Methods of Determining the Longitudinal Weight Distribution of a Ship

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Abstract

Approximation methods for weight distribution of ships are surveyed. Grouping methods such as the "Bucket" and station method are also explored. Detail based methods are explained. Finally, an improved method of distribution based on details is proposed. Guidance for the requirements of a weight database for this method is given and an alternative summary method is suggested to overcome difficulties caused by failure to meet certain database requirements of the detail method. Extensive appendices provide necessary figures and equations for using these methods.

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Introduction

Determination of the longitudinal weight distribution is vital to the proper calculation of the longitudinal strength of a ship. The longitudinal weight distribution also affects speed loss in a seaway [1]. Weight distributions of all three principal axes can also be used to calculate the ship's gyradii [2] which have a profound effect on the seakeeping performance of a vessel. Before the advent of computers, determination of a ship's weight distribution was a "rather laborious process" [3]. Due to the amount of labor involved, approximation methods were developed over the years. With the advent of computers, methods of collecting all of the weights with centers between given locations became less labor intensive giving rise to grouping methods.

For longitudinal strength calculations, various levels of detail are acceptable. However, the standard is a "Twenty Station Weight Distribution" which actually consists of 22 weight segments divided by 21 stations, (Stations 0 through 20).

The Weight Distribution Problem

Weight distributions are needed for numerous uses however weight data is stored in databases as large numbers of discrete details. These details are essentially lumped masses and can represent items which extend for large portions of the length of the vessel.

The traditional response to the need for weight distributions is to use a stock approximation appropriate for the ship type and improve it by distributing the large weight items separately. After the computer revolutionized the storing of weight data, the goal of assigning individual weight items to each station of the weight distribution began to be feasible leading to the grouping methods. However, even highly detailed weight databases often use weight details which are too long longitudinally for such methods to be wholly effective. This gave rise to the goal of distributing each weight record and then combining these distributions. Realization of this goal requires inclusion of the extents of each weight record in the database. Unfortunately the extents are not always included for a multitude of reasons.

This paper surveys all of the methods mentioned thus far and recommends an ideal detail distribution method and guidelines for a database which would make this method feasible. It also explains a summary based method which enables the user to use the same weight distribution techniques proposed for the detail distribution method with a database which does not contain all of the data necessary to distribute individual details.

Approximation Methods

Numerous approximation methods for distributing hull weight have been proposed in the past. Hull weight is traditionally defined as lightship minus the weight of the anchor, chain, anchor handling gear, steering gear and main propulsion machinery. Determination of the exact breakdown of hull weight should be made based on the relative density of the object in question. Items left out of hull weight should be independently distributed as rectangles or trapezoids and combined with the hull weight distribution to determine the total weight distribution for the ship.

Most approximation methods are based on combinations of a midship rectangle with forward and after trapezoids. More sophisticated methods base a portion of the weight curve on the ship's buoyancy curve. Approximation methods are presented in works by Smith [3], Comstock [4], and Hughes [5] as well as in *Principles of Naval Architecture* [6]. Appendix A: Survey of Approximate Methods of Weight Distribution contains details and equations for several of these approximations. These approximations are general and appropriate only for initial stage design due to their low fidelity. An example of such an approximation is depicted in Figure 1.

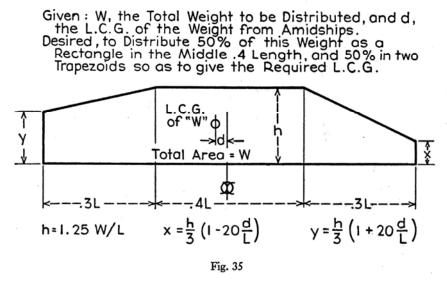


Figure 1: One Example of an Approximate Weight Distribution. Source [4]

Marine Vehicle Weight Engineering [7] contains longitudinal weight distributions by type for various military and support vessels. This segregation of distributions by type allows for improved fidelity. Such base distributions can be grossly modified to quickly arrive at a weight distribution of new designs similar to the provided types in the concept exploration stage of design. However, the resultant distributions are still not accurate enough for final longitudinal strength calculations.

Grouping Methods

The original grouping method is the "Bucket" method. The "Bucket" method derives its name from the fact that the weight details are metaphorically placed in buckets based on the location of their longitudinal center of gravity. If a line item's longitudinal center of gravity falls in the extents of a bucket, it is included in that bucket. This method is illustrated in Figure 2.

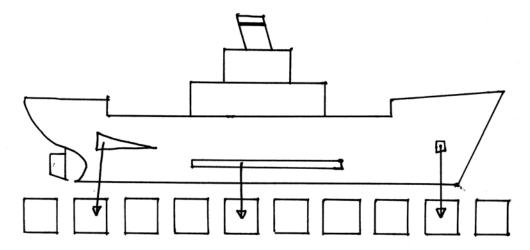


Figure 2: An Illustration of the "Bucket Method"

The flaw of this method is that weight details are lumped representations of distributed weights. One line item can represent 1 or 1000 feet longitudinally. Thus just because the center reported in the line falls in one bucket does not necessarily mean that all or even the majority of the weight reported in that line falls in that bucket. Generally the most offending line items in such a method are items such as paint, weld, and mill tolerance weights as they generally reflect the weight of these items across the entire ship. Weights such as these are generally corrected by hand to improve the distribution. However, most other weights that belong in multiple buckets remain uncorrected. Distributed systems such as piping, electrical and ventilation systems often have weight records that have extents that span multiple stations. Thus the accuracy of this method is limited.

The Ship Design Weight Estimate program used by NAVSEA utilizes an extra field in the weight record allowing the weight calculator to indicate whether the weight represented by that line resides in only the station it is in, is distributed over a number of stations about the record's center, or is spread over the entire ship [8]. This station method improves the quality of the weight distribution and reduces the amount of rework needed to yield a reasonable distribution. It should be noted that this improvement only masks the inherent flaw of this approach; it does not eliminate it.

Direct Distribution Methods

General Philosophy of Distribution

The approach that offers the most promise is distributing the individual weight records directly. The distribution of each weight record can then be summed to determine the weight distribution of the entire ship at a fairly high level of fidelity. This whole ship distribution can then be used to create any representation of the weight distribution, such as the Twenty Station Weight Distribution.

Mechanics of Distribution

The fundamental representative shape of direct distribution methods is the trapezoid. Representing a weight record as a trapezoid requires knowing the weight, the longitudinal extents, and the longitudinal center of the weight being represented. The computer program ShipWeight created by BAS engineering uses such an approach [9].

The fundamental problem with trapezoidal representations is that they are limited to weight records where the center resides in the middle one third of the length. Attempts to represent weight records whose center falls outside of the middle one third using the equations for trapezoidal representations result in part of the weight distributions being negative. Such an "inverted" trapezoid is portrayed in Figure 3.

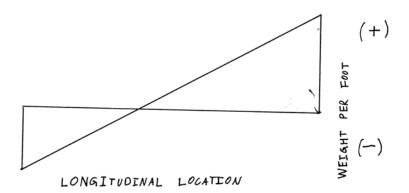


Figure 3: The result of attempting to use a trapezoid to represent a record whose center is outside the center 1/3.

Such a representation is clearly flawed as it subtracts weight from a location that the record should be adding weight to. This can be overcome by requiring that the user adjust the inputs so that the center falls in the middle one third of the extents. This is accomplished by dividing the offending records into acceptable pieces or combining them with other records to create acceptable records and can involve a large amount of user interaction with the tool. Another option is to simply adjust the extents outward until the center is in the middle one third. Unfortunately, this option often reduces the accuracy of the resultant distribution.

An Improved Direct Distribution Method

Before exploring refinements to the direct distribution method, it is informative to consider what is desired from a method and its resultant distribution. First, the method should require a minimum of inputs. A reasonable set of inputs consists of a description of the item, the weight, longitudinal center of gravity and the forward and after longitudinal extents. Obviously a high degree of fidelity in the resulting distribution is desired; particularly, the weight per foot evaluation interval should be far smaller than the station length. Finally a method of validating the resultant weight distribution should be inherent to the method.

The author briefly presented the mechanics of a method that satisfies these requirements in a previous paper [2]. This proposed method will be expounded upon herein. The full set of equations required for this method are given in Appendix B: Equations for the Direct Calculation of Weight Distributions.

The difficulty of failed trapezoids can be overcome by resorting to compound shapes. Five such shapes combining a triangle and a trapezoid are detailed in Appendix B: Equations for the Direct Calculation of Weight Distributions. These shapes extend the allowable center location to the middle 80% of the record's length. The boundaries of a weight record should be able to be chosen such that the center falls within this region. Figure 4 demonstrates a representative compound distribution. These compound shapes give an adequate representation of the weight distribution of records whose center falls outside the middle one third because they roughly mimic the shape of the actual objects that cause such centers. Generally such objects consist of a long part of relatively similar weight per foot and then an abrupt change to a heavier weight region. The compound shapes result in a similar weight distribution. Because the relative weight and length of the triangle and trapezoid in the distribution are chosen parametrically, this agreement is only approximate. However, it conceptually matches the shape of the object which is an improvement over the alternative of using a longer trapezoid to represent such an item.

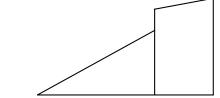


Figure 4: An Example of a Compound Weight Distribution

As with the trapezoidal direct distribution method, the weight per foot of each weight record is summed for each location to determine the weight per foot curve for the entire ship. This is done at a high level of fidelity, say at every half foot, and then the resultant curve is summed to determine the 20 station weight distribution.

Validating the Distribution

The mechanics of this distribution method can be verified by comparing hand calculations of sample inputs with the results from the distribution tool. Such a check of the programming is vital to ensuring accuracy; however, it only verifies one of the two potential sources of error. The larger and more insidious source of error is problems with the inputs.

For complex vessels, the large number of weight records renders user checking of each individual record impractical. It is important to be able to question the peaks and valleys of a detailed weight curve. If the intermediate steps in the weight distribution calculation method are stored in the program, it is a trivial matter to set up a search to query the tool and determine all of the records that contribute weight to a given location on the ship and how much weight they contribute. This allows the user to determine the cause of a weight spike; e.g., the anchor chain. Such validation by inspection is far more useful than plotting the centers of each record against a profile of the vessel, a validation method used in some commercial programs with weight distribution functionality.

Database Requirements for Direct Distribution Methods

It has already been discussed that the weight database must contain the weight, longitudinal center of gravity, and the extents for each weight record in order to calculate the weight distribution directly. There are a few other requirements that this method requires.

Weight records should not be composites of a small number of widely separated items. For example, generators from multiple auxiliary machinery rooms should not appear in the same weight record. This is not to say that sets of transverse stiffeners should not be combined into single records. This sort of combination will be represented well by the distribution method.

The other requirement for a weight database intended for use with a direct distribution method is that weight records must represent actual shipboard items. That is to say impact records must not be used. Impact records are sometimes entered in databases when the impact of a potential design decision is being considered. In this case rather than removing the original weight records and replacing them with new ones detailing the new configuration, one or two weight records are added which when combined with the original weight records results in the proper cumulative weight and center. The problem with impact records is that they rarely are located anywhere near the location of the actual weight they represent and often are outside the bounds of the ship. Such a shortcut is acceptable for a weight database intended to track the weight and center of the ship, but is not acceptable for direct record-by-record calculation of the weight distribution.

Summary Methods

Sometimes the requirements for direct calculation of a weight distribution discussed above are not met. Generally this is because the extents of each record are not included in the database or there is extensive use of impact records. These problems preclude the direct calculation of the weight distribution record-by-record. However, they do not render the general approach behind the direct method unworkable. The problems caused by impact lines and the lack of extents can be overcome through the use of summaries. In particular, summaries at the most detailed level under a given work breakdown structure (WBS) should be used. In the Expanded Ship Work Breakdown Structure used by the US Navy this is the 5 digit level.

The use of these summaries requires user interaction. Discontinuities in a given WBS group must be represented by using a different weight record for each piece of a WBS group. The decision about how many pieces to use to represent a given WBS group is a matter of judgment. It should be considered that most WBS groups should be represented by at least two entries as many WBS groups have a coffin shaped distribution. Neglecting the discontinuities of WBS groups has varying effects on the accuracy of the final distribution. Only experience with this method can show the user how much effect a given choice will make.

At early stages of a ship design, higher levels of abstraction in this summary method may be used to limit the effort required to prepare a distribution. For example, most weight groups can be roughly approximated by two summaries with extents based on the farthest forward and aft detail centers. This approximation will cause the length of the weight group to be underestimated. Such abstraction would raise the uncertainty of the distribution, but trading accuracy for speed of calculation may be preferable or necessary due to lack of detailed information during concept exploration or feasibility studies.

Accuracy of Weight Distributions

The more advanced methods of weight distribution make extensive use of numerical integration of irregular weight curves to determine the weight of each section. This inevitably leads to a mismatch between the total weight of the weight distribution and the total weight in the weight database. This error should be small by percentage and can be corrected by smearing the difference by percentage across the distribution.

The same mechanisms cause slight errors in the longitudinal center of gravity of the weight distribution. This can be corrected by using a triangular distribution of the difference in weight between the distribution and the database to adjust the center of the distribution. However, this method tends to disproportionately change the weight of one end of the ship. Therefore it is preferable to accept a slight error in the longitudinal center of the distribution so long as it is less than

1/1000 of the ship's length between perpendiculars. This value is the result of some sensitivity studies performed by the author and could be conservative.

Conclusion

Approximate methods can be useful in concept exploration and still have much to recommend them for early stage feasibility analysis. However, the improved direct method and its summary simplifications presented in this paper have attained a functionality that allows for far greater accuracy with a minimal increase in effort even at very early stages of design. Thus it is recommended that such approaches be used universally. Grouping methods such as the "Bucket" and station methods of weight distribution have been superseded by direct and summary methods of distribution. It is recommended that weight databases for new ship designs conform to the requirements for direct weight distribution as this would allow for rapid preparation of weight distributions on a regular basis through the ship design.

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Appendix A: Survey of Approximate Methods of Weight Distribution

Approximation per Comstock [4]

Given: W, the Total Weight to be Distributed, and d, the L.C.G. of the Weight from Amidships.

Desired, to Distribute 50% of this Weight as a Rectangle in the Middle .4 Length, and 50% in two Trapezoids so as to give the Required L.C.G.

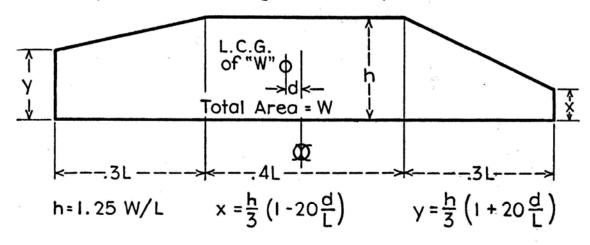


Fig. 35

This sort of representation is typically used to approximate the hull weight, "the steel, woodwork, fittings and outfit except anchors and cables, hull engineering except windlass and steering gear, any spread-out items of deadweight, such as passengers and crew, and designer's margin." Comstock goes on to note that, "The diagram must be proportioned that not only will the area be correct but also the LCG." The cargo should be, "distributed over the length of the cargo holds as trapezoids, and so on until the diagram includes all the weights in the loaded ship."

Approximation per Biles from Munro-Smith [3]

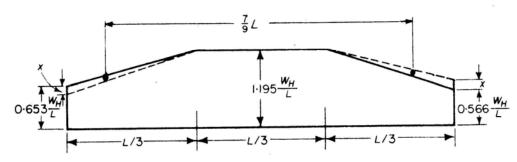
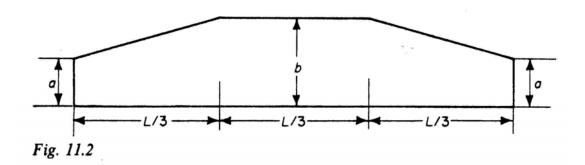


Fig. 11.1

This approximation is appropriate for passenger and cargo vessels. W_H is the weight of the hull in tons and L is the length of the ship in feet. The centroid of the diagram (LCG) is given is 0.0056L abaft midships. The centroid can be shifted by increasing the ordinate at one end of the ship and decreasing the other. The amount to add and subtract (x) is defined as:

$$x = \frac{54}{7} * \frac{W_H}{L} * \frac{Shift_of_Centroid}{L}$$

Approximation according to Prohaska from Munro-Smith [3]



The Table below gives the ordinates for the plot based on Prohaska's work. A method to move the LCG from midships is not provided.

Table 11.1

Type of ship	Prohaska's values			
Type of simp	$a\left \left(\frac{W_{\rm H}}{L}\right)\right $	$b \left \left(\frac{W_{H}}{L} \right) \right $		
Tankers	0.75	1.125		
Full cargo ships without erections	0.65	1.175		
Fine cargo ships without erections	0.60	1.20		
Full cargo ships with erections	0.55	1.225		
Fine cargo ships with erections	0.45	1.275		
Small passenger ships	0.40	1.30		
Large passenger ships	0.30	1.35		

Parabolic Approximation by Cole from Munro-Smith [3] and PNA [6]

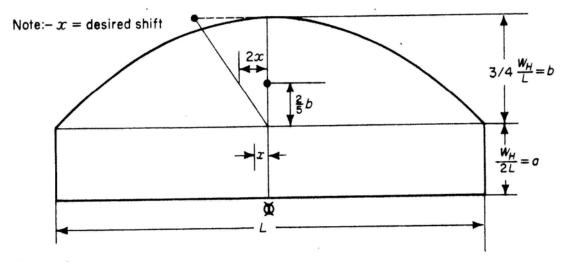


Fig. 11.3

This distribution is intended for vessels which don't have parallel middle body. The centroid of the distribution can be shifted by "swinging the parabola". This method is better depicted in the following figure from PNA [6].

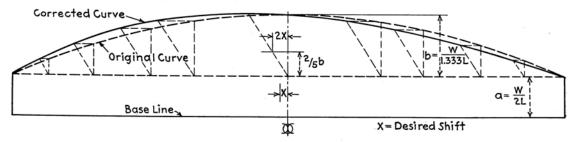


Fig. 4.—Approximation to Hull Weight Curve. No Parallel Middle Body

As PNA states, "Through the centroid of the parabola draw a line parallel to the base and in length equal to twice the shift desired (forward or aft). Through the point thus determined draw a line to the base of the parabola at its mid-length. The intersection of this line with the horizontal drawn from the intersection of the midship ordinate with the original parabolic contour determines the location of on point on the corrected curve. Parallel lines drawn at other ordinates, as indicated in Fig 4, determine the new curve."

Trapezoidal Approximation from PNA [6]

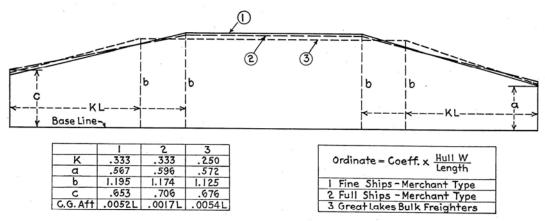


Fig. 5.—Approximation to Hull Weight Curve with Parallel Middle Body

This approximation is useful for ships with parallel midbody.

Approximate Hull Weight Curve based on Buoyancy Curve from Hughes [5]

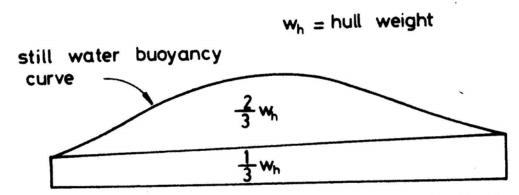


Figure 3.3 Approximation for hull weight distribution.

20 Station Distributions by ship type From Marine Vehicle Weight Engineering [7]

			Light	SHIP								
SHIP	WEIGHT					S	TATION					
		1	2	3	4	5	6	7	8	9	10	11
MHC	840	5	13	26	21	32	38	37	49	38	46	48
	%	0.65	1.54	3.11	2.50	3.85	4.49	4.43	5.78	4.54	5.48	5.74
DDG	6856	71	232	174	176	263	218	356	503	498	499	470
	%	1.03	3.39	2.54	2.57	3.83	3.19	5.20	7.33	7.26	7.28	6.85
LSD	11503	54	239	304	322	505	568	763	858	897	867	693
	%	0.47	2.08	2.64	2.80	4.39	4.94	6.63	7.46	7.80	7.54	6.03
$_{ m LPD}$	17495	60	368	351	477	884	984	1059	1188	1356	1385	1108
	%	0.34	2.11	2.01	2.72	5.05	5.63	6.05	6.79	7.75	7.92	6.34
$_{ m LHD}$	28027	377	828	722	650	926	1190	1135	1215	1447	1455	2589
	%	1.34	2.95	2.57	2.32	3.30	4.25	4.05	4.34	5.16	5.19	9.24
TAO	15917	34	231	446	371	554	497	648	766	991	914	791
	%	0.22	1.45	2.80	2.33	3.48	3.12	4.07	4.81	6.23	5.75	4.97
							STATION					
		12	13	14	15	16	17	18	19	20	21	22
	2.40		0.		00				40	00	10	0
MHC	840	55	87	71	63	57	38	27	43	29	13	2
DD.G	%	6.56	10.34	8.40	7.51	6.84	4.54	3.22	5.09	3.49	1.60	0.29
DDG	6856	457	568	405	346	302	486	205	183	207	217	20
T. 07	%	6.67	8.28	5.91	5.05	4.40	7.08	2.99	2.68	3.02	3.17	0.29
$_{ m LSD}$	11503	747	810	885	540	419	397	347	502	358	310	116
	%	6.50	7.04	7.69	4.70	3.64	3.45	3.02	4.36	3.12	2.69	1.01
$_{ m LPD}$	17495	1146	1336	1263	1157	626	588	526	454	590	454	135
	%	6.55	7.64	7.22	6.62	3.58	3.36	3.01	2.60	3.37	2.59	0.77
$_{ m LHD}$	28027	2703	1928	1871	1750	1273	1074	1106	1180	1200	974	436
	%	9.64	6.88	6.67	6.25	4.54	3.83	3.95	4.21	4.28	3.48	1.55
TAO	15917	634	655	708	1263	1801	1732	1135	657	569	460	60
	%	3.98	4.12	4.45	7.93	11.31	10.88	7.13	4.13	3.57	2.89	0.38

Table 8.17: Longitudinal weight distribution data for various ships' light ship conditions. Top row for each ship is the weight for that station in long tons, the bottom row is the percentage of light ship for that station.

			FULL	LOAD								
SHIP	WEIGHT						STATIO	N				
		1	2	3	4	5	6	7	8	9	10	11
MHC	889	5	15	26	22	32	42	42	56	46	56	58
	%	0.62	1.69	2.97	2.43	3.65	4.72	4.72	6.32	5.17	6.29	6.48
DDG	8797	71	232	196	238	386	309	453	576	626	593	625
	%	0.81	2.64	2.23	2.71	4.38	3.51	5.15	6.55	7.11	6.74	7.10
$_{ m LSD}$	16317	54	251	324	401	616	640	1024	998	1241	1290	1173
	%	0.33	1.54	1.99	2.46	3.77	3.92	6.27	6.12	7.61	7.90	7.19
$_{ m LPD}$	25476	66	395	379	574	987	1243	1444	1804	2066	1963	1935
	%	0.26	1.55	1.49	2.25	3.87	4.88	5.67	7.08	8.11	7.71	7.59
$_{ m LHD}$	40420	399	882	807	734	1333	1706	1772	2379	3012	2440	3634
	%	0.99	2.18	2.00	1.82	3.30	4.22	4.38	5.89	7.45	6.04	8.99
TAO	40461	34	335	1002	1361	1855	2098	1877	1926	2520	3177	2818
	%	0.08	0.83	2.48	3.36	4.58	5.19	4.64	4.76	6.23	7.85	6.97
						5	STATIO	N				
		12	13	14	15	16	17	18	19	20	21	22
MHC	889	57	76	74	65	59	39	29	43	30	15	2
	. %	6.38	8.51	8.35	7.29	6.61	4.44	3.21	4.82	3.33	1.74	0.27
DDG	8797	612	670	543	458	518	682	309	244	217	218	20
	%	6.96	7.62	6.18	5.21	5.89	7.75	3.52	2.78	2.46	2.48	0.23
LSD	16317	1296	1190	1096	1021	598	742	747	617	572	311	116
	%	7.94	7.30	6.72	6.25	3.66	4.55	4.58	3.78	3.50	1.91	0.71
$_{ m LPD}$	25476	1990	2147	1845	1638	1198	1129	726	559	715	530	144
	%	7.81	8.43	7.24	6.43	4.70	4.43	2.85	2.20	2.81	2.08	0.57
$_{ m LHD}$	40420	3631	3026	2763	2721	2224	1522	1323	1227	1415	1024	446
	%	8.98	7.49	6.84	6.73	5.50	3.76	3.27	3.03	3.50	2.53	1.10
TAO	40461	3154	3649	3188	2948	2195	2014	1659	1439	690	460	59
	%	7.80	9.02	7.88	7.29	5.42	4.98	4.10	3.56	1.71	1.14	0.15

Table 8.18: Longitudinal weight distribution data for various ships' full load conditions. Top row for each ship is the weight for that station in long tons, the bottom row is the percentage of full ship for that station.

Appendix B: Equations for the Direct Calculation of Weight Distributions

Note: This Appendix is largely reprinted from Reference [2].

This appendix is included in order to enable the reader to create a spreadsheet to calculate the weight distribution of a ship based on details or summaries. The format and functionality of the various tabs required to perform the calculations are explained and the equations are given. Reproducing the code of the Excel spreadsheet used by the author is not practical, but this presentation is intended to enable the reader to create a similar tool based on the same trapezoid and compound shape reasoning.

Nomenclature:

CG	Center of Gravity			
CG MAX	Maximum Center of Gravity in any detail in a group			
CG MIN	Minimum Center of Gravity in any detail in a group			
Weight	Weight of group			
TRAP	Trapezoid style representation			
TH	Height of Triangle part of a trapezoid in TRAP representation			
RH	Height of rectangular part of trapezoid in TRAP representation			
WZ	Weight of the trapezoidal part of a compound distribution			
WA	Weight of the triangular part of a compound distribution			
LZ	Length of the trapezoidal part of a compound distribution			
LA	Length of the triangular part of a compound distribution			
S	Height of the Triangle part of a compound distribution			
ZH	Height of triangular part of a trapezoid in a compound			
RHC	Height of the rectangular part of a trapezoid in a compound			
Weight Z	Weight of the Trapezoid part of a compound (internal check)			
Weight A	Weight of the Triangle part of a compound (internal check)			
Z slope	Slope of Trapezoid part of compound			
Z intercept	Y intercept for trapezoid part of compound			
	Point where triangle part of compound ends and trapezoid			
Break Point	part begins			
A slope	Slope of triangular part of compound			
A intercept	Y intercept of triangular part of compound			
Compound	Compound Style Representation			
LDIST	Longitudinal distance between CG min and CG max			

Entry sheet Headings:

Column	Heading			
D	Group			
Е	Description			
F	Weight			
G	CG			
Н	MAX CG			
	MIN CG			
J	LDIST			
K	FA (0 if LCG is in fwd half 1 if aft)			
L	Virtual Center (Local Center)			
М	Representation Type			
N	TH			
0	RH			
Р	weight check			
Q	Н			
R	TRAP Slope			
S	TRAP Intercept			
Т	Center Check			
U	Virtual Center %			
V	Compound Method			
W	WZ			
Х	WA			
Y	Virtual CZ			
Z	Local CZ			
AA	LZ			
AB	LA			
AC	S			
AD	ZH			
AE	RHC			
AF	Weight Z			
AG	Weight A			
AH	Z slope			
Al	Z intercept			
AJ	Break Point			
AK	A slope			
AL	A intercept			

Equations:

$$TH = WT \times \frac{Center - Ldist/2}{Ldist^2/3 - Ldist^2/4}$$

$$RH = \frac{WT}{LDist} - \frac{TH}{2}$$

$$weight_check = LDist \times \frac{2 \times RH + TH}{2}$$

$$H = TH + RH$$

If
$$FA = 1$$
:

$$\frac{H - RH}{MAXCG - MINCG}$$

Else:

$$\frac{H - RH}{MINCG - MAXCG}$$

TRAP Intercept =

If
$$FA = 1$$
:

$$H - TRAP _ slope \times MAXCG$$

Else:

$$RH - TRAP _ slope \times MAXCG$$

Center Check =

If
$$FA = 1$$

$$\frac{TH \times LDist^{2}/_{3} + RH \times LDist^{2}/_{2}}{Weight_check} + MINCG$$

Else:

$$\frac{TH \times \frac{LDist^{2}}{6} + RH \times \frac{LDist^{2}}{2}}{Weight_check} + MINCG$$

Virtual Center % = Virtual Center / LDist

If
$$FA = 1$$
:

$$Virtual_Center \times weight - \frac{\frac{2}{3} \times LA \times WA}{WZ}$$

Else:

$$Virtual_Center \times weight - \frac{\frac{1}{3} \times LA \times WA}{WZ}$$

If
$$FA = 1$$

Else

Virtual CZ

$$LZ = LDist - LA$$

$$WA = Weight - WZ$$

$$S = \frac{2 \times WA}{IA}$$

$$ZH =$$

If
$$FA = 1$$

$$\frac{WZ \times Local _CZ - LZ/2}{LZ^2/3 - LZ^2/4}$$

Else

$$\frac{WZ \times Local _CZ - LZ/2}{LZ^2/6 - LZ^2/4}$$

$$RHC = \frac{WZ}{LZ} - \frac{ZH}{2}$$

$$Weight _Z = \frac{LZ \times RCH + ZH}{2}$$

Weight
$$A = S \times LA/LZ$$

Z slope =

If FA = 1

$$ZH/LZ$$
Else
 $-ZH/LZ$

Z intercept =

If FA = 1

 $RHC - Z _ slope \times (MINCG + LA)$
Else
 $RHC - Z _ slope \times (MINCG + LZ)$

Break Point =

If FA = 1

MINCG + LA

Else
MINCG + LZ

A Slope =

If FA = 1

 S/LA
Else
 $-S/LA$

A intercept
If FA = 1

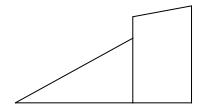
 $S - A _ slope \times (LFWD + LA)$
Else
 $S - A _ slope \times (LAFT - LA)$

Compound Shape Calculations:

Compound Method	Center Locations Covered	WZ	LA
1	66 - 73 %	0.8 x Wt	LDist / 2
2	73 - 79 %	0.93 x Wt	LDist / 2
3	79 - 84 %	0.89 x Wt	2 x LDist / 3
4	84 - 87.5 %	0.9 x Wt	3 x LDist / 4
5	87.5 - 91.25 %	0.99 x Wt	3 x LDist / 4

Calculation Tool:

The equations presented in this appendix enable creation of a spreadsheet that takes weight, extent and center inputs and calculates a representative weight distribution for each group. If the center of the group falls in the middle third of the group's length, the representation is a trapezoid. If the center is outside the middle third, a compound consisting of a triangle and a trapezoid represents the weight distribution. This calculator incorporates five compound combinations to represent shapes where the center is between 66 and 91.25 % of the length of the group from either extent. (The five compound shapes, applicable range, trapezoidal shape weight and the relative length of the triangular part of the compound appear in a table in the Equations section of this appendix.) The weight and length parameters were chosen to provide the greatest coverage.



A Compound Weight Distribution

After the distribution is calculated, the equation of the line along the top of the shape is calculated.

The inputs and calculations described above all take place on a sheet labeled "Entry". Sheets labeled "A", "Z", "TRAP" calculate the weight per foot from the three equations of the lines: the "Sort" sheet selects the correct weight for each location. This "Sort" sheet sums the total weight per foot at each location and then transfers this data to a sheet that stores the weight distribution. The weight is then calculated by Simpson's Rule and the center is calculated directly by summing the moments in order to verify the weight distribution.

Segmenting and integrating the ship's weight in this manner is accurate; however, the use of Simpson's Rule introduces slight integration errors. The difference in total ship weight is generally on the order of less than a half of a percent; this can be improved by increasing the number of samples taken along the axis.