_0 is taken to be zero.

The values of j_s, j'_s, and j''_s at different values of X_s of the current waveform in Figure 1-1 are shown in Table 1-I.

TABLE 1-I

FOURIER ANALYSIS USING JUMP FUNCTIONS

s	X_s	j_s	j_s'	j_s''
1	α_p	0	$+\frac{I_m}{u}$	0
2	$\alpha_p + u$	0	$-\frac{I_m}{u}$	0
3	$\pi - \alpha_p$	0	$-\frac{I_m}{u}$	0
4	$\pi - \alpha_p + u$	0	$+\frac{I_m}{u}$	0
5	$\pi + \alpha_n + \beta$	0	$-\frac{I_m}{u}$	0
6	$\pi + \alpha_n + \beta + u$	0	$+\frac{I_m}{u}$	0
7	$2\pi + \beta - \alpha_n$	0	$+\frac{I_m}{u}$	0
8	$2\pi + \beta - \alpha_n + u$	0	$-\frac{I_m}{u}$	0

The components a_n and b_n are calculated from:

$$a_n = \frac{1}{n\pi}\left[- \sum_{s=1}^{8} j_s \sin(nX_s) - \frac{1}{n} \sum_{s=1}^{8} j_s' \cos(nX_s) + \frac{1}{n^2} \sum_{s=1}^{8} j_s'' \sin(nX_s) + \ldots.\right]$$

$$b_n = \frac{1}{n\pi}\left[\sum_{s=1}^{8} j_s \cos(nX_s) - \frac{1}{n} \sum_{s=1}^{8} j_s' \sin(nX_s) - \frac{1}{n^2} \sum_{s=1}^{8} j_s'' \cos(nX_s) + \ldots\right]$$

In the above equations, $j_s = 0$ because the function is continuous. All j_s'' and higher derivatives are zero. Therefore:

$$a_n = - \frac{1}{n^2\pi} \sum_{s=1}^{8} j_s' \cos(nX_s)$$

$$b_n = - \frac{1}{n^2\pi} \sum_{s=1}^{8} j_s' \sin(nX_s)$$

Where:

$$j_1' = + \frac{I_m}{u}, \quad j_2' = - \frac{I_m}{u} \ldots, \quad j_8' = - \frac{I_m}{u}$$

And:

$X_1 = \alpha_p$, $X_2 = \alpha_p + u$ ... as per Table 1-I.

$$a_n = -\frac{I_m}{un^2\pi}\{\cos(n\alpha_p) - \cos n(\alpha_p + u) - \cos n(\pi - \alpha_p)$$

$$+ \cos n(\pi - \alpha_p + u) - \cos n(\pi + \alpha_n + \beta)$$

$$+ \cos n(\pi + \alpha_n + \beta + u) + \cos n(2\pi + \beta - \alpha_n)$$

$$- \cos n(2\pi + \beta - \alpha_n + u)\}$$

Expand the multiple angle arguments, then combine terms:

$$a_n = -\frac{I_m}{un^2\pi}\{\cos(n\alpha_p) - \cos n(\alpha_p + u) + \cos n(\beta - \alpha_n)$$

$$- \cos n(\beta - \alpha_n + u) + \cos(n\pi)[\cos n(u - \alpha_p)$$

$$- \cos(n\alpha_p) - \cos n(\alpha_n + \beta) + \cos n(\alpha_n + \beta + u)]\}, \quad (100)$$

Likewise:

$$b_n = -\frac{I_m}{un^2\pi}\{\sin(n\alpha_p) - \sin n(\alpha_p + u) - \sin n(\pi - \alpha_p)$$

$$+ \sin n(\pi - \alpha_p + u) - \sin n(\pi + \alpha_n + \beta)$$

$$+ \sin n(\pi + \alpha_n + \beta + u) + \sin n(2\pi + \beta - \alpha_n)$$

$$- \sin n(2\pi + \beta - \alpha_n + u)\}$$

Expand, then combine terms:

$$b_n = -\frac{I_m}{un^2\pi}\{\sin(n\alpha_p) - \sin n(\alpha_p + u) + \sin n(\beta - \alpha_n)$$

$$- \sin n(\beta - \alpha_n + u) + \cos(n\pi)[\sin(n\alpha_p) + \sin n(u - \alpha_p)$$

$$- \sin n(\alpha_n + \beta) + \sin n(\alpha_n + \beta + u)]\}, \quad (101)$$

The above equations reduce under certain conditions as discussed below:

1. If $\alpha_n = \alpha_p$ and n is even, then $a_n = b_n = 0$. There are no even harmonics for positive and negative waveform symmetry.

2. If $\alpha_n = \alpha_p$, $u = 0°$, and n is odd, $a_n = 0$, and only b_n exists.

3. **Standard Square Wave Representation**

If $\alpha_p = \alpha_n = \alpha = 30°$, $u = 0°$, $\beta = 0°$, the waveform is a square wave. For n even, the components a_n, and b_n are both equal zero. For n odd, then $a_n = 0$, $b_n = \frac{4I_m}{n\pi}\left(\Sigma \cos(n\alpha)\sin(n\Theta)\right)$, and the waveform is represented by equation:

$$F(\Theta) = \frac{2\sqrt{3}I_m}{\pi}\left[\sin(\Theta) - \frac{1}{5}\sin(5\Theta) - \frac{1}{7}\sin(7\Theta) + \frac{1}{11}\sin(11\Theta) + \frac{1}{13}\sin(13\Theta) \ \ldots \right],$$

This equation is found in many references on rectifier harmonic analysis.

4. **Standard Trapezoid Wave Representation**

Further simplification of equations (100) and (101) are possible. Let $\beta = 0°$, $\alpha_p = \alpha_n = \alpha = 30°$ so that a 6-pulse rectifier is simulated. Then for characteristic harmonics only:

$$a_n = -\frac{I_m}{un^2\pi}\{4\cos(n\alpha)[1 - \cos(nu)]\}$$

$$b_n = -\frac{I_m}{un^2\pi}[-4\sin(nu)\cos(n\alpha)]$$

With $\alpha = 30°$, the Fourier component c_n is:

$$c_n = \sqrt{a_n^2 + b_n^2}$$

$$c_n = \frac{2\sqrt{3}\ I_m}{un^2\pi}\sqrt{[1 - \cos(nu)]^2 + \sin^2(nu)}$$

$$c_n = \frac{2\sqrt{3}I_m}{un^2\pi}\sqrt{1 - 2\cos(nu) + \cos^2(nu) + \sin^2(nu)}$$

$$= \frac{4\sqrt{3}I_m}{un^2\pi}\sqrt{\frac{1 - \cos(nu)}{2}}$$

$$= \frac{4\sqrt{3}I_m}{un^2\pi}\sin(nu/2)$$

Since $I_f = \dfrac{2\sqrt{3}I_m}{\pi}$, the normalized current harmonic $I_n(\%)$ is :

$$I_n(\%) = \frac{c_n}{I_f} * 100$$

$$= \frac{2\sin(nu/2)}{un^2} * 100$$

$$I_n(\%) = \frac{1}{n}\ \frac{\sin(nu/2)}{(nu/2)} * 100 \quad , \qquad (102)$$

which has the limit of 1/n as $u \to 0°$. **Note that equation (102) applies only to characteristic current harmonics.**

5. Asymmetrical Square Wave Having Triplen Current Harmonics.

Using figure 1-3, with $u = 0°$, equations for triplen current harmonics can be derived. Reference (h) shows the Fourier series for the positive and negative pulses.

Let ε_p = variation from 120° of the positive pulse width.

Let ε_n = variation from 120° of the negative pulse width

In Figure 1-3, with $\varepsilon_p = 6°$, $\varepsilon_n = 2°$.

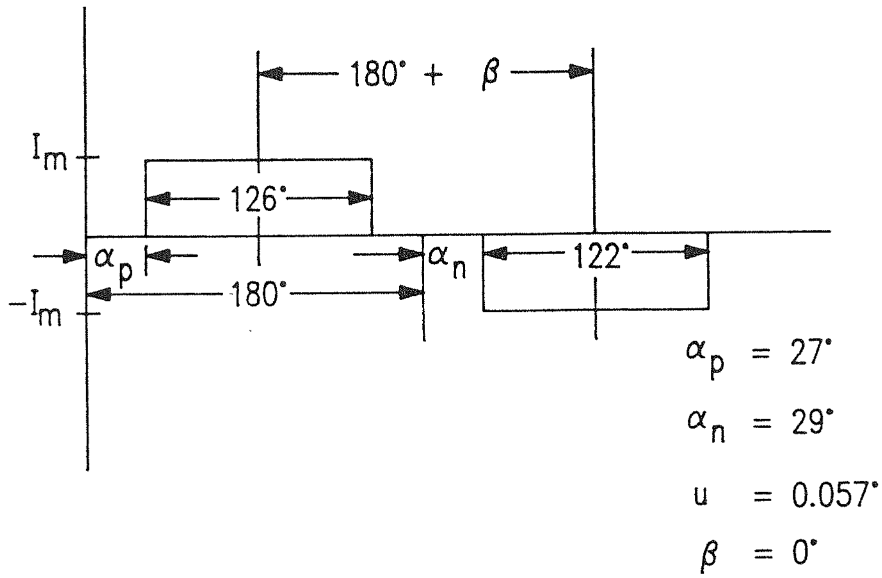


Figure 1-3: Asymmetrical Current Waveform.

The equation for triplen current waveform is:

$$F_n(\Theta) = \frac{2I_m}{n\pi}\left[\sin\left(\frac{n(120° + \varepsilon_p)}{2}\right) + \sin\left(\frac{n(120° + \varepsilon_n)}{2}\right)\right]$$

$$= \frac{2I_m}{n\pi}\left[\sin\left(\frac{n\varepsilon_p}{2}\right) + \sin\left(\frac{n\varepsilon_n}{2}\right)\right]$$

Then the normalized current harmonic I_n in percent is:

$$I_n(\%) = \frac{F_n}{I_f} * 100$$

$$= \left(\frac{1}{n\sqrt{3}}\right)\left[\sin\left(\frac{n\varepsilon_p}{2}\right) + \sin\left(\frac{n\varepsilon_n}{2}\right)\right] * 100 \quad , \qquad (103)$$

If $n\varepsilon_p$ and $n\varepsilon_n$ are small, equation (103) reduces to:

$$I_n(\%) = \frac{\varepsilon_p + \varepsilon_n}{2\sqrt{3}} * 100$$

Then for $\varepsilon_p = 6°$, $\varepsilon_n = 2°$, and $n = 3$

$$I_3(\%) = \frac{(6° + 2°)}{2\sqrt{3} * 57.3°/\text{radian}} * 100$$

$$= 4.03$$

which is the value calculated in Table I on page 34.

6. **Asymmetrical Square Wave Having Even Current Harmonics.**

Using Figure 1-3, with u = 0°, the equation for even current harmonic(n is a multiple of 2) is simplified as follows:

$$F_n(\Theta) = \frac{2I_m}{n\pi}\left[\sin\left(\frac{n(120° + \varepsilon_p)}{2}\right) - \sin\left(\frac{n(120° + \varepsilon_n)}{2}\right)\right]$$

$$F_n(\Theta) = \frac{2I_m}{n\pi}\left[\cos(60n)\left(\sin(\frac{n\varepsilon_p}{2}) - \sin(\frac{n\varepsilon_n}{2})\right)\right.$$

$$\left. + \sin(60n)\left(\cos(\frac{n\varepsilon_p}{2}) - \cos(\frac{n\varepsilon_n}{2})\right)\right], \qquad (104)$$

7. **Asymmetrical Square Wave with Negative pulse Displaced.**

If the waveform has positive and negative pulses which are identical but displaced by (180° + β), then the even current harmonic is:

$$F_n = \frac{2I_m}{n\pi} \sin(\frac{120n}{2})[\cos n(\Theta + \beta) - \cos(n\Theta)]$$

$$= \pm \frac{\sqrt{3}I_m}{n\pi} \sin(n\beta)\sin(n\Theta)$$

and the normalized current harmonic I_n in percent is:

$$I_n(\%) = \left|\frac{F_n}{I_f}\right| * 100$$

$$= \frac{\sin(n\beta)}{2n} * 100 , \qquad (105)$$

8. Representation of Maximum Trapezoidal Current Waveform.

The single-phase line-to-line short circuit current is:

$$I_m = \frac{(\overline{V}_a - \overline{V}_b)}{(X_a + X_b)}$$

Where:

X_a and X_b are the commutation reactances which is the sum of generator, cable, and transformer reactances.

$$\overline{V}_a - \overline{V}_b = \sqrt{V_a^2 + V_b^2 - 2V_aV_b\cos(\Theta)}$$

If $\Theta = 120°$ ==> $\overline{V}_a - \overline{V}_b = \sqrt{3}\,\overline{V}_a$

For the general case, $\overline{V}_a \neq \overline{V}_b \neq \overline{V}_c$ and $X_a \neq X_b \neq X_c$.

Variations of several percent can be expected between the line voltages or between commutation reactances. Likewise, the firing angles may vary by several degrees. There are six firing angles -- the positive and negative ones for each of the three phases.

APPENDIX 2

CALCULATION OF CURRENT HARMONIC REDUCTION BY MULTIPHASE TRANSFORMERS

For the purpose of this analysis, the following assumptions were made:

- o The leakage reactance of the transformer affects the commutation angle.
- o The primary line current is phase shifted by 30°, since the tansformer is connected in delta-wye.
- o For a multiphase transformer arrangement, the secondary winding phase-shifting angle required for each transformer is 360° divided by the pulse number.

1. Current Harmonic Reduction by a 6-phase Secondary Delta Wye Transformer.

Phase shifting for secondary delta-wye transformers merely requires shifting currents by +30° and -30°. The current harmonic is obtained by adding the resulting Fourier series components of the current waveform. Figure 2-2 shows the transformer connection.

The secondary winding currents are calculated as follows:

Currents in Wye Windings.

Phase-A current $\overline{I}_{SYAn}$ at the nth harmonic is:

$$\overline{I}_{SYAn} = \sum_{n=1}^{37} [a_{nA}\cos(n\Theta) + b_{nA}\sin(n\Theta)]$$

Where components a_{nA} and b_{nA} are calculated from equations (100) and (101) derived in Appendix 1.

Phase-B current $\overline{I}_{SYBn}$ at the nth harmonic is:

$$\overline{I}_{SYBn} = \sum_{n=1}^{37} [a_{nB}\cos(n\Theta) + b_{nB}\sin(n\Theta)]$$

Where the components a_{nB} and b_{nB} are calculated as follow:

$$a_{nB} = - \frac{I_m}{un^2\pi}\Big(\cos n(\alpha_p + 30°) - \cos n(\alpha_p + 30° + u)$$

$$+ \cos n(\beta + 30° - \alpha_n) - \cos n(\beta + 30° - \alpha_n + u)$$

$$+ \cos(n\pi)[\cos n(u + 30° - \alpha_p) - \cos n(\alpha_p + 30°)$$

$$- \cos n(\alpha_n + 30° + \beta) + \cos n(\alpha_n + 30° + \beta + u)]\Big), \quad (200)$$

Similarly:

$$b_{nB} = - \frac{Im}{un^2\pi}\Big(\sin n(\alpha_p + 30°) - \sin n(\alpha_p + 30° + u)$$

$$+ \sin n(\beta + 30° - \alpha_n) - \sin n(\beta + 30° - \alpha_n + u)$$

$$+ \cos(n\pi)[\sin n(\alpha_p + 30°) + \sin n(u + 30° - \alpha_p)$$

$$- \sin n(\alpha_n + 30° + \beta) + \sin n(\alpha_n + 30° + \beta + u)]\Big), \quad (201)$$

Phase-C current $\overline{I}_{SYCn}$ at the nth harmonic is:

$$\overline{I}_{SYCn} = \sum_{n=1}^{37} [a_{nC}\cos(n\Theta) + b_{nC}\sin(n\Theta)]$$

With:

$$a_{nC} = - \frac{I_m}{un2\pi}\Big(\cos n(\alpha_p - 30°) - \cos n(\alpha_p - 30° + u)$$

$$+ \cos n(\beta - 30° - \alpha_n) - \cos n(\beta - 30° - \alpha_n + u)$$

$$+ \cos(n\pi)[\cos n(u - 30° - \alpha_p) - \cos n(\alpha_p - 30°)$$

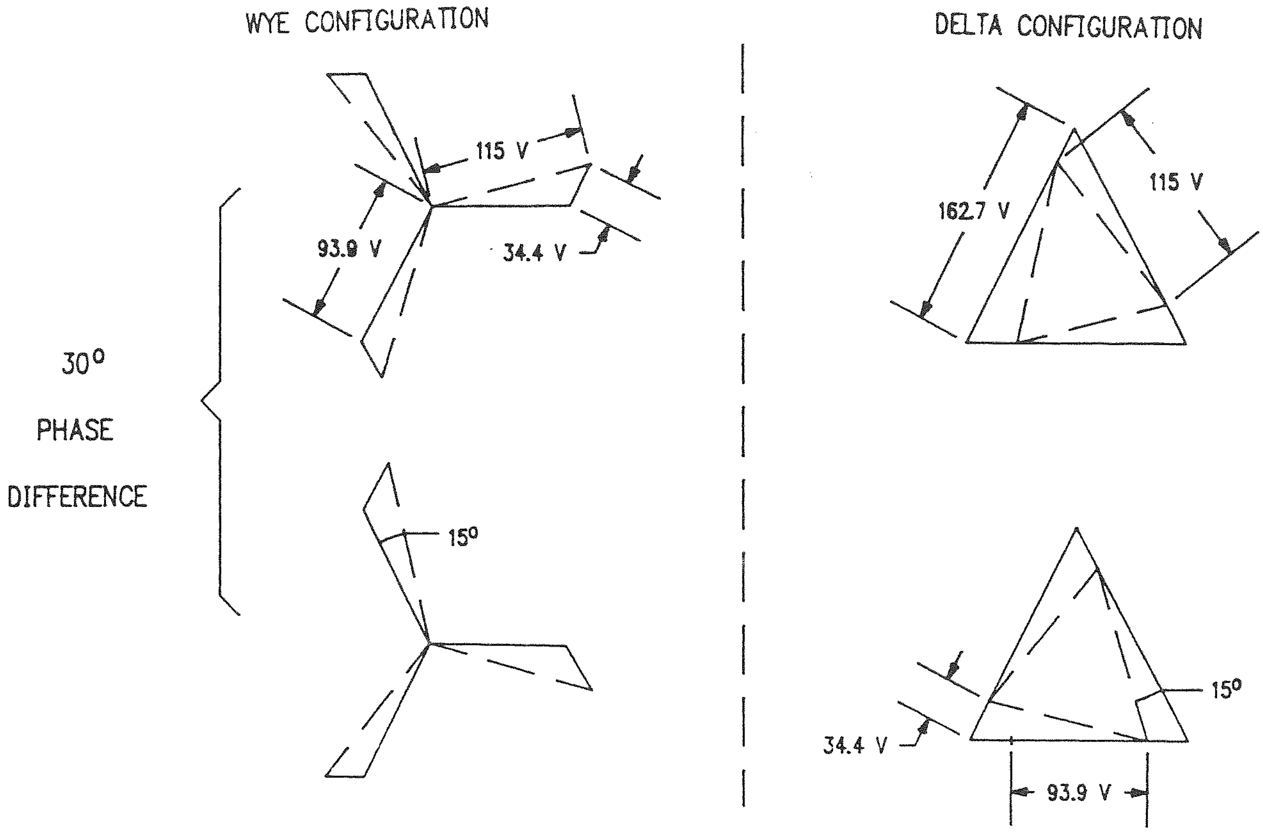
$$- \cos n(\alpha_p - 30° + \beta) + \cos n(\alpha_n - 30° + \beta + u)]\Big), \quad (202)$$

$$b_{nC} = - \frac{I_m}{un^2\pi}\Big(\sin n(\alpha_p - 30°) - \sin n(\alpha_p - 30° + u)$$

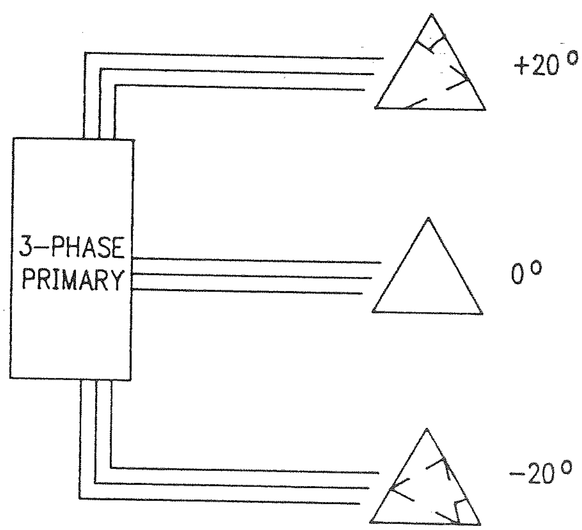
$$+ \sin n(\beta - 30° - \alpha_n) - \sin n(\beta - 30° - \alpha_n + u)$$

$$+ \cos(n\pi)[\sin n(\alpha_p - 30°) + \sin n(u - 30° - \alpha_p)$$

$$- \sin n(\alpha_n - 30° + \beta) + \sin n(\alpha_n - 30° + \beta + u)]\Big), \quad (203)$$



6-Phase(12-Pulse)Transformer.



9-Phase(18-Pulse)Transformer.

Figure 2-1: Multiphase Transformer Connections

Currents in Delta Windings.

Phase-A current $\bar{I}_{SDAn}$ is calculated from:

$$\bar{I}_{SDAn} = \frac{(\bar{I}_{SYBn} + \bar{I}_{SYCn})}{\sqrt{3}}$$

The algebraic sum of currents $\bar{I}_{SYBn}$ and $\bar{I}_{SYCn}$ is divided by $\sqrt{3}$ because the voltage in Delta transformer must be $\sqrt{3}$ times the voltage in Wye transformer for the transformer power rating to remain the same.

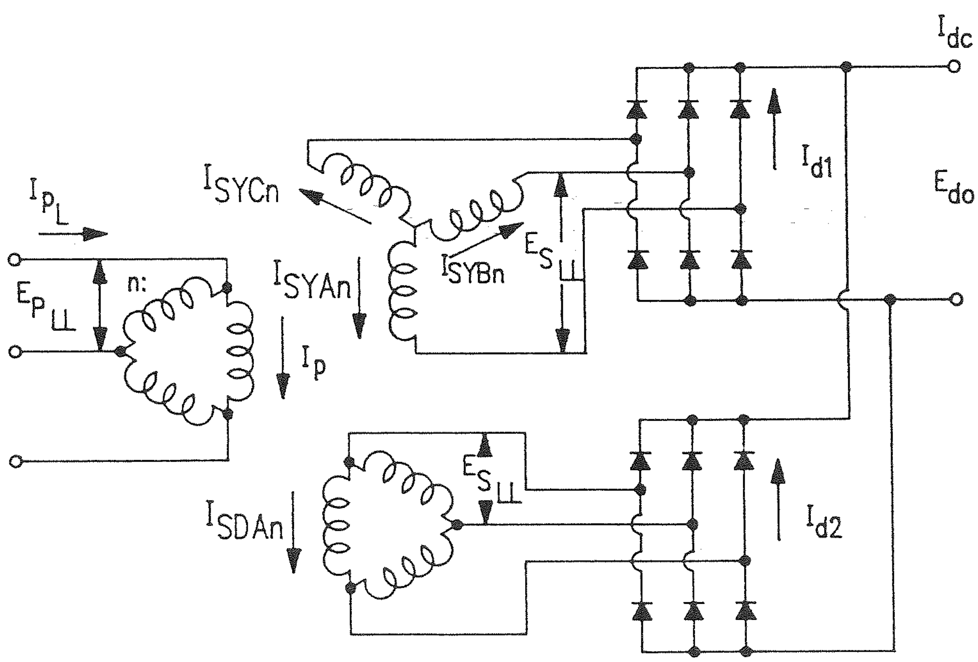


Figure 2-2: 6-Phase Secondary Delta Wye Transformer Connections.

2. Current Harmonic Reduction by Multiphase Zig-Zag Transformers.

Current harmonics in the primary winding of a multiphase transformer can be eliminated or reduced by considering a transformer connected in a ziz-zag configuration. The transfomer is defined as having primary winding with unity turns N_1, and secondary winding with two parts of N_2 and N_3 turns. Secondary windings from two separated legs are connected together to provide a phase-shifted current. For the purpose of this analysis, the secondary phase-A leg with N_{2k} turns and phase-C leg with N_{3k} turns are connected together. The primary phase-A winding is chosen.

The primary and secondary winding currents are related by:

$$\overline{I}_{nAk} = N_{2k}\overline{I}_{nak} - N_{3k}\overline{I}_{nck}, \qquad (204)$$

The positive integers k indicate number of phase-shifting transformers. The vector diagram called the current compensation triangle is shown in figure 2-3. A compensation triangle should be drawn for each phase-shifting transformer.

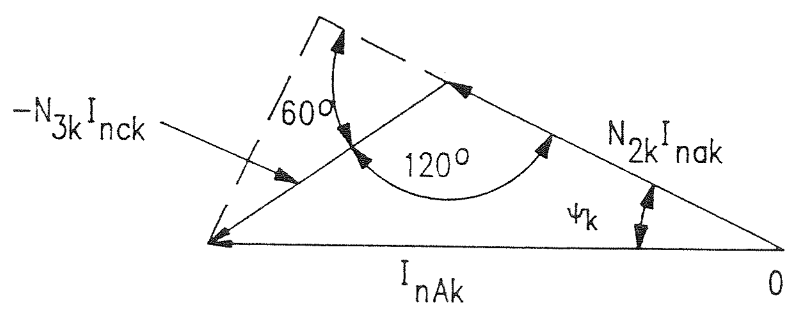


Figure 2-3: Current Compensation Triangle.

If the magnitudes of currents $\overline{I}_{nak}$, $\overline{I}_{nck}$, and $\overline{I}_{nAk}$ are equal to 1.0 pu(the system is balanced so that certain current harmonics will cancel), then the number of turns N_{2k} and N_{3k} can be computed as follows:

$$\frac{N_{3k}}{\sin(\psi_k)} = \frac{1}{\sin(120°)}$$

Then,

$$N_{3k} = \frac{2\sin(\psi_k)}{\sqrt{3}}, \qquad (206)$$

Also

$$\frac{N_{2k}}{\sin(60° - \psi_k)} = \frac{1}{\sin(120°)}$$

$$N_{2k} = (2/\sqrt{3})\sin(60° - \psi_k)$$

Expand the sine term:

$$N_{2k} = \cos(\psi_k) - \frac{\sin(\psi_k)}{\sqrt{3}}, \qquad (206)$$

The angle, ψ_k, between the primary and the secondary windings is fixed once the pulse number(converter type)is chosen. This angle is determined as follow:

$$\psi_k = \frac{360°}{(\text{pulse number})}, \qquad (207)$$

The computed values of N_{2k}, N_{3k}, and ψ_k for 6, 9, and 12-phase transformers which correspond respectively to 12, 18, and 24 pulse converter systems are shown in Table 2-I.

Note that for a system with a small voltage unbalance, the angle between phases will be different from 120°. If it is 119° on one phase(and 120° on the others) then there will be no perfect cancellation of currents for balanced loads. For example, in the case of a 6-phase transformer(12-pulse)if one of the compensation triangle angles is not equal to 120°, the current $\overline{I}_{nA2}$ will not completely cancel with $\overline{I}_{nA1}$.

Likewise, if the currents $\overline{I}_{na2}\ldots\overline{I}_{nak}$, and $\overline{I}_{nc2}\ldots\overline{I}_{nck}$, in phase-A and phase-C legs respectively are not equal to 1.0 pu, then there will be an incomplete cancellation of currents, and for example, some 5th current harmonic will appear in a 6, 9, or 12 phase-shifting transformer.

The terms positive and negative sequences arise as follows: In the secondary windings, at the fundamental frequency, phase-C leg is shifted from the phase-A leg by 240°. Consequently, currents $\overline{I}_{nck}$ have an added phase-shift of n240° at each harmonic. This characteristic gives rise to the positive and negative sequence components. At the 5th harmonic, current $\overline{I}_{5ck}$ is shifted by 5*240° from current $\overline{I}_{5ak}$. The normal rotation of phases at the fundamental frequency is abc. Since 5*240° is the same as 120°, the phase-C current $\overline{I}_{5ck}$, not the phase-B current, follows phase-A current $\overline{I}_{5ak}$ when the vectors rotate counterclockwise. If the vectors rotate clockwise, then the phase rotation will be in the normal abc sequence. Therefore, the (6n-1) characteristic for 6-pulse harmonics, the 5th, 11th, 17th, ... are considered to be negative sequence. The 7th, 13th, ... are considered as positive sequence quantities.

The equation (204) for the currents at nth harmonic can be rewritten in polar form as follows:

$$\bar{I}_{nAk} = N_{2k}\bar{I}_{nak} - N_{3k}\bar{I}_{nck}$$

$$\bar{I}_{nAk} = N_{ak}I_{nak}\underline{/n\psi_k} - N_{ck}I_{nck}\underline{/n\psi_k - n240°} \quad , \qquad (208)$$

The total current harmonic in the primary windings is then:

$$\bar{I}_n = \sum_{k=1}^{\infty} \bar{I}_{nAk} \quad , \qquad (209)$$

6-Phase(12-Pulse)Transformer.

There will be two phase-shifting transformers in which angles ψ_1 and ψ_2 are equal respectively to 0° and 30°. One compensation triangle should be drawn. From equation (209), the total primary current harmonic $\bar{I}_n$ in phase-A is:

$$\bar{I}_n = \bar{I}_{nA1} + \bar{I}_{nA2}$$

For the phase-shifting transformer number one (k=1):

$$\bar{I}_{nA1} = N_{21}I_{na1}\underline{/n\psi_1} - N_{31}I_{nc1}\underline{/n\psi_1 - n240°}$$

$$\bar{I}_{nA1} = N_{21}I_{na1}\cos(n\psi_1) - N_{31}I_{nc1}\cos(n\psi_1 - n240°)$$
$$+j[N_{21}I_{na1}\sin(n\psi_1) - N_{31}I_{nc1}\sin(n\psi_1 - n240°)]$$

For k = 2:

$$\overline{I}_{nA2} = N_{22} I_{na2} \underline{/n\psi_2} - N_{32} I_{nc2} \underline{/n\psi_2 - n240^\circ}$$

$$\overline{I}_{nA2} = N_{22} I_{na2} \cos(n\psi_2) - N_{32} I_{nc2} \cos(n\psi_2 - n240^\circ)$$
$$+J[N_{22} I_{na2} \sin(n\psi_2) - N_{32} I_{nc2} \sin(n\psi_2 - n240^\circ)]$$

TABLE 2-I
TRANSFORMER PULSE NUMBERS VS. PHASE-SHIFTING ANGLES AND NUMBER OF TURNS

PULSES	ANGLES				NUMBER OF TURNS							
	ψ_k				N_{2k}				N_{3k}			
	ψ_1	ψ_2	ψ_3	ψ_4	N_{21}	N_{22}	N_{23}	N_{24}	N_{31}	N_{32}	N_{33}	N_{34}
12	0°				1.00				0.0			
		30°				0.577				0.577		
18	0°				1.00				0.0			
		20°				0.742				0.395		
			40°				0.395				0.742	
24	0°				1.00				0.0			
		15°				0.817				0.299		
			30°				0.577				0.577	
				45°				0.299				0.817

Resolve into real and imaginary components:

$$I_{nr1} = N_{21}I_{na1}\cos(n\psi_1) - N_{31}I_{nc1}\cos(n\psi_1 - n240°), \quad (210)$$

$$I_{ni1} = N_{21}I_{na1}\sin(n\psi_1) - N_{31}I_{nc1}\sin(n\psi_1 - n240°), \quad (211)$$

$$I_{nr2} = N_{22}I_{na2}\cos(n\psi_2) - N_{32}I_{nc2}\cos(n\psi_2 - n240°), \quad (212)$$

$$I_{ni2} = N_{22}I_{na2}\sin(n\psi_2) - N_{32}I_{nc2}\sin(n\psi_2 - n240°), \quad (213)$$

From the above Table 2-I for the 6-phase(12-pulse)transformer:

For $\psi_1 = 0° \implies N_{21} = 1.0,\ N_{31} = 0$

For $\psi_2 = 30° \implies N_{22} = 0.577,\ N_{32} = 0.577$

Then equations (211) through (213) can be rewritten as follows:

$$I_{nr1} = I_{na1} = I_{nA1}, \quad I_{ni1} = 0$$

$$I_{nr2} = 0.577I_{na2}\cos(n30°) - 0.577I_{nc2}\cos(n30° - n240°), \quad (214)$$

$$I_{ni2} = 0.577I_{na2}\sin(n30°) - 0.577I_{nc2}\sin(n30° - n240°), \quad (215)$$

The magnitude of primary current harmonic $\overline{I}_n$ in percent in phase-A is:

$$I_n(\%) = \sqrt{(I_{nr1} + I_{nr2})^2 + (I_{ni1} + I_{ni2})^2} * \frac{100}{2n}, \quad (216)$$

Figure 2-4 shows a 6-phase(12-pulse)transformer with secondary windings connected in a zig-zag configuration. Figure 2-5 shows the elimination of the 5th current harmonic components in each phase of the primary windings. In phase-A, the 5th current harmonic component $\overline{I}_{A1}$ is cancelled with $\overline{I}_{A2}$.

9-Phase (18-Pulse) Transformer.

In this case there are three phase-shifting transformers. Two compensation triangles should be drawn, one with a phase-shift of 20° the other with a phase-shift of 40°. Using equation (209), the total primary current harmonic $\bar{I}_n$ in phase-A is:

$$\bar{I}_n = \bar{I}_{nA1} + \bar{I}_{nA2} + \bar{I}_{nA3}, \qquad (217)$$

Where:

For k = 1

$$\bar{I}_{nA1} = N_{21} I_{na1} \underline{/n\psi_1} - N_{31} I_{nc1} \underline{/n\psi_1 - n240^\circ}$$

$$\bar{I}_{nA1} = N_{21} I_{na1} \cos(n\psi_1) - N_{31} I_{nc1} \cos n(\psi_1 - 240^\circ)$$
$$+ j[N_{21} I_{na1} \sin(n\psi_1) - N_{31} I_{nc1} \sin n(\psi_1 - 240^\circ)], \qquad (218)$$

For k = 2

$$\bar{I}_{nA2} = N_{22} I_{na2} \underline{/n\psi_2} - N_{32} I_{nc2} \underline{/n\psi_2 - n240^\circ}$$

$$\bar{I}_{nA2} = N_{22} I_{na2} \cos(n\psi_2) - N_{32} I_{nc2} \cos n(\psi_2 - 240^\circ)$$
$$+ j[N_{22} I_{na2} \sin(n\psi_2) - N3_2 I_{nc2} \sin n(\psi_2 - 240^\circ)], \qquad (219)$$

For k = 3

$$\bar{I}_{nA3} = N_{23} I_{na3} \underline{/n\psi_3} - N_{33} I_{nc3} \underline{/n\psi_3 - n240^\circ}$$

$$\bar{I}_{nA3} = N_{23} I_{na3} \cos(n\psi_3) - N_{33} I_{nc3} \cos n(\psi_3 - 240^\circ)$$
$$+ j[N_{23} I_{na3} \sin(n\psi_3) - N_{33} I_{nc3} \sin n(\psi_3 - 240^\circ)], \qquad (220)$$

From Table 2-I, by substituting the values of ψ_k, N_{2k}, and N_{3k} (with k = 1, 2, 3) into equations (218) through (220), the real and imaginary components of the currents are:

$$I_{nr1} = I_{nA1}\ ,\ I_{ni1} = 0$$

$$I_{nr2} = 0.742 I_{na2}\cos(n20°) - 0.395 I_{nc2}\cos(n20° - n240°), \quad (221)$$

$$I_{ni2} = 0.742 I_{na2}\sin(n20°) - 0.395 I_{nc2}\sin(n20° - n240°), \quad (222)$$

$$I_{nr3} = 0.395 I_{na3}\cos(n40°) - 0.742 I_{nc3}\cos(n40° - n240°), \quad (223)$$

$$I_{ni3} = 0.395 I_{na3}\sin(n40°) - 0.742 I_{nc3}\sin(n40° - n240°), \quad (224)$$

The magnitude of the primary current harmonic $\overline{I}_n$ in percent in phase-A is:

$$I_n(\%) = \sqrt{(I_{nr1} + I_{nr2} + I_{nr3})^2 + (I_{ni1} + I_{ni2} + I_{ni3})^2} * \frac{100}{(n3)}, \quad (225)$$

Figure 2-6 shows a 9-phase(18-pulse)transformer with secondary windings connected in a zig-zag configuration.

12-Phase(24-Pulse)Transformer.

There are four phase-shifting transformers, and three compensation triangles should be drawn, with phase-shift angles of 15°, 30°, and 45°. Four current vectors must be added to find the resultant:

$$\overline{I}_n = \overline{I}_{nA1} + \overline{I}_{nA2} + \overline{I}_{nA3} + \overline{I}_{nA4}, \quad (226)$$

With similar procedures , equations for calculating the current harmonics are:

$$I_{nr1} = I_{nA1},\ I_{ni1} = 0$$

$$I_{nr2} = 0.817 I_{na2}\cos(n15°) - 0.299 I_{nc2}\cos(n15° - n240°), \qquad (227)$$

$$I_{ni2} = 0.817 I_{na2}\sin(n15°) - 0.299 I_{nc2}\sin(n15° - n240°), \qquad (228)$$

$$I_{nr3} = 0.577 I_{na3}\cos(n30°) - 0.577 I_{nc3}\cos(n30° - n240°), \qquad (229)$$

$$I_{ni3} = 0.577 I_{na3}\sin(n30°) - 0.577 I_{nc3}\sin(n30° - n240°), \qquad (230)$$

$$I_{nr4} = 0.299 I_{na4}\cos(n45°) - 0.817 I_{nc4}\cos(n45° - n240°), \qquad (231)$$

$$I_{ni4} = 0.299 I_{na4}\sin(n45°) - 0.817 I_{nc4}\sin(n45° - n240°), \qquad (232)$$

The magnitude of the primary current harmonic $\overline{I}_n$ in percent in phase-A is:

$$I_n(\%) = \sqrt{(I_{nr1}+I_{nr2}+I_{nr3}+I_{nr4})^2 + (I_{ni1}+I_{ni2}+I_{ni3}+I_{ni4})^2} * \frac{100}{(4n)}, \qquad (233)$$

Figure 2-7 shows a typical arrangement for a 12-phase(24-pulse) transformer with secondary windings connected in a zig-zag configuration. Figure 2-8 shows a vectorial elimination of the 11th current harmonic components in the transformer primary windings.

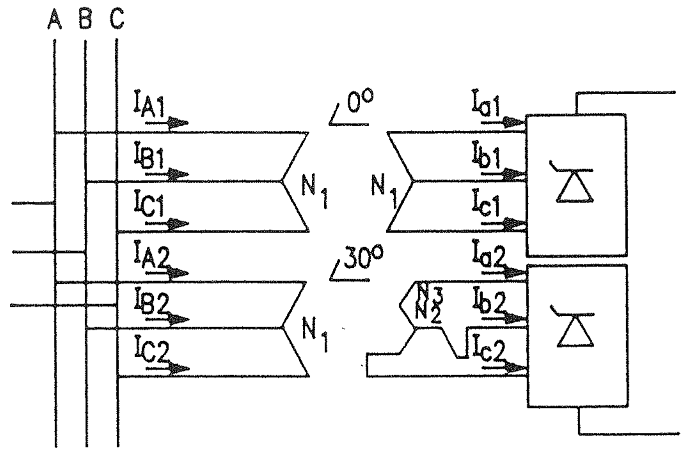


Figure 2-4: 6-Phase(12-Pulse)Transformer.

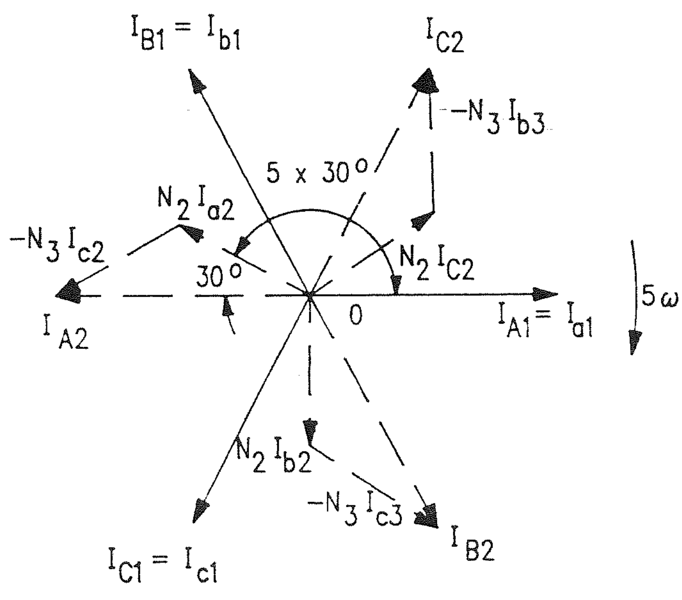


Figure 2-5: 5th Current Harmonic Component Elimination.

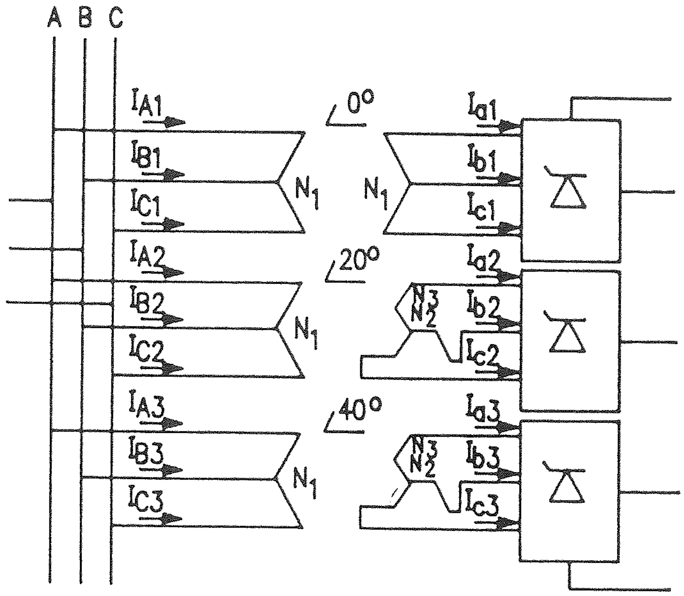


Figure 2-6: 9-Phase(18-Pulse)Transformer.

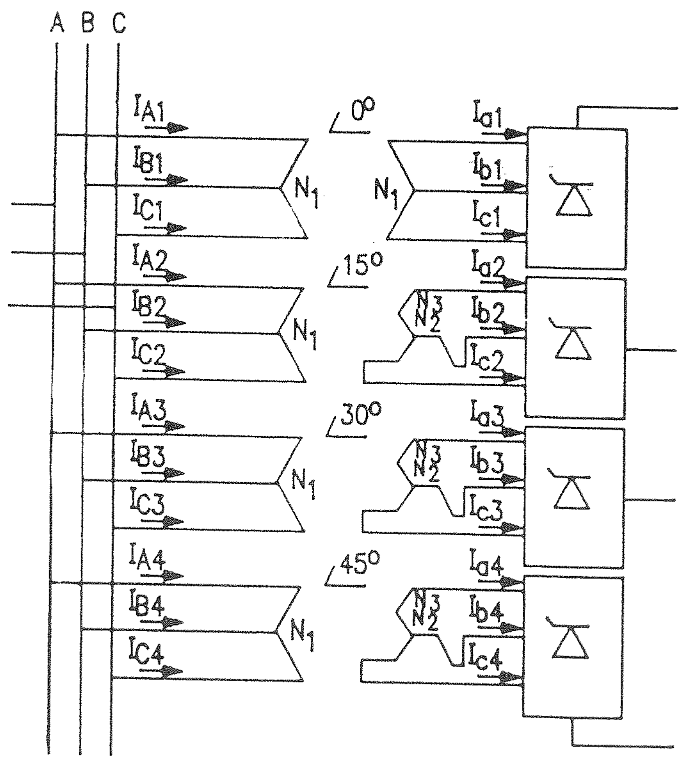


Figure 2-7: 12-Phase(24-Pulse)Transformer.

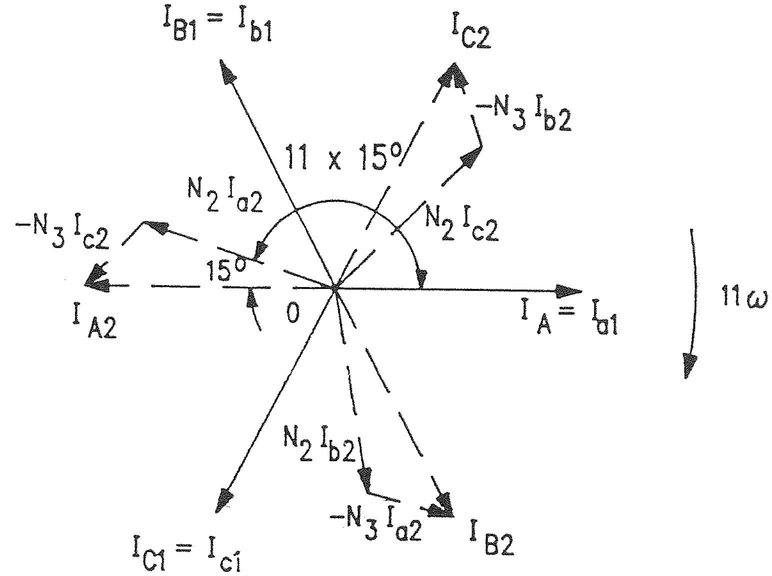


Figure 2-8: 11th Current Harmonic Component Elimination.

APPENDIX 3

CALCULATIONS OF VOLTAGE UNBALANCES DUE TO UNBALANCED LOADS.

For ungrounded systems, the zero sequence current $\overline{I}_o$ is zero.

Derive boundary conditions from Figure 3-1:

$$\overline{V}_a = e_n + \overline{Z}_a\overline{I}_a$$

$$= e_n + \overline{Z}_a(\overline{I}_1 + \overline{I}_2)$$

$$\overline{V}_b = e_n + \overline{Z}_b\overline{I}_b$$

$$= e_n + \overline{Z}_b(a^2\overline{I}_1 + a\overline{I}_2)$$

$$\overline{V}_c = e_n + \overline{Z}_c\overline{I}_c$$

$$= e_n + \overline{Z}_c(a\overline{I}_1 + a^2\overline{I}_2)$$

Positive sequence voltage is:

$$\overline{V}_1 = \frac{\overline{V}_a + a\overline{V}_b + a^2\overline{V}_c}{3}$$

$$= (1/3)[e_n\ (1 + a + a^2) + \overline{I}_1(\overline{Z}_a + \overline{Z}_b + \overline{Z}_c)$$
$$+ \overline{I}_2\ (\overline{Z}_a + a^2\overline{Z}_b + a\overline{Z}_c)]$$

$$\overline{V}_1 = \overline{I}_1\frac{(\overline{Z}_a + \overline{Z}_b + \overline{Z}_c)}{3} + \overline{I}_2\frac{(\overline{Z}_a + a^2\overline{Z}_b + a\overline{Z}_c)}{3}$$

Assume the unbalance occurs in one phase only.

Let $\overline{Z}_c = \overline{Z}_b$, then $(a^2 + a)\overline{Z}_b = -\overline{Z}_b$

Then,

$$\overline{V}_1 = \overline{I}_1\left(\frac{\overline{Z}_a + 2\overline{Z}_b}{3}\right) + \overline{I}_2\left(\frac{\overline{Z}_a - \overline{Z}_b}{3}\right), \qquad (300)$$

Negative sequence voltage is:

$$\overline{V}_2 = \frac{\overline{V}_a + a^2\overline{V}_b + a\overline{V}_c}{3}$$

$$= (1/3)\left[e_n(1 + a + a^2) + \overline{I}_1\left(\frac{\overline{Z}_a + a\overline{Z}_b + a^2\overline{Z}_c}{3}\right) + \overline{I}_2\left(\frac{\overline{Z}_a + \overline{Z}_b + \overline{Z}_c}{3}\right)\right]$$

$$\overline{V}_2 = \overline{I}_1\left(\frac{\overline{Z}_a + a\overline{Z}_b + a^2\overline{Z}_c}{3}\right) + \overline{I}_2\left(\frac{\overline{Z}_a + \overline{Z}_b + \overline{Z}_c}{3}\right)$$

If $\overline{Z}_c = \overline{Z}_b$, then $(a + a^2)\overline{Z}_b = -\overline{Z}_b$

Then,

$$\overline{V}_2 = \overline{I}_1\left(\frac{\overline{Z}_a - \overline{Z}_b}{3}\right) + \overline{I}_2\left(\frac{\overline{Z}_a + 2\overline{Z}_b}{3}\right), \qquad (301)$$

From the above equations, the term $(\overline{Z}_a - \overline{Z}_b)/3$ can be considered as a mutual impedance between the positive and negative sequence networks. If $\overline{Z}_c$ is not equal to $\overline{Z}_b$ (or $\overline{Z}_a$), the symmetrical component solution method does not simplify the problem. Figure 3-2 shows the sequence connection which satisfies equations (300) and (301).

If $\overline{Z}_a \rightarrow \infty$, the system reduces to a single-phase load of $2\overline{Z}_b$. The sequence connection diagram for this case is shown in Figure 3-3.

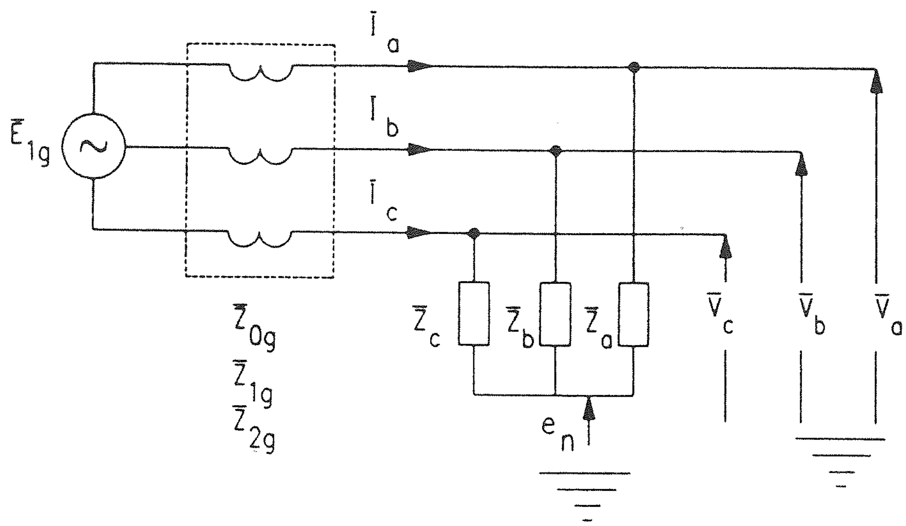


Figure 3-1: Three-Phase Unbalanced Load Schematic.

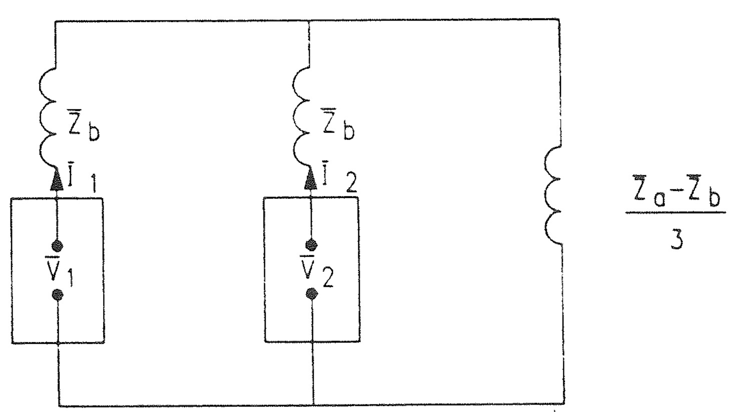


Figure 3-2: Three-Phase Unbalanced Load Sequence Network Connection.

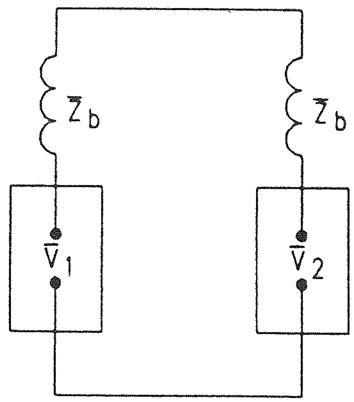


Figure 3-3: Single-Phase Load Sequence Network Connection.

Voltage Calculations for Single-phase Unbalanced load.

Method 1 -- The Single-phase Unbalanced Current $\overline{I}_u$ is known.

Let $\overline{V}_1 = 1.0 + j0.0$ pu

In Figure 3-4, with $R + jX = 0$, then $\overline{I}_u = \overline{I}_1 = - \overline{I}_2$

$$\overline{V}_a = \overline{V}_1 + \overline{V}_2$$

$$= \overline{V}_1 - \overline{I}_u\overline{Z}_2$$

$$\overline{V}_b = a^2\overline{V}_1 + a\overline{V}_2$$

$$= a^2\overline{V}_1 - a\overline{I}_u\overline{Z}_2$$

$$\overline{V}_c = a\overline{V}_1 + a^2\overline{V}_2$$

$$= a\overline{V}_1 - a^2\overline{I}_u\overline{Z}_2$$

$$\overline{V}_{ab} = \overline{V}_a - \overline{V}_b$$

$$= 1.0 + j0 - \overline{I}_u\overline{Z}_2 - a^2 + a\overline{I}_u\overline{Z}_2$$

$$= (1 - a^2) - \overline{I}_u\overline{Z}_2(1 - a)$$

$$= \frac{3}{2} + j\frac{\sqrt{3}}{2} - \frac{3}{2}\overline{I}_u\overline{Z}_2 + j\frac{\sqrt{3}}{2}\overline{I}_u\overline{Z}_2$$

$$= \frac{3}{2}(1 - \overline{I}_u\overline{Z}_2) + j\frac{\sqrt{3}}{2}(1 + \overline{I}_u\overline{Z}_2), \quad (302)$$

$$\overline{V}_{bc} = \overline{V}_b - \overline{V}_c$$

$$= a^2 - a\overline{I}_u\overline{Z}_2 - a + a^2\overline{I}_u\overline{Z}_2$$

$$= -j\sqrt{3} + \overline{I}_u\overline{Z}_2(a^2 - a)$$

$$= -j\sqrt{3}(1 + \overline{I}_u\overline{Z}_2), \quad (303)$$

$$\overline{V}_{ca} = \overline{V}_c - \overline{V}_a$$

$$= a - a^2\overline{I}_u\overline{Z}_2 - 1 + \overline{I}_u\overline{Z}_2$$

$$= (a - 1) + \overline{I}_u\overline{Z}_2(1 - a^2)$$

$$= \frac{3}{2}(\overline{I}_u\overline{Z}_2 - 1) + j\frac{\sqrt{3}}{2}(1 + \overline{I}_u\overline{Z}_2), \quad (304)$$

Method 2.-- Single-phase Unbalanced Impedance is known.

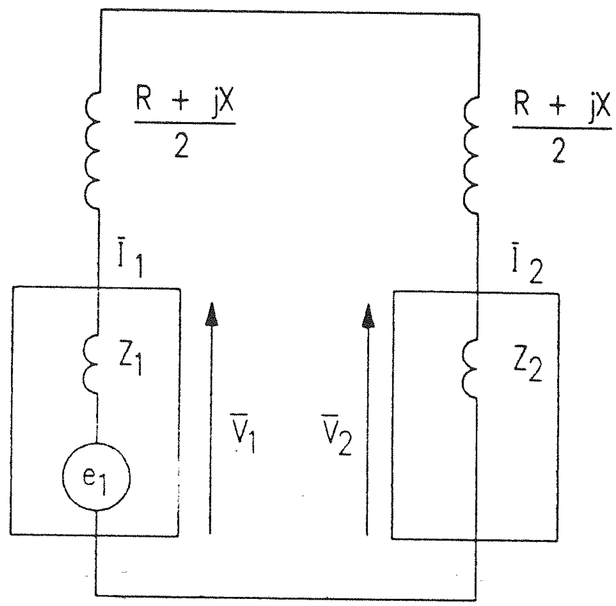


Figure 3-4: Single-phase Load Sequence Connection-Example.

In the above Figure 3-4, let consider:

$\overline{Z} = R + jX$, the total single-phase unbalanced impedance.

$\overline{Z}_2 = 0 + jX_2$, the negative sequence impedance of the network equal to generator negative sequence impedance $\overline{Z}_{2g}$.

$$\overline{V}_a = \overline{V}_1 + \overline{V}_2$$

$$= \overline{V}_1 - j\overline{V}_1\left(\frac{X_2}{R + j(X + X_2)}\right)$$

$$= \left(\frac{\overline{V}_1}{R + j(X + X_2)}\right)(R + jX), \qquad (305)$$

$$\overline{V}_b = a^2\overline{V}_1 + a\overline{V}_2$$

$$= \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)\overline{V}_1 - j\overline{V}_1\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)\left(\frac{X_2}{R + j(X + X_2)}\right)$$

$$= \left(\frac{\overline{V}_1}{R + j(X + X_2)}\right)\left((-\frac{R}{2} + \frac{\sqrt{3}}{2}X + \sqrt{3}X_2)\right.$$

$$\left. - j(\frac{\sqrt{3}}{2}R + \frac{X}{2})\right), \qquad (306)$$

$$\overline{V}_c = a\overline{V}_1 + a^2\overline{V}_2$$

$$= \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)\overline{V}_1 - j\overline{V}_1\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)\left(\frac{X_2}{R + j(X + X_2)}\right)$$

$$= \left(\frac{\overline{V}_1}{R + j(X + X_2)}\right)\left((-\frac{R}{2} - \frac{\sqrt{3}X}{2} - \sqrt{3}X_2) + j(\frac{\sqrt{3}R}{2} - \frac{X}{2})\right), \qquad (307)$$

Let

$$K = \left(\frac{\overline{V}_1}{R + j(X + X_2)}\right)$$

Equations (305), (306), and (307) reduce to:

$$\overline{V}_a = K(R + jX), \tag{308}$$

$$\overline{V}_b = K\left[(-\frac{R}{2} + \frac{\sqrt{3}X}{2} + \sqrt{3}X_2) - j(\frac{\sqrt{3}R}{2} + \frac{X}{2})\right], \tag{309}$$

$$\overline{V}_c = K\left[(-\frac{R}{2} - \frac{\sqrt{3}X}{2} - \sqrt{3}X_2) + j(\frac{\sqrt{3}R}{2} - \frac{X}{2})\right], \tag{310}$$

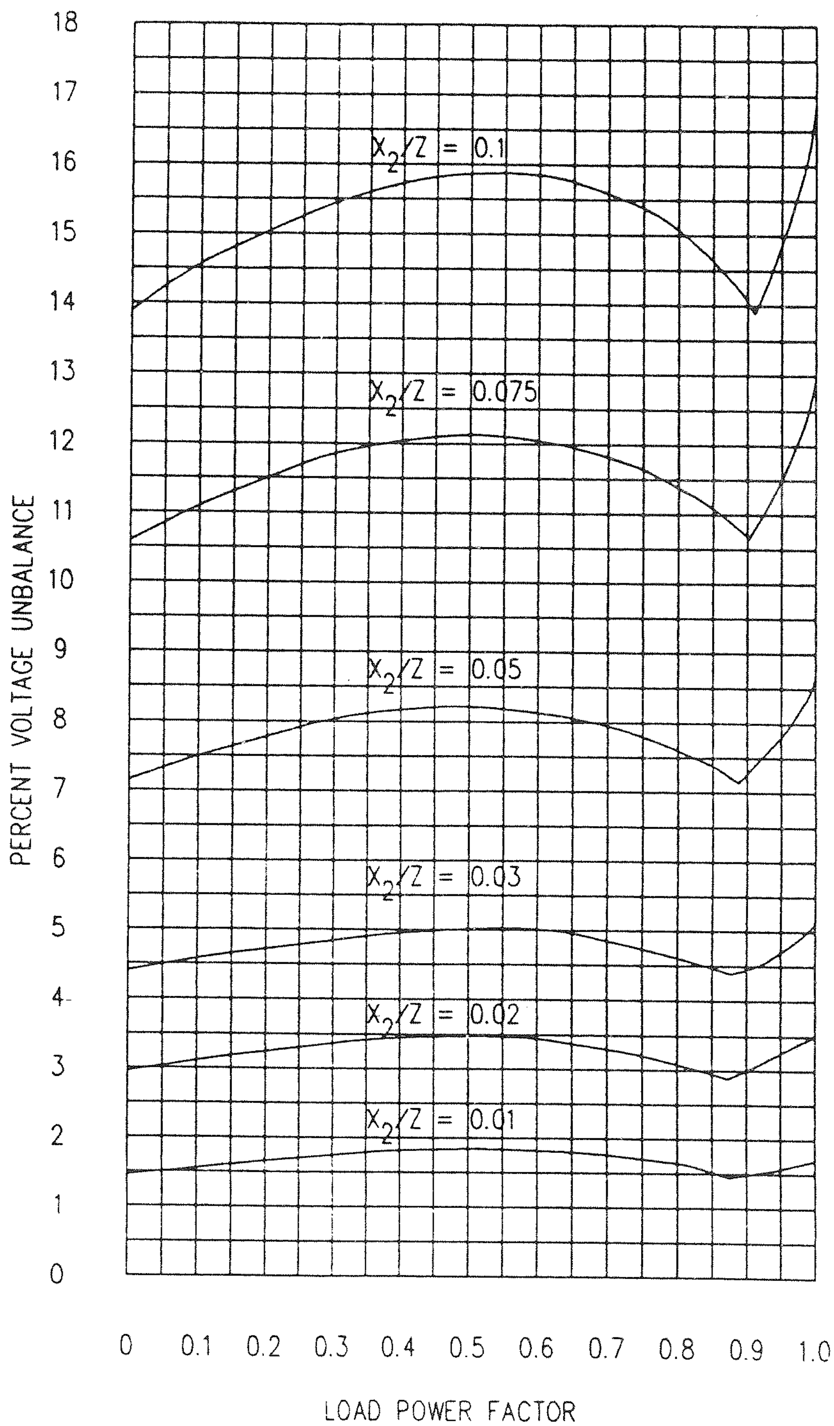
Finally the line-to-line voltages $\overline{V}_{ab}$, $\overline{V}_{bc}$, and $\overline{V}_{ca}$ can be calculated from the following equations:

$$\overline{V}_{ab} = \overline{V}_a - \overline{V}_b \quad , \tag{311}$$

$$\overline{V}_{bc} = \overline{V}_b - \overline{V}_c \quad , \tag{312}$$

$$\overline{V}_{ca} = \overline{V}_c - \overline{V}_a \quad , \tag{313}$$

Figure 3-5 shows a plot of percent voltage unbalances vs. load power factors for a number of ratios of $X_2/\ \overline{Z}$, calculated from equations (311), (312), and (313).



X_2 : Line-to-line Single-phase Negative Sequence Reactance.

$\overline{Z}$: Line-to-line Single-phase Impedance.

Figure 3-5: Voltage Unbalances Due to Single-phase Unbalanced Load.

Voltage Calculations for Three-Phase Unbalanced Load.

The three-phase unbalanced load analysis is more complicated than the single-phase load analysis.

The zero sequence currents do not exist because the system is not grounded. $\overline{I}_{a0} = \overline{I}_{b0} = \overline{I}_{c0} = 0$

$$\overline{I}_1 = \frac{\overline{E}_{1g} - \overline{V}_1}{\overline{Z}_{1g}}$$

Then:

$$\overline{V}_1 = \overline{E}_{1g} - \overline{I}_2\overline{Z}_{1g}, \qquad (314)$$

$$\overline{I}_2 = - \frac{\overline{V}_2}{\overline{Z}_{2g}}$$

Then:

$$\overline{V}_2 = - \overline{I}_2\overline{Z}_{2g}, \qquad (315)$$

Where:

$\overline{E}_{1g}$: Generator positive sequence voltage.

$\overline{Z}_{1g}$: Generator positive sequence impedance.

$\overline{Z}_{2g}$: Generator negative sequence impedance.

From reference (e), page 377, equation (484), for unbalanced loads:

$$\overline{V}_1 = \overline{Z}_{0u}\overline{I}_1 + \overline{Z}_{2u}\overline{I}_2 \quad , \qquad (316)$$

$$\overline{V}_2 = \overline{Z}_{1u}\overline{I}_1 + \overline{Z}_{0u}\overline{I}_2 \quad , \qquad (317)$$

Where:

$$\overline{Z}_{0u} = (1/3)(\overline{Z}_a + \overline{Z}_b + \overline{Z}_c)$$

$$\overline{Z}_{1u} = (1/3)(\overline{Z}_a + a\overline{Z}_b + a^2\overline{Z}_c)$$

$$\overline{Z}_{2u} = \frac{1}{3}(\overline{Z}_a + a^2\overline{Z}_b + a\overline{Z}_c)$$

Equate (314) and (316):

$$\overline{I}_1(\overline{Z}_{0u} + \overline{Z}_{1g}) + \overline{Z}_{2u}\overline{I}_2 = \overline{E}_{1g} \qquad (318)$$

Equate (315) and (317):

$$\overline{I}_1\overline{Z}_{1u} + \overline{I}_2(\overline{Z}_{0u} + \overline{Z}_{2g}) = 0 \qquad (319)$$

Solve for $\overline{I}_1$ and $\overline{I}_2$:

$$\overline{I}_1 = -\frac{\overline{E}_{1g}(\overline{Z}_{0u} + \overline{Z}_{2g})}{\overline{Z}_{1u}\overline{Z}_{2u} - (\overline{Z}_{0u} + \overline{Z}_{1g})(\overline{Z}_{0u} + \overline{Z}_{2g})}$$

$$\overline{I}_2 = \frac{\overline{E}_{1g}\overline{Z}_{1u}}{\overline{Z}_{1u}\overline{Z}_{2u} - (\overline{Z}_{0u} + \overline{Z}_{1g})(\overline{Z}_{0u} + \overline{Z}_{2g})}$$

Write equations for the phase currents:

$$\overline{I}_a = \overline{I}_1 + \overline{I}_2$$
$$= K_1(\overline{Z}_{1u} - \overline{Z}_{0u} - \overline{Z}_{2g})$$

$$\overline{I}_b = a^2\overline{I}_1 + a\overline{I}_2$$
$$\overline{I}_b = K_1(a\overline{Z}_{1u} - a^2\overline{Z}_{0u} - a^2\overline{Z}_{2g})$$

$$\overline{I}_c = a\overline{I}_1 + a^2\overline{I}_2$$
$$\overline{I}_c = K_1(a^2\overline{Z}_{1u} - a\overline{Z}_{0u} - a\overline{Z}_{2g})$$

Where:

$$K_1 = \left(\frac{\overline{E}_{1g}}{\overline{Z}_{1u}\overline{Z}_{2u} - (\overline{Z}_{0u} + \overline{Z}_{1g})(\overline{Z}_{0u} + \overline{Z}_{2g})}\right)$$

Write equations for phase voltages:

$$\overline{V}_a = \overline{I}_a\overline{Z}_a$$

$$\overline{V}_b = \overline{I}_b\overline{Z}_b$$

$$\overline{V}_c = \overline{I}_c\overline{Z}_c$$

And line-to-line voltages:

$$\overline{V}_{ab} = \overline{V}_a - \overline{V}_b$$

$$= \overline{I}_a\overline{Z}_a - \overline{I}_b\overline{Z}_b$$

$$\overline{V}_{ab} = K_1\left(\overline{Z}_a(\overline{Z}_{1u} - \overline{Z}_{0u} - \overline{Z}_{2g}) - \overline{Z}_b(a\overline{Z}_{1u} - a^2\overline{Z}_{0u} - a^2\overline{Z}_{2g})\right)$$

Substitute the equivalences of $\overline{Z}_{0u}$ and $\overline{Z}_{1u}$:

$$\overline{V}_{ab} = (K_1/3)\left(\overline{Z}_a[\overline{Z}_a + a\overline{Z}_b + a^2\overline{Z}_c - \overline{Z}_a - \overline{Z}_b - \overline{Z}_c - \overline{Z}_{2g}]\right.$$

$$\left. - \overline{Z}_b[a\overline{Z}_a + a^2\overline{Z}_b + \overline{Z}_c - a^2\overline{Z}_a - a^2\overline{Z}_b - a^2\overline{Z}_c - 3a\ \overline{Z}_{2g}]\right)$$

$$\overline{V}_{ab} = (K_1/3)\left(\overline{Z}_a[\overline{Z}_b(a-1) + \overline{Z}_c(a^2-1) - 3\overline{Z}_{2g}]\right.$$

$$\left. + \overline{Z}_b[\overline{Z}_a(a^2-a) + \overline{Z}_c(a^2-1) + 3a^2\overline{Z}_{2g}]\right)$$

Set $\overline{Z}_b = \overline{Z}_c$:

$$\overline{V}_{ab} = (K_1/3)[2\overline{Z}_b\overline{Z}_a(a^2-1) - 3\overline{Z}_a\overline{Z}_{2g} + 3a^2\overline{Z}_{2g}\overline{Z}_b + \overline{Z}_b{}^2(a^2-1)]$$

Let $K_2 = K_1/3$

$$\overline{V}_{ab} = K_2[(2\overline{Z}_b\overline{Z}_a + \overline{Z}_b{}^2)(a^2 -1) + 3\overline{Z}_{2g}(a^2\overline{Z}_b - \overline{Z}_a)], \qquad (320)$$

Similarly,

$$\overline{V}_{bc} = \overline{V}_b - \overline{V}_c$$

$$\overline{V}_{bc} = \overline{I}_b\overline{Z}_b - \overline{I}_c\overline{Z}_c$$

$$= K_2\{\overline{Z}_b(a\overline{Z}_{1u} - a^2\overline{Z}_{0u} - 3a^2\overline{Z}_{2g}) - \overline{Z}_c(a^2\overline{Z}_{1u} - a\overline{Z}_{0u} - 3a\overline{Z}_{2g})\}$$

$$= K_2\{\overline{Z}_b(a\overline{Z}_a + \overline{Z}_c - a^2\overline{Z}_a - a^2\overline{Z}_c - 3a^2\overline{Z}_{2g})$$

$$- \overline{Z}_c(a^2\overline{Z}_a + \overline{Z}_b - a\overline{Z}_a - a\overline{Z}_b - 3a\overline{Z}_{2g})\}$$

Again, let $\overline{Z}_b = \overline{Z}_c$:

$$\overline{V}_{bc} = K_2\{(\overline{Z}_b{}^2 + 3\overline{Z}_{2g}\overline{Z}b + 2\overline{Z}_a\overline{Z}_b)(a - a^2)\}, \qquad (321)$$

Finally,

$$\overline{V}_{ca} = \overline{V}_c - \overline{V}_a$$

$$\overline{V}_{ca} = \overline{I}_c\overline{Z}_c - \overline{I}_a\overline{Z}_a$$

$$= K_2\{\overline{Z}_c(a^2\overline{Z}_{1u} - a\overline{Z}_{0u} - a\overline{Z}_{2g}) - \overline{Z}_a(\overline{Z}_{1u} - \overline{Z}_{0u} - \overline{Z}_{2g})\}$$

$$= K_2\{ \overline{Z}_c(a^2\overline{Z}_a + \overline{Z}_b - a\overline{Z}_a - a\overline{Z}_b - 3a\overline{Z}_{2g})$$

$$- \overline{Z}_a(a\overline{Z}_b + a^2\overline{Z}_c - \overline{Z}_b - \overline{Z}_c - 3a\overline{Z}_{2g})\}$$

With $\overline{Z}_b = \overline{Z}_c$, the equation reduces to:

$$\overline{V}_{ca} = K_2\{(\overline{Z}_b{}^2 + 2\overline{Z}_a\overline{Z}_b)(1 - a) + 3\overline{Z}_{2g}(\overline{Z}_a - a\overline{Z}_b)\}, \qquad (322)$$

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